Particle Physics

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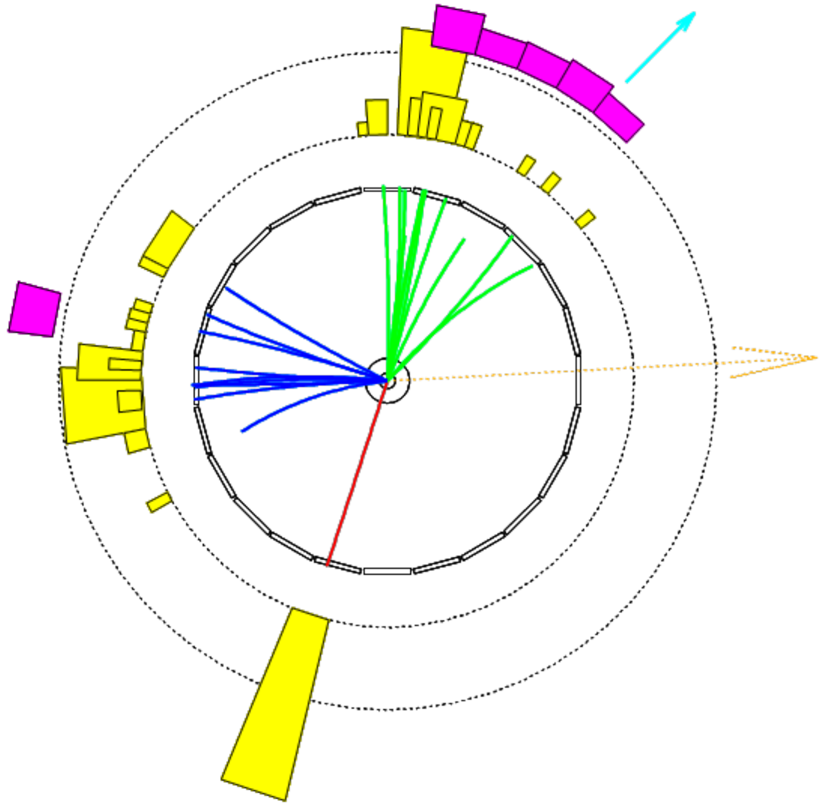


Handout 13 : Electroweak Unification and the W and Z Bosons

Boson Polarization States

★ In this handout we are going to consider the decays of W and Z bosons, for this we will need to consider the polarization. Here simply quote results although the justification is given in Appendices I and II

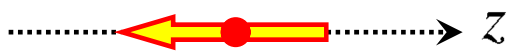
★ A real (i.e. not virtual) massless spin-1 boson can exist in two transverse polarization states, a massive spin-1 boson also can be longitudinally polarized

★ Boson wave-functions are written in terms of the polarization four-vector ε^{μ}

$$B^{\mu} = \varepsilon^{\mu} e^{-ip.x} = \varepsilon^{\mu} e^{i(\vec{p}.\vec{x}-Et)}$$

★ For a spin-1 boson travelling along the z-axis, the polarization four vectors are:

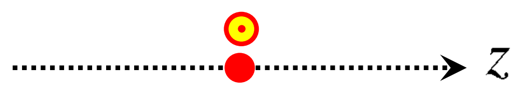
$$\varepsilon^{\mu}_{-} = \frac{1}{\sqrt{2}}(0,1,-i,0);$$



$$S_z = -1$$

transverse

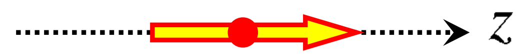
$$\varepsilon_L = \frac{1}{m}(p_z,0,0,E)$$



$$S_z = 0$$

longitudinal

$$\varepsilon^{\mu}_{+} = -\frac{1}{\sqrt{2}}(0,1,i,0)$$



$$S_z = +1$$

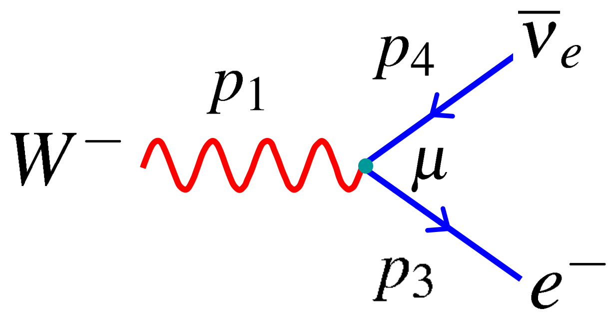
transverse

Longitudinal polarization isn't present for on-shell massless particles, the photon can exist in two helicity states $h = \pm 1$ (LH and RH circularly polarized light)

W-Boson Decay

★ To calculate the W-Boson decay rate first consider $W^- \rightarrow e^- \overline{\nu}_e$

★ Want matrix element for :



Incoming W-boson : $\varepsilon_\mu(p_1)$

Out-going electron : $\overline{u}(p_3)$

Out-going $\overline{\nu}_e$: $v(p_4)$

Vertex factor : $-i\frac{g_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$

$$-iM_{fi} = \varepsilon_\mu(p_1).\overline{u}(p_3).-i\frac{g_W}{\sqrt{2}}\gamma^\mu\tfrac{1}{2}(1-\gamma^5).v(p_4)$$

Note, no propagator

$$\Rightarrow \quad M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)\overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)$$

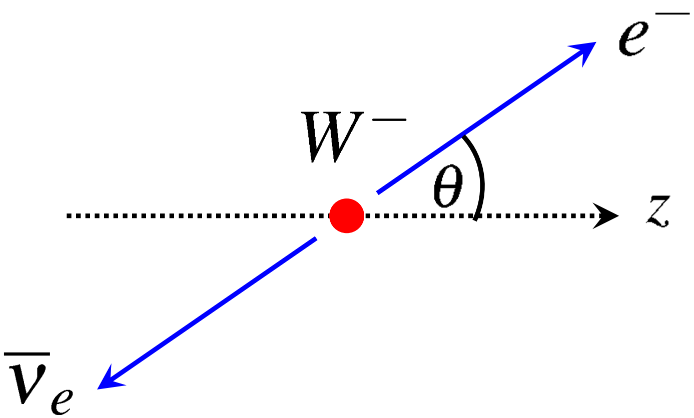
★ This can be written in terms of the four-vector scalar product of the W-boson polarization $\varepsilon_\mu(p_1)$ and the weak charged current j^μ

$$M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1).j^\mu \quad \text{with} \quad j^\mu = \overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)$$

W-Decay : The Lepton Current

★ **First consider the lepton current** $j^\mu = \overline{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)v(p_4)$

★ **Work in Centre-of-Mass frame**



$$p_1 = (m_W, 0, 0, 0);$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta)$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

with $E = \dfrac{m_W}{2}$

★ **In the ultra-relativistic limit only LH particles and RH anti-particles participate in the weak interaction so**

$$j^\mu = \overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

Note: $\frac{1}{2}(1-\gamma^5)v(p_4) = v_\uparrow(p_4)$ $\qquad$ $\overline{u}(p_3)\gamma^\mu v_\uparrow(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$

Chiral projection operator, e.g. see p.131 or p.294

"Helicity conservation", e.g. see p.133 or p.295

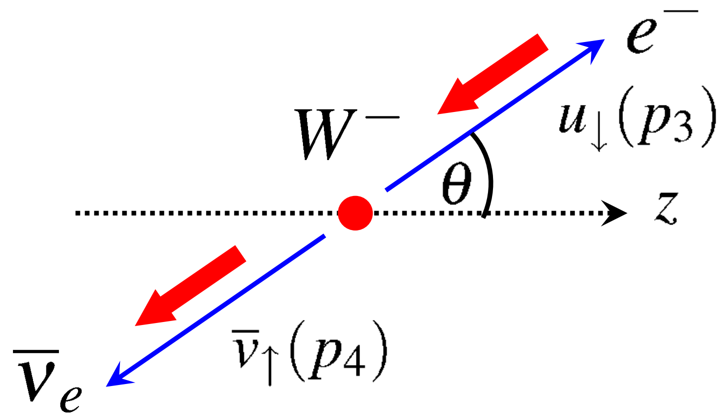
- We have already calculated the current

$$j^\mu = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

when considering $e^+e^- \to \mu^+\mu^-$

- From page 128 we have for $\mu_L^-\mu_R^+$

$$j_{\uparrow\downarrow}^\mu = 2E(0, -\cos\theta, -i, \sin\theta)$$

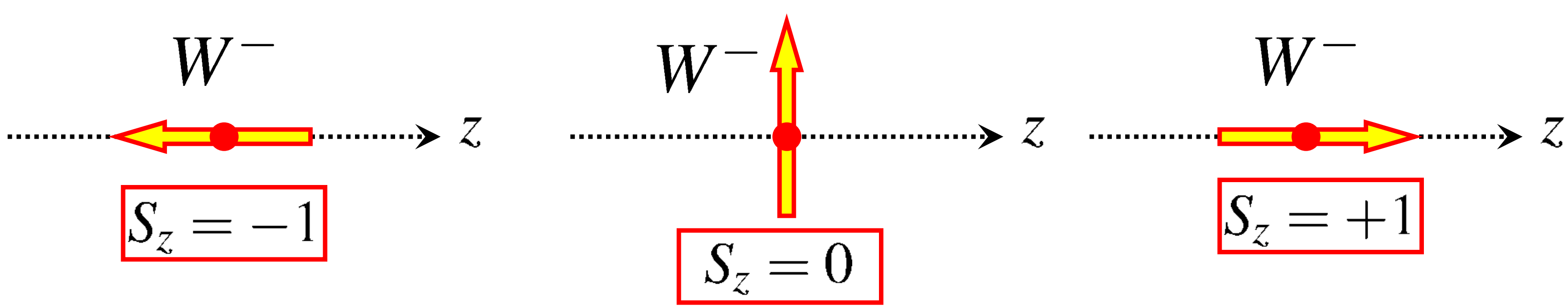


- For the charged current weak Interaction we only have to consider this single combination of helicities

$$j^\mu = \overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

and the three possible W-Boson polarization states:

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = \frac{1}{m}(p_z,0,0,E) \quad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$



★ **For a W-boson at rest these become:**

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = (0,0,0,1) \quad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ **Can now calculate the matrix element for the different polarization states**

$$M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)j^\mu \quad \textbf{with} \quad j^\mu = 2\frac{m_W}{2}(0,-\cos\theta,-i,\sin\theta)$$

Decay at rest : $E_e = E_\nu = m_W/2$

★ **giving**

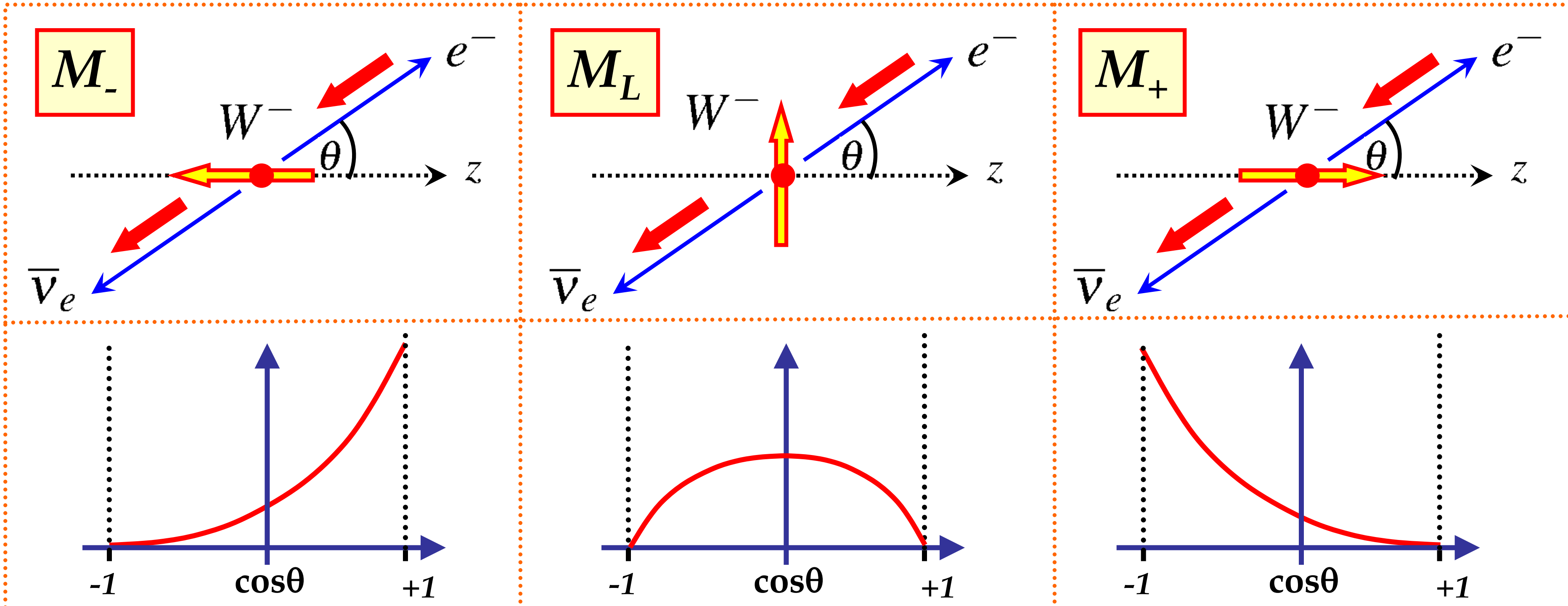
ε_- $$M_- = \frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,-i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \frac{1}{2}g_W m_W(1+\cos\theta)$$

ε_L $$M_L = \frac{g_W}{\sqrt{2}}(0,0,0,1).m_W(0,-\cos\theta,-i,\sin\theta) = -\frac{1}{\sqrt{2}}g_W m_W\sin\theta$$

ε_+ $$M_+ = -\frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \frac{1}{2}g_W m_W(1-\cos\theta)$$

$$|M_-|^2 = g_W^2 m_W^2 \tfrac{1}{4}(1+\cos\theta)^2$$
$$|M_L|^2 = g_W^2 m_W^2 \tfrac{1}{2}\sin^2\theta$$
$$|M_+|^2 = g_W^2 m_W^2 \tfrac{1}{4}(1-\cos\theta)^2$$

★ The angular distributions can be understood in terms of the spin of the particles



$\frac{1}{4}(1+\cos\theta)^2$ $\frac{1}{2}\sin^2\theta$ $\frac{1}{4}(1-\cos\theta)^2$

★ The differential decay rate (see page 27) can be found using:

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

where p* is the C.o.M momentum of the final state particles, here $p^* = \frac{m_W}{2}$

★ **Hence for the three different polarisations we obtain:**

$$\frac{d\Gamma_-}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1+\cos\theta)^2 \qquad \frac{d\Gamma_L}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{2}\sin^2\theta \qquad \frac{d\Gamma_+}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1-\cos\theta)^2$$

★ **Integrating over all angles using**

$$\int \tfrac{1}{4}(1\pm\cos\theta)^2 d\phi\, d\cos\theta = \int \tfrac{1}{2}\sin^2\theta\, d\phi\, d\cos\theta = \frac{4\pi}{3}$$

★ **Gives**

$$\boxed{\Gamma_- = \Gamma_L = \Gamma_+ = \frac{g_W^2 m_W}{48\pi}}$$

★ **The total W-decay rate is independent of polarization; this has to be the case as the decay rate cannot depend on the arbitrary definition of the z-axis**

★ **For a sample of unpolarized W boson each polarization state is equally likely, for the average matrix element sum over all possible matrix elements and average over the three initial polarization states**

$$\begin{aligned}\langle |M_{fi}|^2 \rangle &= \tfrac{1}{3}(|M_-|^2 + |M_L|^2 + |M_+|^2) \\ &= \tfrac{1}{3} g_W^2 m_W^2 \left[\tfrac{1}{4}(1+\cos\theta)^2 + \tfrac{1}{2}\sin^2\theta + \tfrac{1}{4}(1-\cos\theta)^2\right] \\ &= \tfrac{1}{3} g_W^2 m_W^2\end{aligned}$$

★ **For a sample of unpolarized W-bosons, the decay is isotropic (as expected)**

★ **For this isotropic decay**

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle \;\Rightarrow\; \Gamma = \frac{4\pi|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle$$

$$\Rightarrow \quad \boxed{\Gamma(W^- \to e^-\overline{\nu}) = \frac{g_W^2 m_W}{48\pi}}$$

★ **The calculation for the other decay modes (neglecting final state particle masses) is same. For quarks need to account for colour and CKM matrix. No decays to top – the top mass (175 GeV) is greater than the W-boson mass (80 GeV)**

$$\begin{array}{lll|l|ll|l} W^- \to e^-\overline{\nu}_e & & W^- \to d\overline{u} & \times 3|V_{ud}|^2 & & W^- \to d\overline{c} & \times 3|V_{cd}|^2 \\ W^- \to \mu^-\overline{\nu}_\mu & & W^- \to s\overline{u} & \times 3|V_{us}|^2 & & W^- \to s\overline{c} & \times 3|V_{cs}|^2 \\ W^- \to \tau^-\overline{\nu}_\tau & & W^- \to b\overline{u} & \times 3|V_{ub}|^2 & & W^- \to b\overline{c} & \times 3|V_{cb}|^2 \end{array}$$

★ **Unitarity of CKM matrix gives, e.g.** $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

★ **Hence** $BR(\mathrm{W} \to qq') = 6BR(\mathrm{W} \to \mathrm{e}\nu)$

and thus the total decay rate :

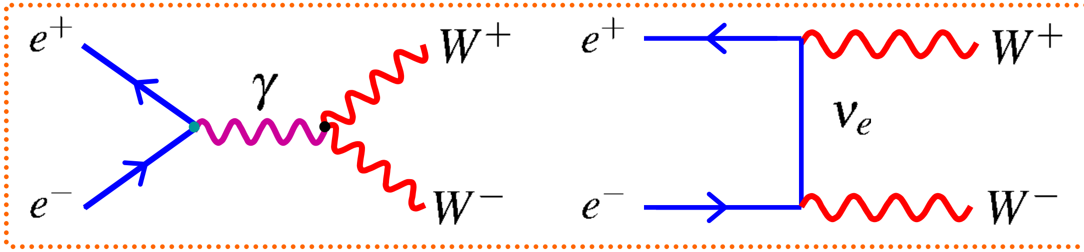
$$\boxed{\Gamma_W = 9\Gamma_{W\to e\nu} = \frac{3g_W^2 m_W}{16\pi} = 2.07\,\mathrm{GeV}}$$

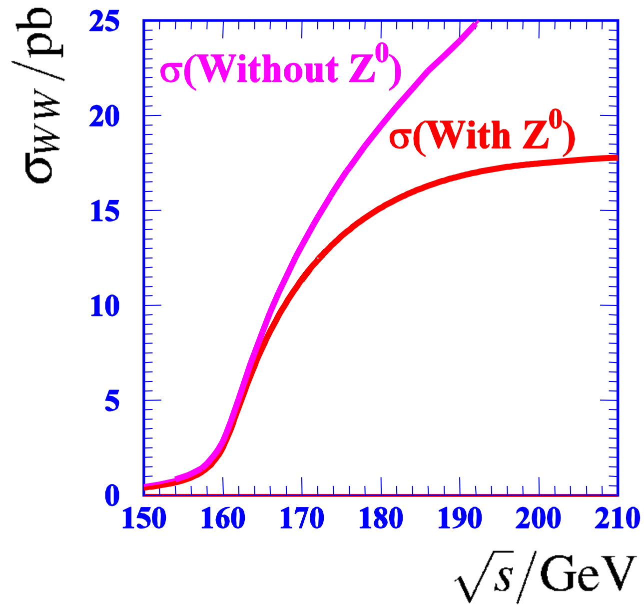
Experiment: 2.14±0.04 GeV
(our calculation neglected a 3% QCD correction to decays to quarks)

From W to Z

★ The W± bosons carry the EM charge - suggestive Weak are EM forces are related.

★ W bosons can be produced in e⁺e⁻ annihilation

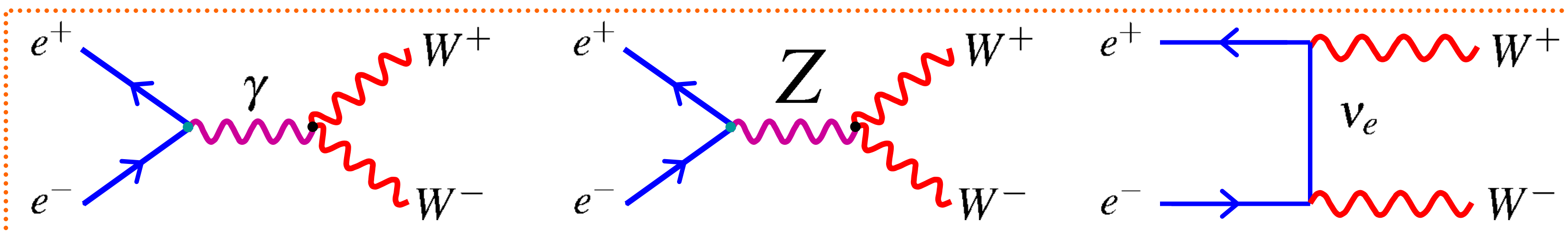




★ With just these two diagrams there is a problem: the cross section increases with C.o.M energy and at some point violates **QM unitarity**

UNITARITY VIOLATION: when QM calculation gives larger flux of W bosons than incoming flux of electrons/positrons

★ Problem can be "fixed" by introducing a new boson, the Z. The new diagram interferes negatively with the above two diagrams fixing the unitarity problem



$$|M_{\gamma WW} + M_{ZWW} + M_{\nu WW}|^2 < |M_{\gamma WW} + M_{\nu WW}|^2$$

★ Only works if **Z, γ, W** couplings are related: need **ELECTROWEAK UNIFICATION**

SU(2)$_L$: The Weak Interaction

★ The Weak Interaction arises from **SU(2)** local phase transformations

$$\psi \rightarrow \psi' = \psi e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e the three Pauli spin matrices ⇒ **3 Gauge Bosons** $W_1^\mu, \; W_2^\mu, \; W_3^\mu$

★ The wave-functions have two components which, in analogy with isospin, are represented by **"weak isospin"**

★ The fermions are placed in isospin doublets and the local phase transformation corresponds to

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}' = e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

★ Weak Interaction only couples to **LH particles**/**RH anti-particles**, hence only place **LH particles**/**RH anti-particles** in weak isospin doublets: $I_W = \frac{1}{2}$
RH particles/**LH anti-particles** placed in weak isospin singlets: $I_W = 0$

Weak Isospin

$I_W = \frac{1}{2}$ $\quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$ ← $I_W^3 = +\frac{1}{2}$ (upper), $I_W^3 = -\frac{1}{2}$ (lower)

$I_W = 0$ $\quad (\nu_e)_R, \; (e^-)_R, \; \ldots (u)_R, \; (d)_R, \ldots$

Note: RH/LH refer to chiral states

★ For simplicity only consider $\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$

•The gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) – [note: here include interaction strength in current]

$$j_\mu^1 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_1 \chi_L \qquad j_\mu^2 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_2 \chi_L \qquad j_\mu^3 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_3 \chi_L$$

★The charged current W^+/W^- interaction enters as a linear combinations of W_1, W_2

$$W^{\pm\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu)$$

★ The $W^\pm$ interaction terms

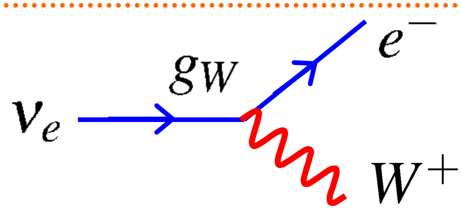
$$j_\pm^\mu = \frac{g_W}{\sqrt{2}}(j_1^\mu \mp i j_2^\mu) = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \tfrac{1}{2}(\sigma_1 \mp i\sigma_2)\chi_L$$

★ Express in terms of the weak isospin ladder operators $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$j_\pm^\mu = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_\mp \chi_L$$

} Origin of $\frac{1}{\sqrt{2}}$ in Weak CC

W^+



corresponds to $j_+^\mu = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_- \chi_L$

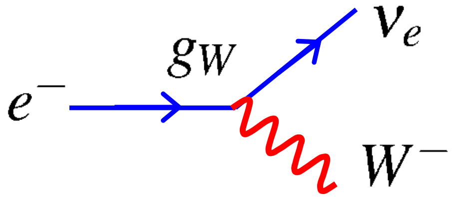
Bars indicates adjoint spinors

which can be understood in terms of the weak isospin doublet

$$j_+^\mu = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_- \chi_L = \frac{g_W}{\sqrt{2}} (\overline{\nu}_L, \overline{e}_L)\gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{g_W}{\sqrt{2}} \overline{e}_L \gamma^\mu \nu_L = \boxed{\frac{g_W}{\sqrt{2}} \overline{e} \gamma^\mu \tfrac{1}{2}(1-\gamma^5)\nu}$$

★ **Similarly**

W⁻



corresponds to

$$j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_+\chi_L$$

$$j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_+\chi_L = \frac{g_W}{\sqrt{2}}(\overline{\nu}_L,\overline{\mathrm{e}}_L)\gamma^\mu\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L = \frac{g_W}{\sqrt{2}}\overline{\nu}_L\gamma^\mu \mathrm{e}_L = \frac{g_W}{\sqrt{2}}\overline{\nu}\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\mathrm{e}$$

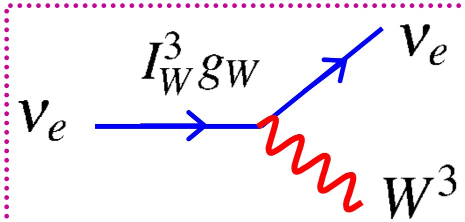
★ **However have an additional interaction due to W^3**

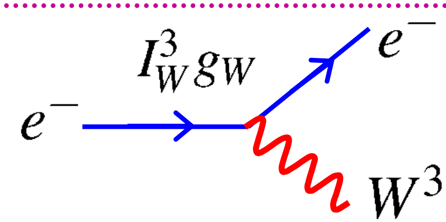
$$j^\mu_3 = g_W\overline{\chi}_L\gamma^\mu\tfrac{1}{2}\sigma_3\chi_L$$

expanding this:

$$I^3_W = \pm\tfrac{1}{2}$$

$$j^\mu_3 = g_W\frac{1}{2}(\overline{\nu}_L,\overline{\mathrm{e}}_L)\gamma^\mu\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L = g_W\tfrac{1}{2}\overline{\nu}_L\gamma^\mu\nu_L - g_W\tfrac{1}{2}\overline{\mathrm{e}}_L\gamma^\mu \mathrm{e}_L$$





⇒ **NEUTRAL CURRENT INTERACTIONS !**

Electroweak Unification

★Tempting to identify the W^3 as the Z

★However this is not the case, have two physical neutral spin-1 gauge bosons, γ, Z and the W^3 is a mixture of the two,

★ Equivalently write the photon and Z in terms of the W^3 and a new neutral spin-1 boson the B

★The physical bosons (the Z and photon field, A) are:

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

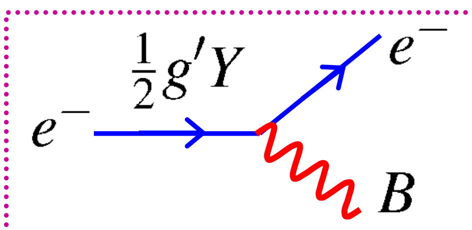
θ_W is the weak mixing angle

★The new boson is associated with a new gauge symmetry similar to that of electromagnetism : $U(1)_Y$

★The charge of this symmetry is called **WEAK HYPERCHARGE** Y

$$Y = 2Q - 2I^3_W$$

Q is the EM charge of a particle
I^3_W is the third comp. of weak isospin



•By convention the coupling to the B_μ is $\frac{1}{2}g'Y$

$e_L: \; Y = 2(-1) - 2(-\frac{1}{2}) = -1 \qquad \nu_L: \; Y = -1$

$e_R: \; Y = 2(-1) - 2(0) = -2 \qquad \nu_R: \; Y = 0$

(this identification of hypercharge in terms of Q and I_3 makes all of the following work out)

★ For this to work the coupling constants of the **W^3, B**, and **photon** must be related

e.g. consider contributions involving the neutral interactions of electrons:

γ $$j_\mu^{em} = e\overline{\psi}Q_e\gamma_\mu\psi = e\bar{e}_L Q_e\gamma_\mu e_L + e\bar{e}_R Q_e\gamma_\mu e_R$$

W^3 $$j_\mu^{W^3} = -\frac{g_W}{2}\bar{e}_L\gamma_\mu e_L$$

B $$j_\mu^Y = \frac{g'}{2}\overline{\psi}Y_e\gamma_\mu\psi = \frac{g'}{2}\bar{e}_L Y_{e_L}\gamma_\mu e_L + \frac{g'}{2}\bar{e}_R Y_{e_R}\gamma_\mu e_R$$

★ The relation $A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W$ is equivalent to requiring

$$j_\mu^{em} = j_\mu^Y\cos\theta_W + j_\mu^{W^3}\sin\theta_W$$

•Writing this in full:

$$e\bar{e}_L Q_e\gamma_\mu e_L + e\bar{e}_R Q_e\gamma_\mu e_R = \frac{1}{2}g'\cos\theta_W[\bar{e}_L Y_{e_L}\gamma_\mu e_L + \bar{e}_R Y_{e_R}\gamma_\mu e_R] - \frac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$$

$$-e\bar{e}_L\gamma_\mu e_L - e\bar{e}_R\gamma_\mu e_R = \frac{1}{2}g'\cos\theta_W[-\bar{e}_L\gamma_\mu e_L - 2\bar{e}_R\gamma_\mu e_R] - \frac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$$

which works if: $$e = g_W\sin\theta_W = g'\cos\theta_W$$ (i.e. equate coefficients of L and R terms)

★ Couplings of electromagnetism, the weak interaction and the interaction of the $U(1)_Y$ symmetry are therefore related.

The Z Boson

★ **In this model we can now derive the couplings of the Z Boson**

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

I^3_W — for the electron $I^3_W = -\frac{1}{2}$

$$j^Z_\mu = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L Y_{e_L} \gamma_\mu e_L + \bar{e}_R Y_{e_R} \gamma_\mu e_R] - \tfrac{1}{2} g_W \cos\theta_W [\bar{e}_L \gamma_\mu e_L]$$

• **Writing this in terms of weak isospin and charge:**

$$j^Z_\mu = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L (2Q - 2I^3_W) \gamma_\mu e_L + \bar{e}_R (2Q) \gamma_\mu e_R] + I^3_W g_W \cos\theta_W [\bar{e}_L \gamma_\mu e_L]$$

For RH chiral states $I_3=0$

• **Gathering up the terms for LH and RH chiral states:**

$$j^Z_\mu = [g' I^3_W \sin\theta_W - g' Q \sin\theta_W + g_W I^3_W \cos\theta_W] \bar{e}_L \gamma_\mu e_L - [g' Q \sin\theta_W] \bar{e}_R \gamma_\mu e_R$$

• **Using:** $e = g_W \sin\theta_W = g' \cos\theta_W$ **gives**

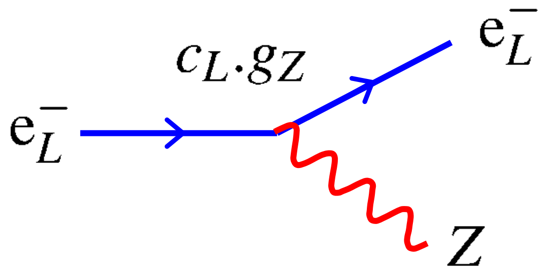
$$j^Z_\mu = \left[g' \frac{(I^3_W - Q\sin^2\theta_W)}{\sin\theta_W} \right] \bar{e}_L \gamma_\mu e_L - \left[g' \frac{Q\sin^2\theta_W}{\sin\theta_W} \right] \bar{e}_R \gamma_\mu e_R$$

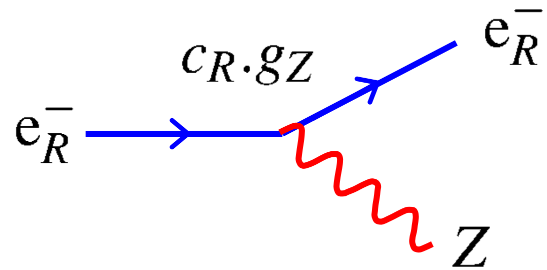
$$j^Z_\mu = g_Z (I^3_W - Q\sin^2\theta_W) [\bar{e}_L \gamma_\mu e_L] - g_Z Q \sin^2\theta_W [\bar{e}_R \gamma_\mu e_R]$$

with $e = g_Z \cos\theta_W \sin\theta_W$ **i.e.** $g_Z = \dfrac{g_W}{\cos\theta_W}$

★ **Unlike for the Charged Current Weak interaction (W) the Z Boson couples to both LH and RH chiral components, but not equally…**

$$j^Z_\mu = g_Z(I^3_W - Q\sin^2\theta_W)[\bar{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W[\bar{e}_R\gamma_\mu e_R]$$
$$= g_Z c_L[\bar{e}_L\gamma_\mu e_L] + g_Z c_R[\bar{e}_R\gamma_\mu e_R]$$





$$c_L = I^3_W - Q\sin^2\theta_W \qquad c_R = -Q\sin^2\theta_W$$

W^3 part of Z couples only to LH components (like $W^\pm$)

B_μ part of Z couples equally to LH and RH components

★ **Use projection operators to obtain vector and axial vector couplings**

$$\bar{u}_L\gamma_\mu u_L = \bar{u}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)u \qquad \bar{u}_R\gamma_\mu u_R = \bar{u}\gamma_\mu \tfrac{1}{2}(1+\gamma_5)u$$

$$j^Z_\mu = g_Z\bar{u}\gamma_\mu\left[c_L\tfrac{1}{2}(1-\gamma_5) + c_R\tfrac{1}{2}(1+\gamma_5)\right]u$$

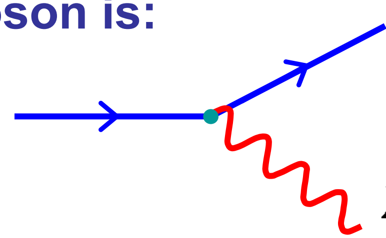
$$j_\mu^Z = \frac{g_Z}{2}\overline{u}\gamma_\mu\left[(c_L + c_R) + (c_R - c_L)\gamma_5)\right]u$$

★ **Which in terms of V and A components gives:** $$j_\mu^Z = \frac{g_Z}{2}\overline{u}\gamma_\mu\left[c_V - c_A\gamma_5\right]u$$

with $$c_V = c_L + c_R = I_W^3 - 2Q\sin^2\theta_W \qquad c_A = c_L - c_R = I_W^3$$

★ **Hence the vertex factor for the Z boson is:**

$$-ig_Z\tfrac{1}{2}\gamma_\mu\left[c_V - c_A\gamma_5\right]$$

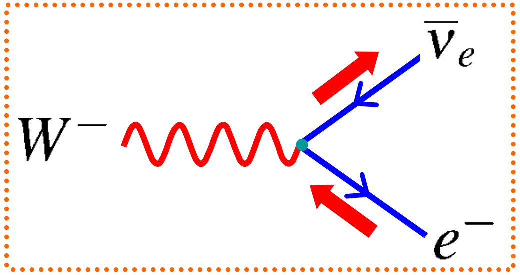


★ **Using the experimentally determined value of the weak mixing angle:** $\sin^2\theta_W \approx 0.23$

Fermion	Q	I_W^3 L	R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	0	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	0	-0.42	0.08	-0.35	$-\frac{1}{2}$

Z Boson Decay : Γ_Z

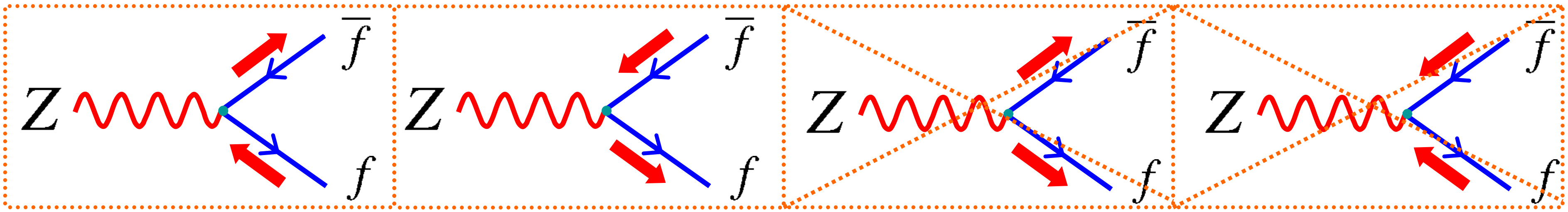
★ In W-boson decay only had to consider one helicity combination of (assuming we can neglect final state masses: helicity states = chiral states)



W-boson couples:
to LH particles
and RH anti-particles

★ But Z-boson couples to LH and RH particles (with different strengths)

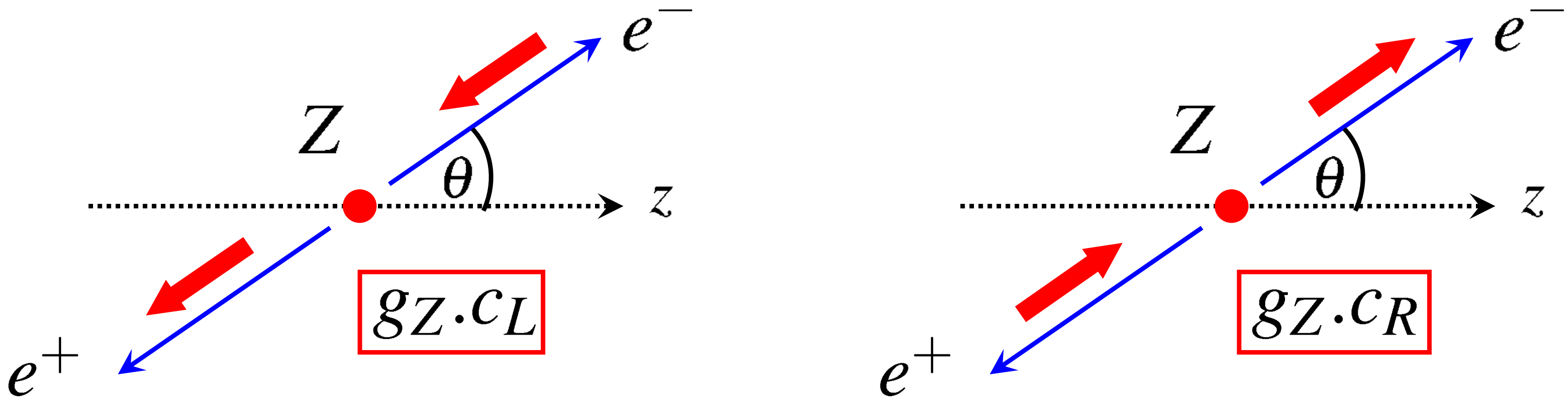
★ Need to consider only two helicity (or more correctly chiral) combinations:



This can be seen by considering either of the combinations which give zero

e.g. $$\overline{u}_R\gamma^\mu(c_V+c_A\gamma_5)v_R = u^\dagger\tfrac{1}{2}(1+\gamma^5)\gamma^0\gamma^\mu(c_V+c_A\gamma^5)\tfrac{1}{2}(1-\gamma^5)v$$
$$= \tfrac{1}{4}u^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)(c_V+c_A\gamma^5)v$$
$$= \tfrac{1}{4}\overline{u}\gamma^\mu(1+\gamma^5)(1-\gamma^5)(c_V+c_A\gamma_5)v = 0$$

★ **In terms of left and right-handed combinations need to calculate:**



★ **For unpolarized Z bosons:** **(Question 26)**

$$\langle |M_{fi}|^2 \rangle = \frac{1}{3}[2c_L^2 g_Z^2 m_Z^2 + 2c_R^2 g_Z^2 m_Z^2] = \frac{2}{3} g_Z^2 m_Z^2 (c_L^2 + c_R^2)$$

average over polarization

★ **Using** $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ **and** $\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$

$$\Rightarrow \quad \Gamma(Z \to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

Z Branching Ratios

(Question 27)

★ (Neglecting fermion masses) obtain the same expression for the other decays

$$\Gamma(Z \to f\overline{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

•Using values for c_V and c_A on page 471 obtain:

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \approx 3.5\%$$

$$Br(Z \to \nu_1\overline{\nu}_1) = Br(Z \to \nu_2\overline{\nu}_2) = Br(Z \to \nu_3\overline{\nu}_3) \approx 6.9\%$$

$$Br(Z \to d\overline{d}) = Br(Z \to s\overline{s}) = Br(Z \to b\overline{b}) \approx 15\%$$

$$Br(Z \to u\overline{u}) = Br(Z \to c\overline{c}) \approx 12\%$$

•The Z Boson therefore predominantly decays to hadrons

$$Br(Z \to \text{hadrons}) \approx 69\%$$

Mainly due to factor 3 from colour

•Also predict total decay rate (total width)

$$\Gamma_Z = \sum_i \Gamma_i = 2.5\,\text{GeV}$$

Experiment:

$$\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}$$

Summary

★ The Standard Model interactions are mediated by spin-1 gauge bosons

★ The form of the interactions are completely specified by the assuming an underlying local phase transformation ➡ GAUGE INVARIANCE

$U(1)_{em}$	⇒	QED
$SU(2)_L$	⇒	Charged Current Weak Interaction + W^3
$SU(3)_{col}$	⇒	QCD

★ In order to "unify" the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry : U(1) hypercharge

$U(1)_Y$ ⇒ B_μ

★ The physical Z boson and the photon are mixtures of the neutral W boson and B determined by the Weak Mixing angle

$$\sin^2\theta_W \approx 0.23$$

★ Have we really unified the EM and Weak interactions ? Well not really…

- Started with two independent theories with coupling constants g_W, e
- Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model θ_W
- Interactions not unified from any higher theoretical principle… but it works!

Appendix I : Photon Polarization

(Non-examinable)

- For a free photon (i.e. $j^\mu = 0$) equation (A7) becomes

$$\Box^2 A^\mu = 0 \qquad \text{(B1)}$$

(note have chosen a gauge where the Lorentz condition is satisfied)

★ Equation (A8) has solutions (i.e. the wave-function for a free photon)

$$A^\mu = \varepsilon^\mu(q)e^{-iq.x}$$

where ε^μ is the four-component polarization vector and q is the photon four-momentum

$$0 = \Box^2 A^\mu = -q^2 \varepsilon^\mu e^{-iq.x}$$

$$\Rightarrow \quad q^2 = 0$$

★ Hence equation (B1) describes a massless particle.

★ But the solution has four components – might ask how it can describe a spin-1 particle which has three polarization states?

★ But for (A8) to hold we must satisfy the Lorentz condition:

$$0 = \partial_\mu A^\mu = \partial_\mu(\varepsilon^\mu e^{-iq.x}) = \varepsilon^\mu \partial_\nu(e^{-iq.x}) = -i\varepsilon^\mu q_\mu e^{-iq.x}$$

Hence the Lorentz condition gives $q_\mu \varepsilon^\mu = 0$ (B2)

i.e. only 3 independent components.

★ However, in addition to the Lorentz condition still have the addional gauge freedom of $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda$ with (A8) $\Box^2 \Lambda = 0$

• Choosing $\Lambda = iae^{-iq.x}$ which has $\Box^2 \Lambda = q^2 \Lambda = 0$

$$
\begin{aligned}
A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda \quad &= \quad \varepsilon_\mu e^{-iq.x} + ia\partial_\mu e^{-iq.x} \\
&= \quad \varepsilon_\mu e^{-iq.x} + ia(-iq_\mu)e^{-iq.x} \\
&= \quad (\varepsilon_\mu + aq_\mu)e^{-iq.x}
\end{aligned}
$$

★ Hence the electromagnetic field is left unchanged by

$$\varepsilon_\mu \rightarrow \varepsilon'_\mu = \varepsilon_\mu + aq_\mu$$

★ Hence the two polarization vectors which differ by a mulitple of the photon four-momentum describe the same photon. Choose a such that the time-like component of ε_μ is zero, i.e. $\varepsilon_0 \equiv 0$

★ With this choice of gauge, which is known as the **COULOMB GAUGE**, the Lorentz condition (B2) gives

$$\boxed{\vec{\varepsilon} \cdot \vec{q} = 0} \qquad \text{(B3)}$$

i.e. only 2 independent components, both transverse to the photons momentum

★ A massless photon has two transverse polarisation states. For a photon travelling in the z direction these can be expressed as the transversly polarized states:

$$\varepsilon_1^\mu = (0,1,0,0); \qquad \varepsilon_2^\mu = (0,0,1,0)$$

★ Alternatively take linear combinations to get the circularly polarized states

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ It can be shown that the ε_+ state corresponds the state in which the photon spin is directed in the **+z** direction, i.e. $S_z = +1$

Appendix II : Massive Spin-1 particles

(Non-examinable)

- For a massless photon we had (before imposing the Lorentz condition) we had from equation **(A5)**

$$\Box^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = j^\mu$$

★ The Klein-Gordon equation for a spin-0 particle of mass m is

$$(\Box^2 + m^2)\phi = 0$$

suggestive that the appropriate equations for a massive spin-1 particle can be obtained by replacing $\Box^2 \rightarrow \Box^2 + m^2$

★ This is indeed the case, and from QFT it can be shown that for a massive spin 1 particle equation **(A5)** becomes

$$(\Box^2 + m^2)B^\mu - \partial^\mu(\partial_\nu B^\nu) = j^\mu$$

★ Therefore a free particle must satisfy

$$(\Box^2 + m^2)B^\mu - \partial^\mu(\partial_\nu B^\nu) = 0 \qquad \textbf{(B4)}$$

•Acting on equation (B4) with ∂_ν gives

$$\begin{aligned}(\Box^2+m^2)\partial_\mu B^\mu - \partial_\mu\partial^\mu(\partial_\nu B^\nu) &= 0\\ (\Box^2+m^2)\partial_\mu B^\mu - \Box^2(\partial_\nu B^\nu) &= 0\\ m^2\partial_\mu B^\mu &= 0\end{aligned}$$

(B5)

★ Hence, for a massive spin-1 particle, unavoidably have $\partial_\mu B^\mu = 0$; note this is not a relation that reflects to choice of gauge.

•Equation (B4) becomes

$$(\Box^2+m^2)B^\mu = 0$$

(B6)

★ For a free spin-1 particle with 4-momentum, p^μ , equation (B6) admits solutions

$$B_\mu = \varepsilon_\mu e^{-ip.x}$$

★ Substituting into equation (B5) gives

$$p_\mu \varepsilon^\mu = 0$$

★The four degrees of freedom in ε^μ are reduced to three, but for a massive particle, equation (B6) **does not** allow a choice of gauge and we can not reduce the number of degrees of freedom any further.

★ **Hence we need to find three orthogonal polarisation states satisfying**

$$\boxed{p_\mu \varepsilon^\mu = 0} \qquad \textbf{(B7)}$$

★ **For a particle travelling in the z direction, can still admit the circularly polarized states.**

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ **Writing the third state as**

$$\varepsilon_L^\mu = \frac{1}{\sqrt{\alpha^2+\beta^2}}(\alpha,0,0,\beta)$$

equation (B7) gives $\alpha E - \beta p_z = 0$

$$\Rightarrow \qquad \varepsilon_L^\mu = \frac{1}{m}(p_z,0,0,E)$$

★ **This longitudinal polarisation state is only present for massive spin-1 particles, i.e. there is no analogous state for a free photon (although off-mass shell virtual photons can be longitudinally polarized – a fact that was alluded to on page 114).**