Particle Physics

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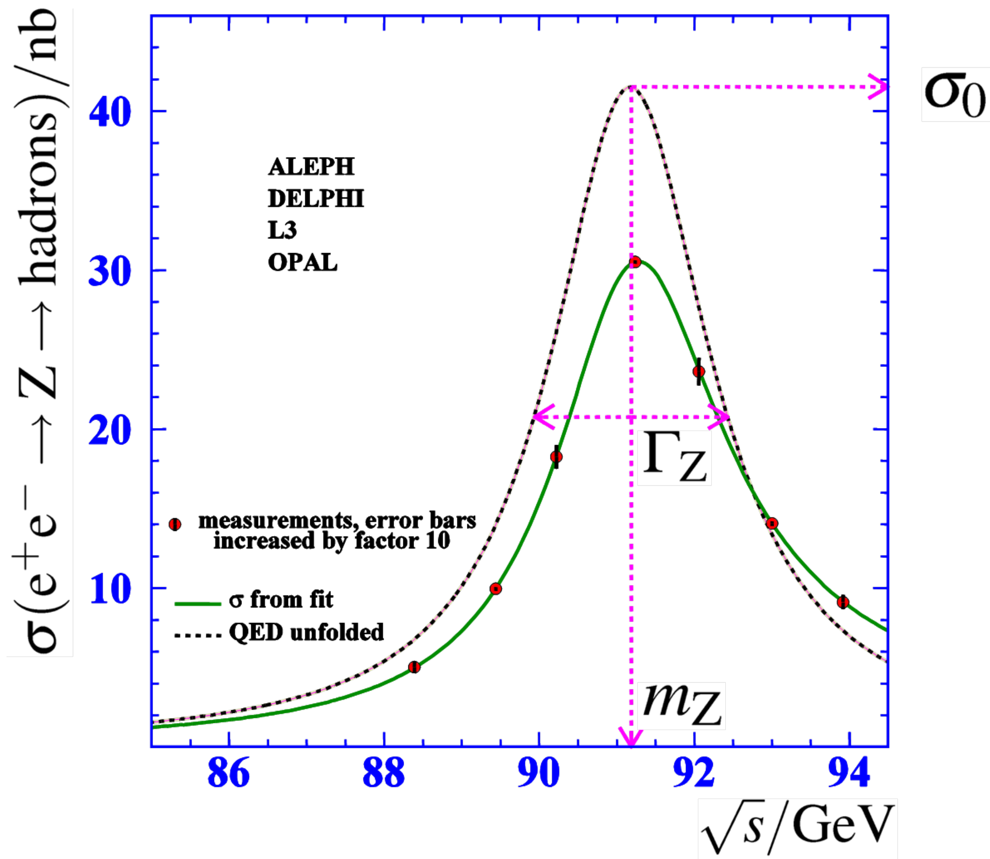
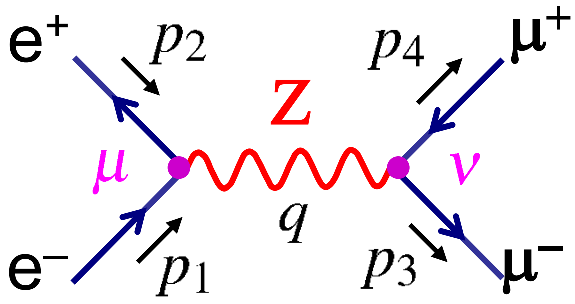


Handout 14 : Precision Tests of the Standard Model

The Z Resonance

★ **Want to calculate the cross-section for** $e^+e^- \to Z \to \mu^+\mu^-$

- **Feynman rules for the diagram below give:**



e⁺e⁻ vertex: $\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

μ⁺μ⁻ vertex: $\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

➡ $$-iM_{fi} = [\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}.[\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

➡ $$\boxed{M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]}$$

★ **Convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)**

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

LH and RH projections operators

hence $c_V = (c_L + c_R), \;\; c_A = (c_L - c_R)$

and $\frac{1}{2}(c_V - c_A\gamma^5) \;=\; \frac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5)$

$$= c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

with $c_L = \frac{1}{2}(c_V + c_A), \;\; c_R = \frac{1}{2}(c_V - c_A)$

★ Rewriting the matrix element in terms of LH and RH couplings:

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)]$$

★ Apply projection operators remembering that in the ultra-relativistic limit

$\frac{1}{2}(1-\gamma^5)u = u_\downarrow; \quad \frac{1}{2}(1+\gamma^5)u = u_\uparrow, \quad \frac{1}{2}(1-\gamma^5)v = v_\uparrow, \quad \frac{1}{2}(1+\gamma^5)v = v_\downarrow$

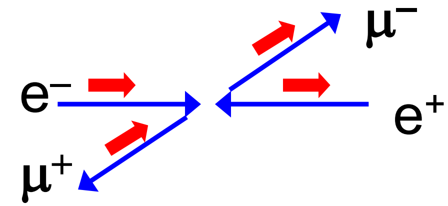
⇨ $$M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ For a combination of **V** and **A** currents, $\overline{u}_\uparrow\gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions

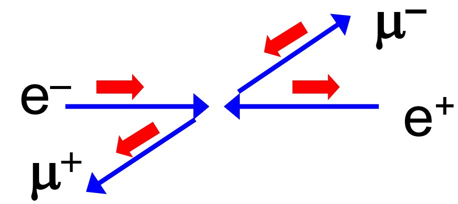
⇨ $$\boxed{\begin{aligned}&-\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]\\ &\qquad\times[c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]\end{aligned}}$$

★ **Sum of 4 terms**

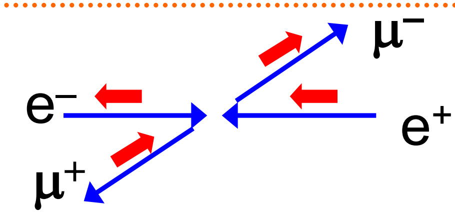
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



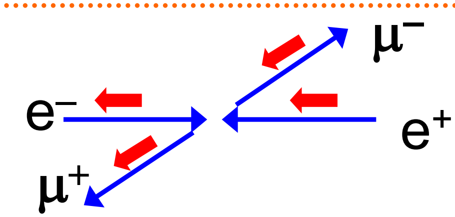
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state <u>particles</u>

★ **Fortunately we have calculated these terms before when considering**

$e^+e^- \to \gamma \to \mu^+\mu^-$ **giving:** **(pages 137-138)**

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta)$$ **etc.**

★ **Applying the QED results to the Z exchange with**

$$\frac{e^2}{q^2} \to \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

gives:

$$|M_{RR}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_R^e)^2 (c_R^\mu)^2 (1+\cos\theta)^2$$

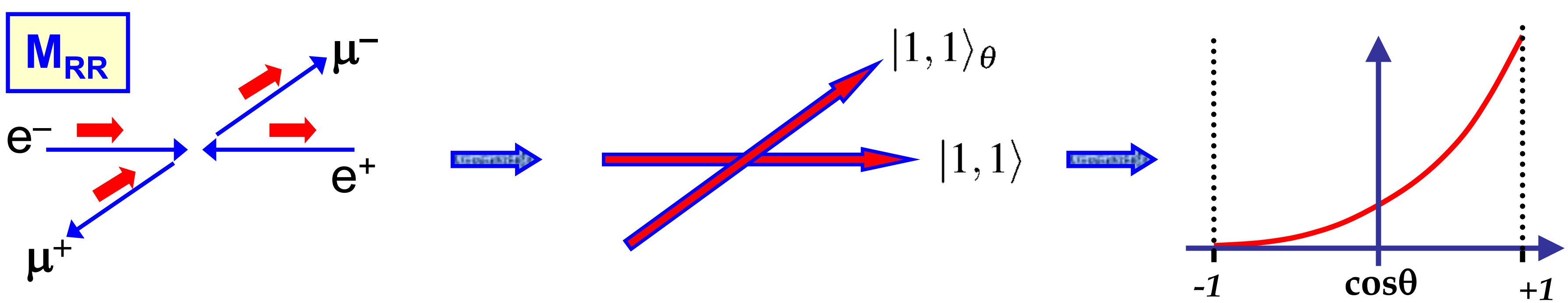
$$|M_{RL}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_R^e)^2 (c_L^\mu)^2 (1-\cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_L^e)^2 (c_R^\mu)^2 (1-\cos\theta)^2$$

$$|M_{LL}|^2 = s^2 \left|\frac{g_Z^2}{s-m_Z^2}\right|^2 (c_L^e)^2 (c_L^\mu)^2 (1+\cos\theta)^2$$

where $q^2 = s = 4E_e^2$

★ **As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.**



The Breit-Wigner Resonance

★ Need to consider carefully the propagator term $1/(s-m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson

★ To do this need to account for the fact that the Z boson is an unstable particle

- For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

- For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

- Equivalent to making the replacement

$$m \to m - i\Gamma/2$$

★ In the Z boson propagator make the substitution:

$$m_Z \to m_Z - i\Gamma_Z/2$$

★ Which gives:

$$(s-m_Z^2) \longrightarrow [s-(m_Z - i\Gamma_Z/2)^2] = s - m_Z^2 + im_Z\Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

★ Which gives

$$\left|\frac{1}{s-m_Z^2}\right|^2 \to \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

★ **And the Matrix elements become**

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$ **etc.**

★ **In the limit where initial and final state particle mass can be neglected:**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

(page 33)

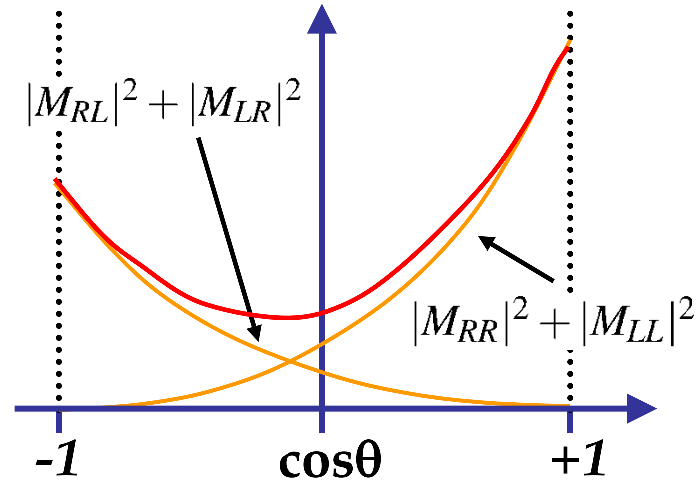
★ **Giving:**

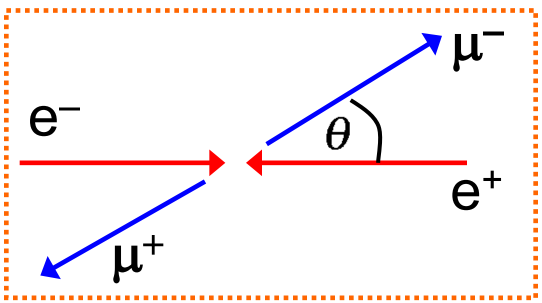
$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_L^\mu)^2(1-\cos\theta)^2$$





★ **Because** $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$ **, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).**

Cross section with unpolarized beams

★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states.** Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2}\cdot\frac{1}{2}\cdot(|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \{[(c_R^e)^2(c_R^\mu)^2 + (c_L^e)^2(c_L^2)^2](1+\cos\theta)^2$$
$$+[(c_L^e)^2(c_R^\mu)^2 + (c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}$$

★ The part of the expression {...} can be rearranged:

$$\{\ldots\} = [(c_R^e)^2 + (c_L^e)^2][(c_R^\mu)^2 + (c_L^\mu)^2](1+\cos^2\theta)$$
$$+2[(c_R^e)^2 - (c_L^e)^2][(c_R^\mu)^2 - (c_L^\mu)^2]\cos\theta \qquad \textbf{(1)}$$

and using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $c_V c_A = c_L^2 - c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2](1+\cos^2\theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta$$

★ **Hence the complete expression for the unpolarized differential cross section is:**

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle \\ &= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times \\ &\quad \left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\}\end{aligned}$$

★ **Integrating over solid angle** $d\Omega = d\phi\, d(\cos\theta) = 2\pi d(\cos\theta)$

$$\int_{-1}^{+1}(1+\cos^2\theta)d(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \tfrac{8}{3} \quad \textbf{and} \quad \int_{-1}^{+1}\cos\theta\, d(\cos\theta) = 0$$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]}$$

★ **Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions**

$$(c_V^f)^2+(c_A^f)^2$$

Connection to the Breit-Wigner Formula

★ **Can write the total cross section**

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]}$$

in terms of the Z boson decay rates (partial widths) from page 478 (question 26)

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right] \quad \textbf{and} \quad \Gamma(Z\to \mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

$$\Rightarrow \quad \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

★ **Writing the partial widths as** $\Gamma_{ee} = \Gamma(Z\to e^+e^-)$ **etc., the total cross section can be written**

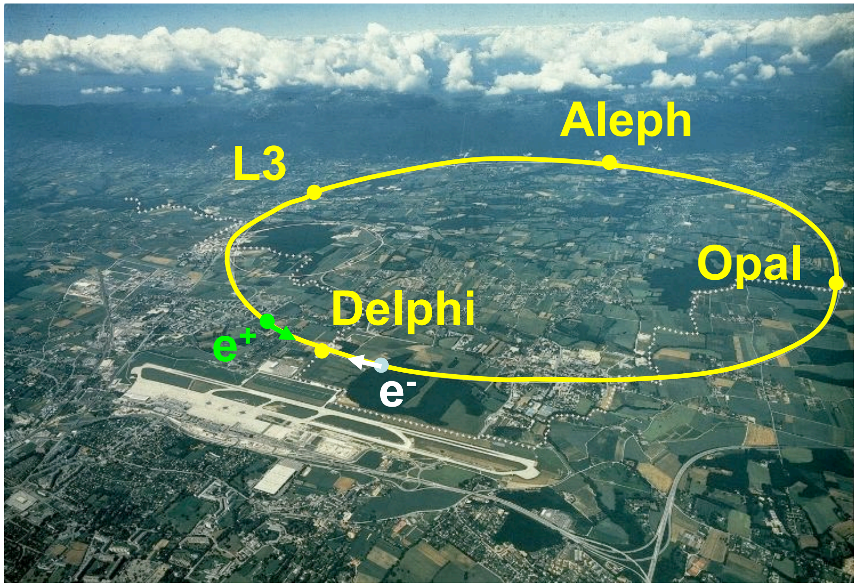
$$\boxed{\sigma(e^+e^-\to Z\to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff}} \qquad \textbf{(2)}$$

where f is the final state fermion flavour:

(The relation to the non-relativistic form of the part II course is given in the appendix)

Electroweak Measurements at LEP

★The **L**arge **E**lectron **P**ositron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.



- 26 km circumference accelerator straddling French/Swiss boarder
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):
  ALEPH,
  DELPHI,
  L3,
  OPAL

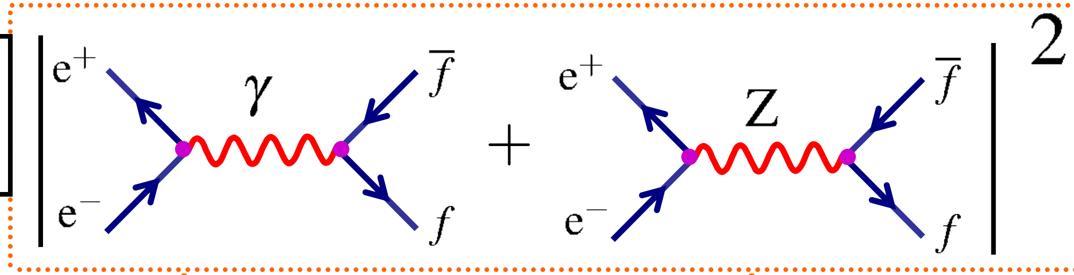
Basically a large Z and W factory:

★ 1989-1995: Electron-Positron collisions at $\sqrt{s}$ = 91.2 GeV
  - 17 Million Z bosons detected

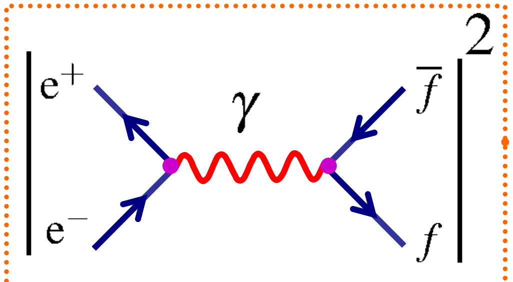
★ 1996-2000: Electron-Positron collisions at $\sqrt{s}$ = 161-208 GeV
  - 30000 W^+W^- events detected

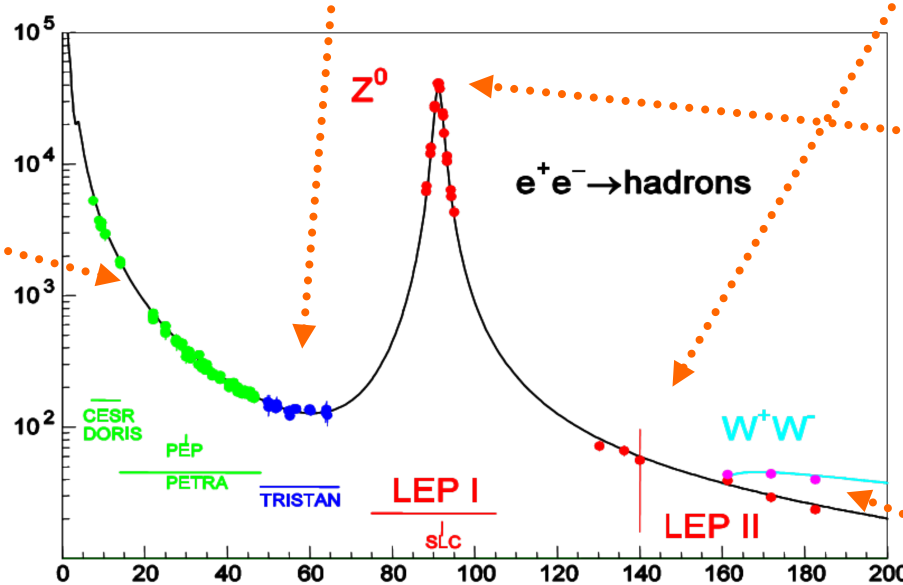
e^+e^- Annihilation in Feynman Diagrams

In general e^+e^- annihilation involves both photon and Z exchange : + interference

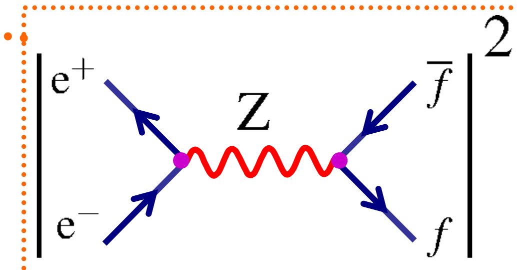


Well below Z: photon exchange dominant

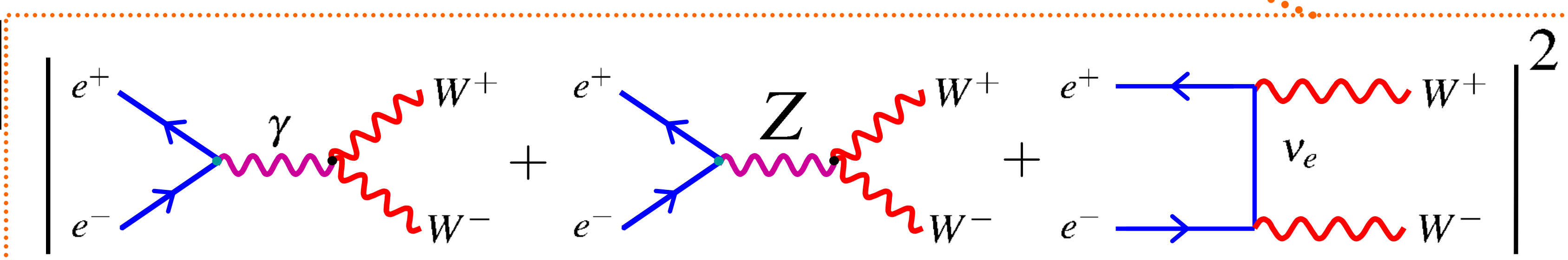




At Z resonance: Z exchange dominant



High energies: WW production



Cross Section Measurements

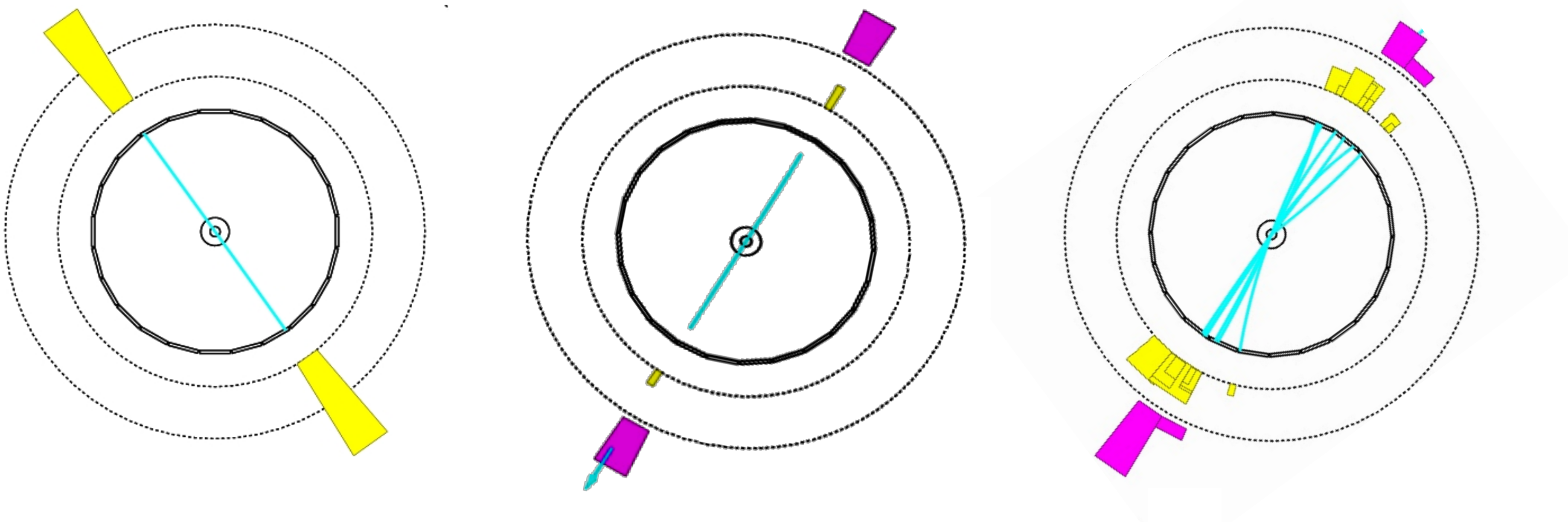
★ **At Z resonance mainly observe four types of event:**

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \tau^+\tau^-$$

$$e^+e^- \to Z \to q\bar{q} \to \text{hadrons}$$

★ **Each has a distinct topology in the detectors, e.g.**

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \text{hadrons}$$



★ **To work out cross sections, first count events of each type**

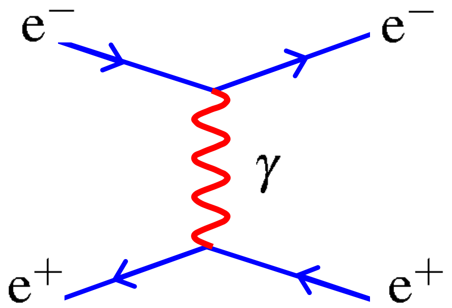
★ **Then need to know "integrated luminosity" of colliding beams, i.e. the relation between cross-section and expected number of interactions**

$$N_{\text{events}} = \mathscr{L}\sigma$$

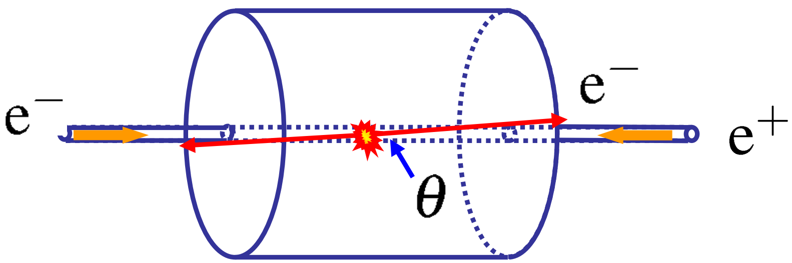
★ To calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile

- very difficult to achieve with precision of better than 10%

★ Instead "normalise" using another type of event:



- Use the QED Bhabha scattering process
- QED, so cross section can be calculated very precisely
- Very large cross section – small statistical errors
- Reaction is very forward peaked – i.e. the electron tends not to get deflected much



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{\sin^4\theta/2} \Rightarrow \boxed{\frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}}$$

Photon propagator

e.g. see handout 5

- Count events where the electron is scattered in the very forward direction

$$N_{\text{Bhabha}} = \mathscr{L}\sigma_{\text{Bhabha}} \Rightarrow \mathscr{L}$$

σ_{Bhabha} known from QED calc.

★ Hence all other cross sections can be expressed as

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}}$$

⇒ **Cross section measurements Involve just event counting !**

Measurements of the Z Line-shape

- ★ Measurements of the Z resonance lineshape determine:
  - m_Z : peak of the resonance
  - Γ_Z : FWHM of resonance
  - Γ_f : Partial decay widths
  - N_ν : Number of light neutrino generations
- ★ Measure cross sections to different final states versus C.o.M. energy $\sqrt{s}$
- ★ Starting from

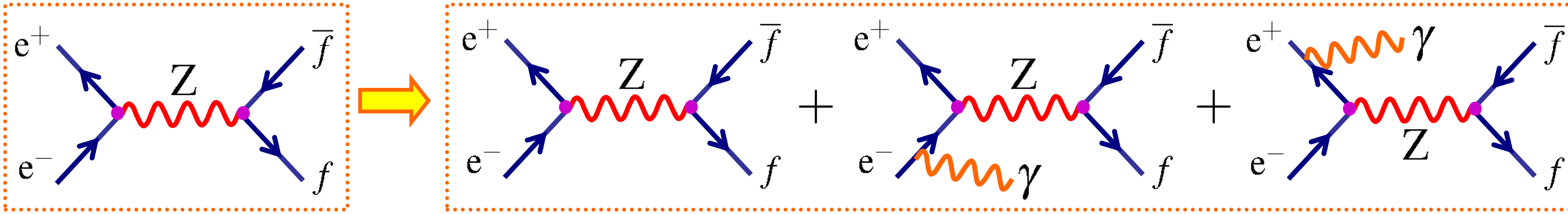
$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad (3)$$

maximum cross section occurs at $\sqrt{s} = m_Z$ with peak cross section equal to

$$\boxed{\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}}$$

- ★ Cross section falls to half peak value at $\sqrt{s} \approx m_z \pm \frac{\Gamma_Z}{2}$ which can be seen immediately from eqn. (3)
- ★ Hence $\Gamma_Z = \frac{\hbar}{\tau_Z} = \text{FWHM of resonance}$

- In practise, it is not that simple, QED corrections distort the measured line-shape
- One particularly important correction: **initial state radiation** (**ISR**)



- Initial state radiation reduces the centre-of-mass energy of the e^+e^- collision

$$e^+ \xrightarrow{E} \xleftarrow{E} e^- \qquad \sqrt{s} = 2E$$

becomes

$$\xrightarrow{E} \quad \xleftarrow{E - E_\gamma} \qquad \sqrt{s}' \approx 2E\left(1 - \frac{E_\gamma}{2E}\right)$$

- Measured cross section can be written:

$$\sigma_{\text{meas}}(E) = \int \sigma(E')f(E',E)\mathrm{d}E'$$

Probability of e+e- colliding with C.o.M. energy E' when C.o.M energy before radiation is E

- Fortunately can calculate $f(E',E)$ very precisely, just QED, and can then obtain Z line-shape from measured cross section

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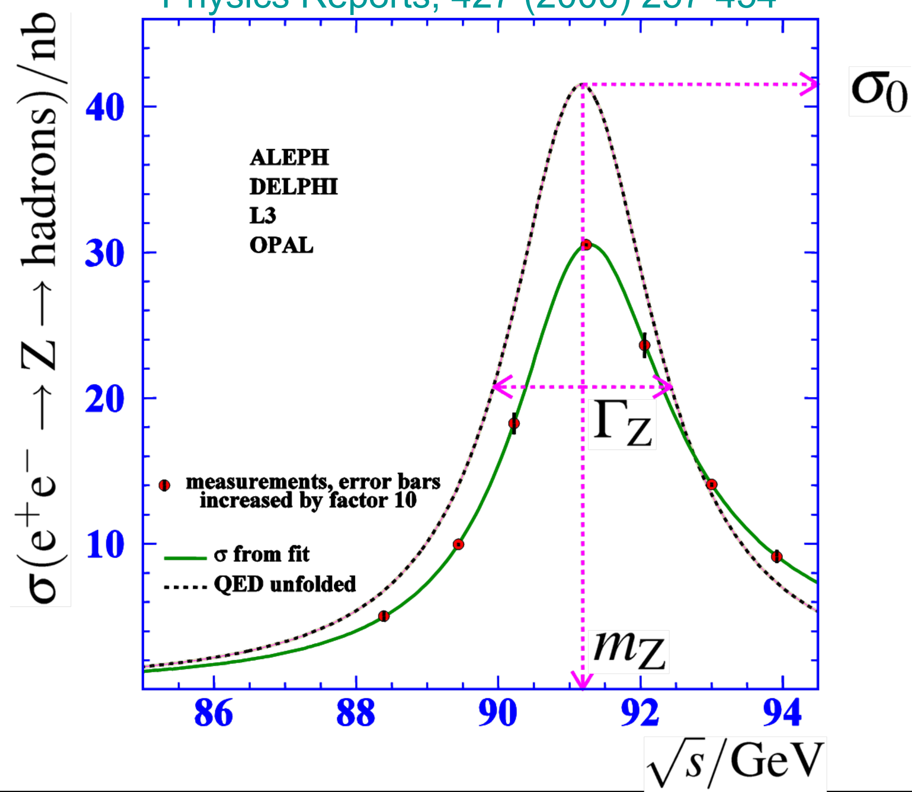


★ In principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM

$$\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}} \qquad \boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}}$$

★ 0.002 % measurement of m_Z !

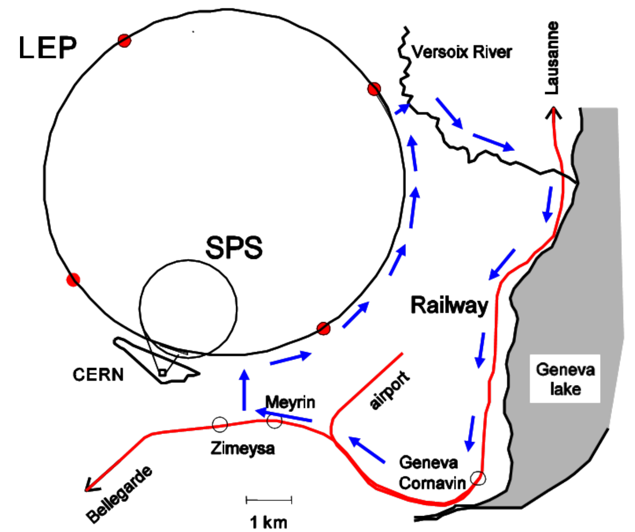
★ To achieve this level of precision – need to know energy of the colliding beams to better than 0.002 % : sensitive to unusual systematic effects…

Moon:

- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm
- Changes beam energy by ~10 MeV : need to correct for tidal effects !

Trains:

- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV



Number of generations

★ Total decay width measured from Z line-shape: $\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$

★ If there were an additional 4th generation would expect $Z \to \nu_4 \overline{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. > $m_Z/2$)

★ Total decay width is the sum of the partial widths:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

★ Although don't observe neutrinos, $Z \to \nu\overline{\nu}$ decays affect the Z resonance shape for all final states

★ For all other final states can determine partial decay widths from peak cross sections:

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

★ Assuming lepton universality:

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + \boxed{N_\nu}\Gamma_{\nu\nu}$$

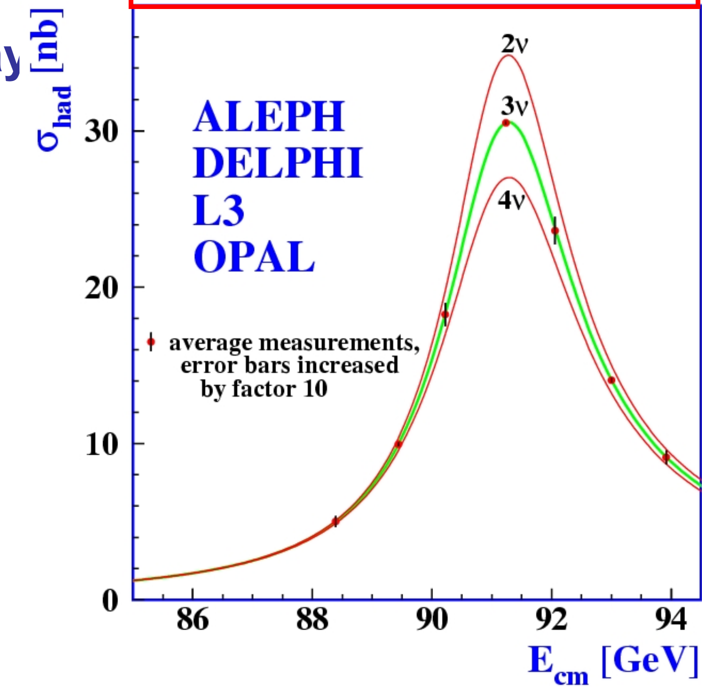
measured from Z lineshape

measured from peak cross sections

calculated, e.g. question 26

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

$e^+e^- \to Z \to \text{hadrons}$



Physics Reports, 427 (2006) 257-454

★ ONLY 3 GENERATIONS (unless a new 4th generation neutrino has very large mass)

Forward-Backward Asymmetry

★ On page 495 we obtained the expression for the differential cross section:

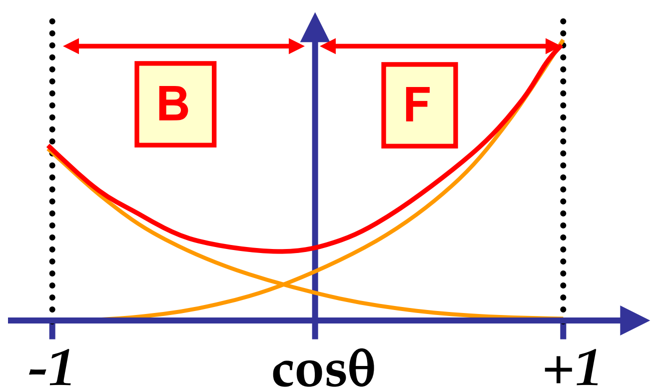
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+\ 2\,[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

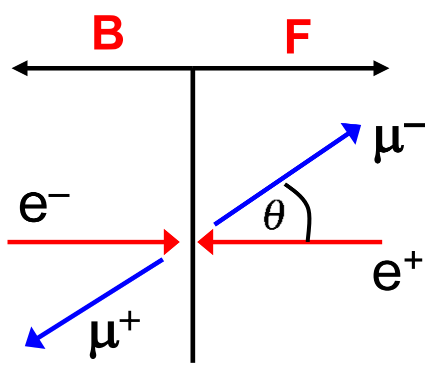
★ The differential cross sections is therefore of the form:

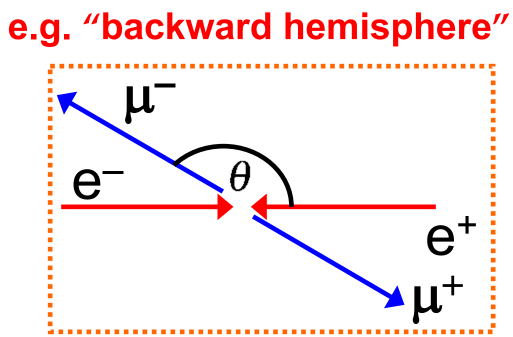
$$\frac{d\sigma}{d\Omega} = \kappa\times\left[A(1+\cos^2\theta)+B\cos\theta\right] \qquad \left\{\begin{array}{l} A = [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B = 2[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{array}\right.$$

★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\boxed{\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta}\,d\cos\theta} \qquad \boxed{\sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta}\,d\cos\theta}$$

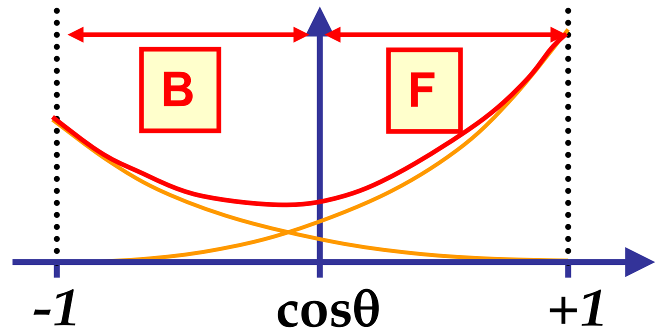






★ The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{\rm FB}=\frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}$$



• Integrating equation (1):

$$\sigma_F=\kappa\int_0^1[A(1+\cos^2\theta)+B\cos\theta]\mathrm{d}\cos\theta=\kappa\int_0^1[A(1+x^2)+Bx]\mathrm{d}x=\kappa\left(\frac{4}{3}A+\frac{1}{2}B\right)$$

$$\sigma_B=\kappa\int_{-1}^0[A(1+\cos^2\theta)+B\cos\theta]\mathrm{d}\cos\theta=\kappa\int_{-1}^0[A(1+x^2)+Bx]\mathrm{d}x=\kappa\left(\frac{4}{3}A-\frac{1}{2}B\right)$$

★ Which gives:

$$A_{\rm FB}=\frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}=\frac{B}{(8/3)A}=\frac{3}{4}\left[\frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2-(c_R^\mu)^2}{(c_L^\mu)^2+(c_R^\mu)^2}\right]$$

★ This can be written as

$$\boxed{A_{\rm FB}=\frac{3}{4}A_eA_\mu}$$

with

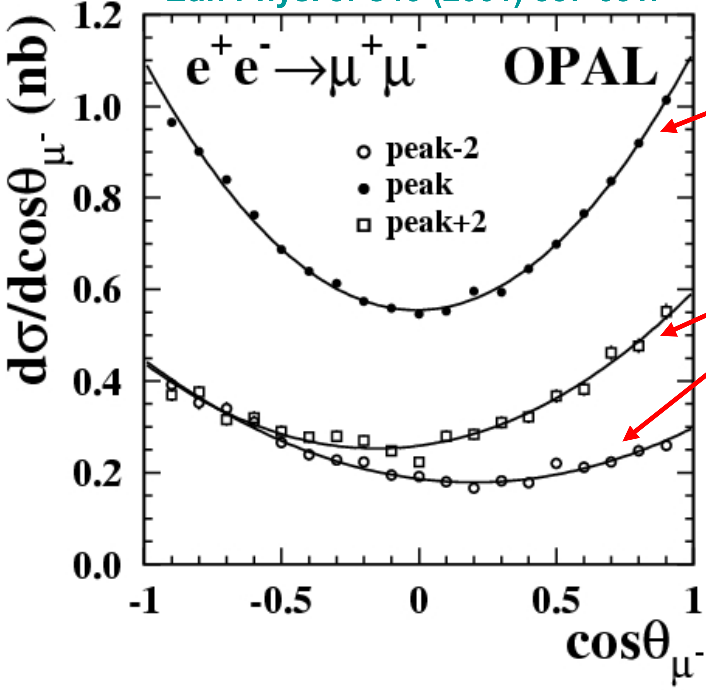
$$\boxed{A_f\equiv\frac{(c_L^f)^2-(c_R^f)^2}{(c_L^f)^2+(c_R^f)^2}=\frac{2c_V^fc_A^f}{(c_V^f)^2+(c_A^f)^2}} \qquad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \to Z \to \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_w \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

★ To relate these measurements to the couplings uses $A_{\rm FB} = \frac{3}{4} A_e A_\mu$

★ In all cases asymmetries depend on A_e

★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4} A_e^2$ (also see Appendix II for A_{LR})

Determination of the Weak Mixing Angle

★ **From LEP :** $A_{FB}^{0,f} = \frac{3}{4} A_e A_f$

★ **From SLC :** $A_{LR} = A_e$

$$\left.\begin{array}{l} A_{FB}^{0,f} = \frac{3}{4} A_e A_f \\ A_{LR} = A_e \end{array}\right\} \quad A_e, A_\mu, A_\tau, \ldots$$

Putting everything together ⇨

$$\boxed{\begin{aligned} A_e &= 0.1514 \pm 0.0019 \\ A_\mu &= 0.1456 \pm 0.0091 \\ A_\tau &= 0.1449 \pm 0.0040 \end{aligned}}$$

includes results from other measurements

with

$$A_f \equiv \frac{2 c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2 \frac{c_V / c_A}{1 + (c_V / c_A)^2}$$

★ **Measured asymmetries give ratio of vector to axial-vector Z coupings.**

★ **In SM these are related to the weak mixing angle**

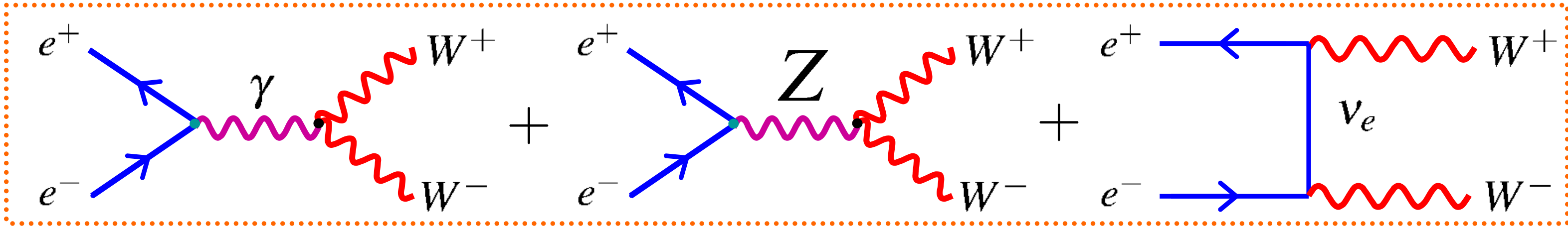
$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q \sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_3} \sin^2\theta_W = 1 - 4|Q| \sin^2\theta_W$$

★ **Asymmetry measurements give precise determination of** $\sin^2\theta_W$

$$\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$$

W^+W^- Production

- From 1995-2000 LEP operated above the threshold for W-pair production
- Three diagrams "CC03" are involved

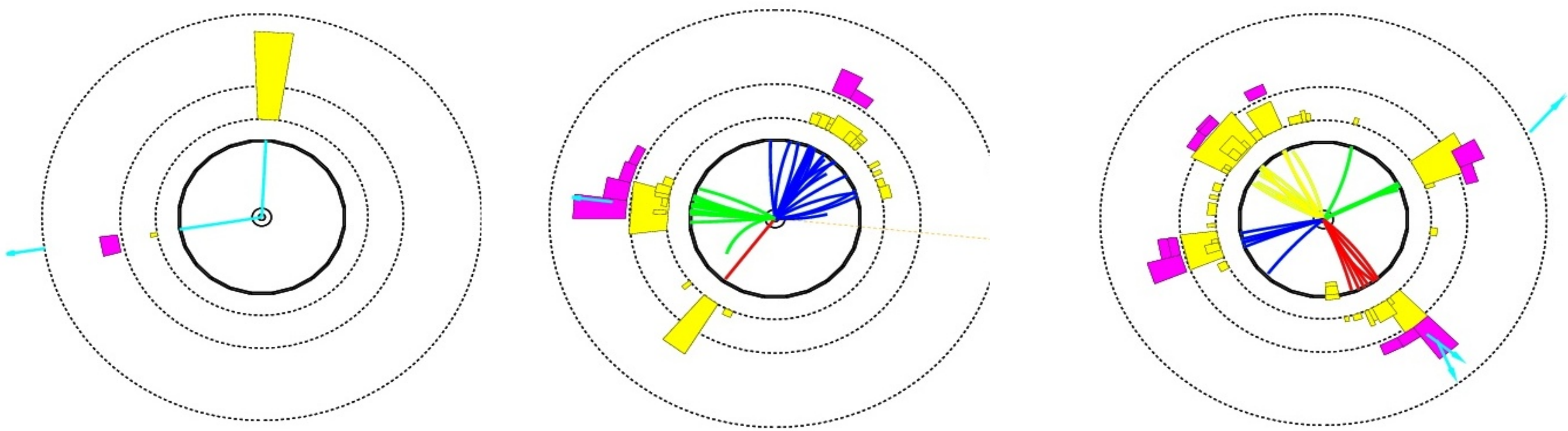


- W bosons decay (p.459) either to leptons or hadrons with branching fractions:

$$Br(W^- \to \text{hadrons}) \approx 0.67 \qquad Br(W^- \to \mathrm{e}^-\overline{\nu}_\mathrm{e}) \approx 0.11$$

$$Br(W^- \to \mu^-\overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \to \tau^-\overline{\nu}_\tau) \approx 0.11$$

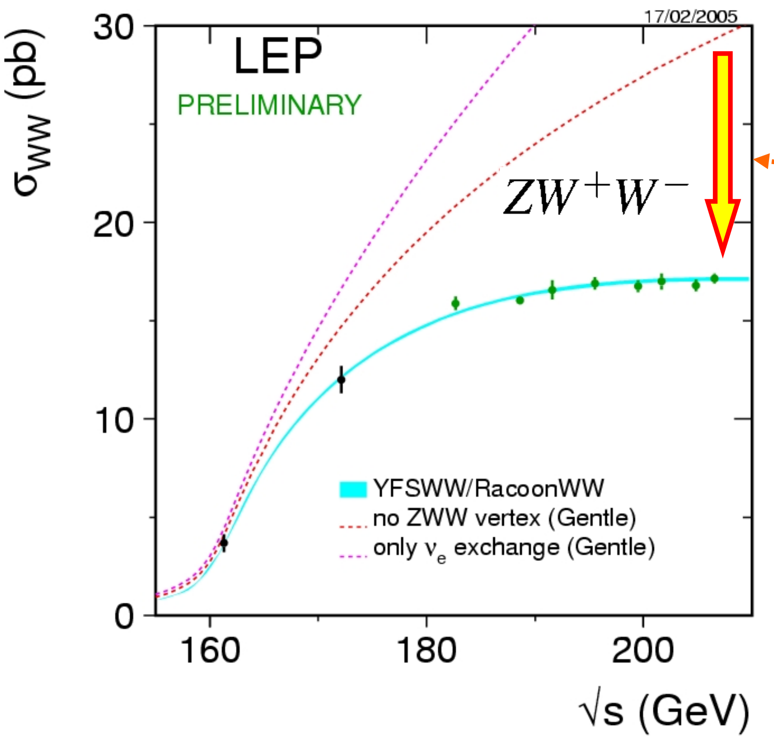
- Gives rise to three **distinct topologies**



$$W^+W^- \to \ell^+\nu\ell^-\overline{\nu} \qquad W^+W^- \to q\overline{q}\ell\nu \qquad W^+W^- \to q\overline{q}q\overline{q}$$

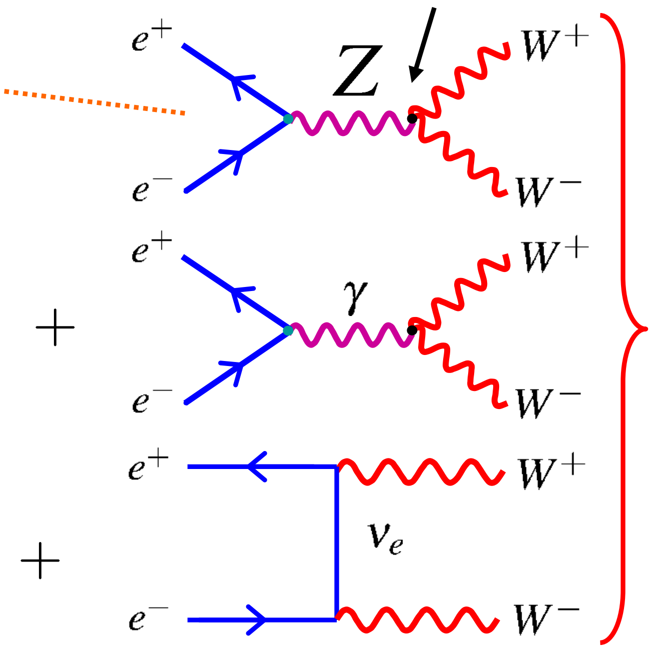
$e^+e^- \to W^+W^-$ Cross Section

★ Measure cross sections by **counting events** and normalising to low angle Bhabha scattering events



★ Data consistent with SM expectation

★ Provides a direct test of ZW^+W^- vertex



★ Recall that without the Z diagram the cross section violates unitarity

★ Presence of Z fixes this problem

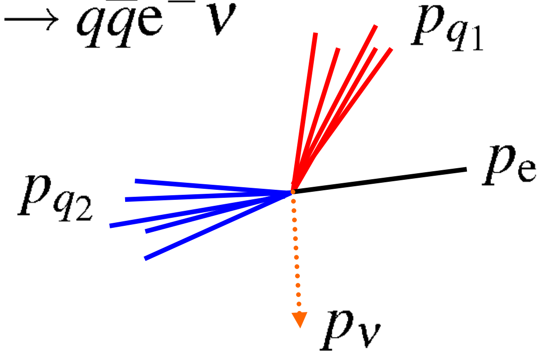
W-mass and W-width

★ Unlike $e^+e^- \to Z$, the process $e^+e^- \to W^+W^-$ is not a resonant process

⇨ Different method to measure W-boson Mass

•Measure energy and momenta of particles produced in the W boson decays, e.g.

$W^+W^- \to q\bar{q}e^-\bar{\nu}$

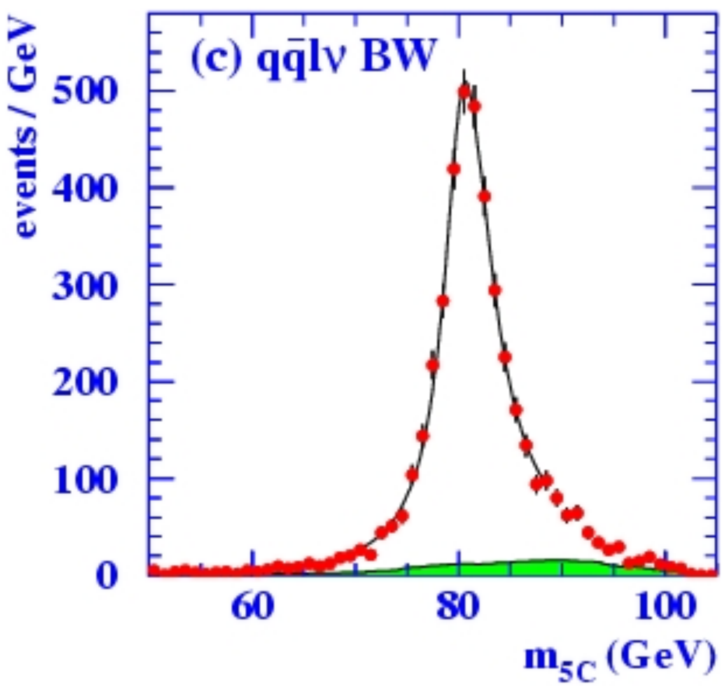


▪ Neutrino four-momentum from energy-momentum conservation !

$$p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$$

▪ Reconstruct masses of two W bosons

$$M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$$
$$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$



$\approx \frac{1}{2}(M_+ + M_-)$

★ Peak of reconstructed mass distribution gives

$$\boxed{m_W = 80.376 \pm 0.033\,\text{GeV}}$$

★ Width of reconstructed mass distribution gives:

$$\boxed{\Gamma_W = 2.196 \pm 0.083\,\text{GeV}}$$

Does not include measurements from Tevatron at Fermilab

The Higgs Mechanism

★ Higgs mechanism can be used to give masses to both fermions and gauge bosons – but mechanism is different in the two cases.

★Explaining how the Higgs mechanism gives the W and Z gauge bosons masses, while leaving the photon massless, is (unfortunately) beyond this course. [See, hopefully, Gauge Field Theory minor option)]

★By way of apology, we instead provide here an attempt to at least describe the way the mechanism gives masses to fermions – that will hopefully whet your appetite.

Higgs Mechanism & Higgs Boson (1)

•Quantum Field Theories (QFTs) are written down in a Lagrangian formalism.

•A scalar field x with a mass m must have a term "$\frac{1}{2}m^2xx$" in the Lagrangian.

•A fermionic field ψ with a mass m must have a term "mψψ" in the Lagrangian.

•QFTs that are "Gauge Field Theories" have a Lagrangian which is also invariant under the action of a "Gauge Group".

•The Standard Model "Gauge Group" is chosen to be $U(1)\times SU(2)_L \times SU(3)$ in order to allow it to model EM, weak and strong interactions in accordance with experiment.

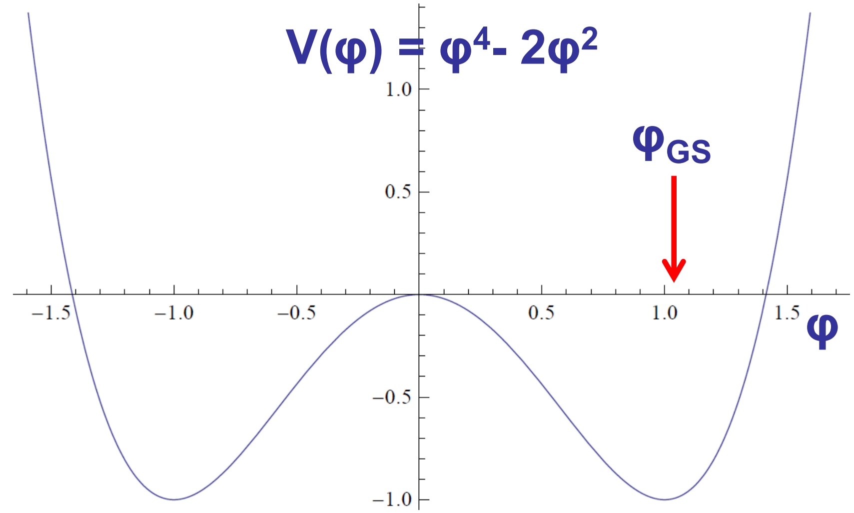
•Terms of the type mψψ are (unfortunately!) not invariant under the above gauge group. So one cannot have massive fermions (eg muon) in the Standard Model ☹

•However, interactions between fields enter the Lagrangian as products of three or more fields. For example, a term proportional to "φψψ" leads to the theory having an interaction vertex connecting one φ to two ψ particles. So:

•IF you could contrive to have a term "φψψ" in the Lagrangian AND could guarantee that φ could spend most of its time taking values near some non-zero value "m", THEN (1) the fermion field ψ would act "as if" there were a term "mψψ" in the Lagrangian, and so would look very much like it had mass m, even if it were actually massless, and (2) the field ψ would have an interaction with the field φ, leading to the testable and falsifiable prediction that an excitation of the field φ (i.e. a "φ particle") should couple to, or decay into, the fermions to which it "gives mass".

Higgs Mechanism & Higgs Boson (2)

- **A field φ could spend a lot of time near a non-zero value if it took a non-zero value in its ground state. Most fields take the value of zero in their ground-state, but this need not always be the case:**



- **For example, a field φ having a potential energy $V(\varphi) = a\varphi^4 - b\varphi^2$ has a ground-state located at $\varphi_{GS}=\pm\sqrt{(b/(2a))}$**
- **So by arranging:**
  - (1) for φ to have a non-zero value φ_{GS} in its ground state by ensuring that the potential $V(\varphi)$ in the Lagrangian is of the right form, and
  - (2) for there to be a (gauge invariant) interaction term "$y\varphi\psi\psi$" in the Lagrangian ("y" being just a constant of proportionality called the "Yukawa Coupling") ...
- **... then the field ψ will look like it has a mass $m=y\varphi_{GS}$! Call φ the "Higgs Field".**
- **Give different fermions different masses by using different Yukawa Couplings.**
- **Note that in the vicinity of the minimum, the potential $V(\varphi)$ necessarily takes the form $V(\varphi_{GS}+x) = V_{min}+\lambda x^2+O(x^3)$ for some constants λ and V_{min}. We already said that terms like λx^2 are banned from the Lagrangian if x is a fermionic field as they break gauge invariance. However, these terms are not banned if x is a scalar field. So this excitation x of the Higgs Field must be a scalar. Call it the "Higgs Boson". We recognise λx^2 as a mass-term for a scalar, so the Higgs Boson has a free (and unknown) mass.**

Higgs theory summary for fermions:

Fermions are intrinsically massless, and need to be so to satisfy "Gauge Invariance".

Nevertheless, interactions with the Higgs field make fermions look like they have mass at "low temperature" (i.e. when the Higgs field is near its ground state, below ~10^{15} K)

Apparent fermion masses are controlled by free parameters called Yukawa Couplings (the strength of the coupling to the Higgs field)

A Higgs Boson is an excitation of the Higgs Field.

The Higgs Boson must be a scalar particle to make everything work.

The Higgs Boson has a mass, but the mass is not predicted by the theory – we have to find it experimentally.

The Higgs Boson has couplings to all the particles it gives mass to (and indeed to gauge bosons too!) and so has many ways it could decay, all fully calculable and determined by the theory as a function of its (as yet unknown) mass

(For proper discussion of the Higgs mechanism see the Gauge Field Theory minor option)

Higgs mechanism for gauge bosons:

★ The Higgs mechanism results in absolute **predictions** for **masses of gauge bosons**

★ In the SM, fermion masses are also ascribed to interactions with the Higgs field

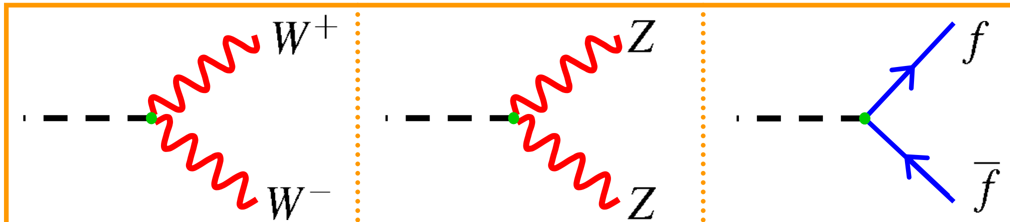
- however, here no prediction of the masses – **just put in by hand**

★ The Higgs is electrically neutral but carries weak hypercharge of **1/2**

★ The photon does not couple to the Higgs field and remains massless

★ The W bosons and the Z couple to weak hypercharge and become massive

Feynman Vertex factors:



$ig_W m_W g^{\mu\nu}$	$ig_Z m_Z g^{\mu\nu}$	$-i\frac{g_W}{2m_W}m_f$

★ Within the SM of Electroweak unification with the Higgs mechanism:

⇨ **Relations between standard model parameters**

$$\boxed{m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W}}$$

$$\boxed{m_Z = \frac{m_W}{\cos\theta_W}}$$

★ Hence, if you know **any three** of : $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ predict the other two.

Precision Tests of the Standard Model

★ From LEP and elsewhere have precise measurements – can test predictions of the Standard Model !

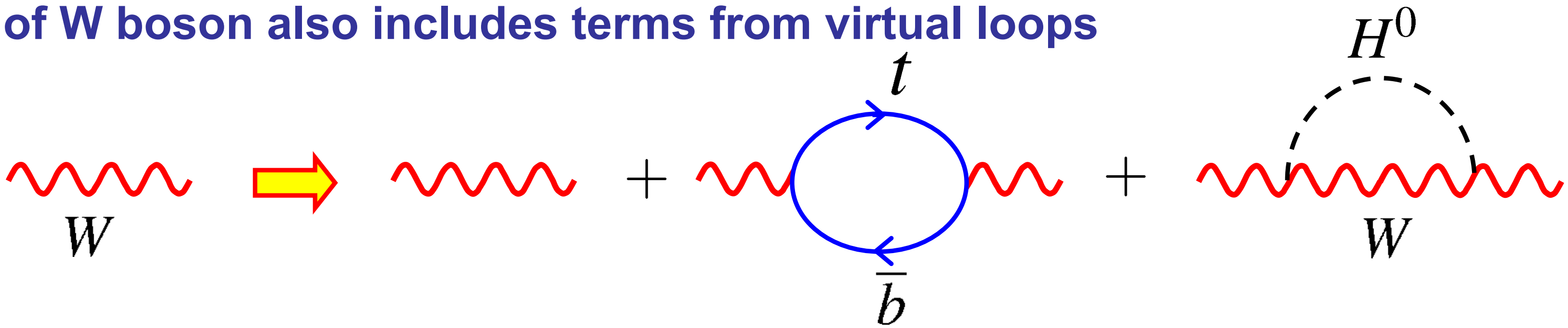
•e.g. predict: $\boxed{m_W = m_Z \cos\theta_W}$ measure $\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}}$ $\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$

•Therefore expect:

$\boxed{m_W = 79.946 \pm 0.008\,\mathrm{GeV}}$ but measure $\boxed{m_W = 80.376 \pm 0.033\,\mathrm{GeV}}$

★ Close, but not quite right – but have only considered lowest order diagrams

★ Mass of W boson also includes terms from virtual loops



$$\boxed{m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)}$$

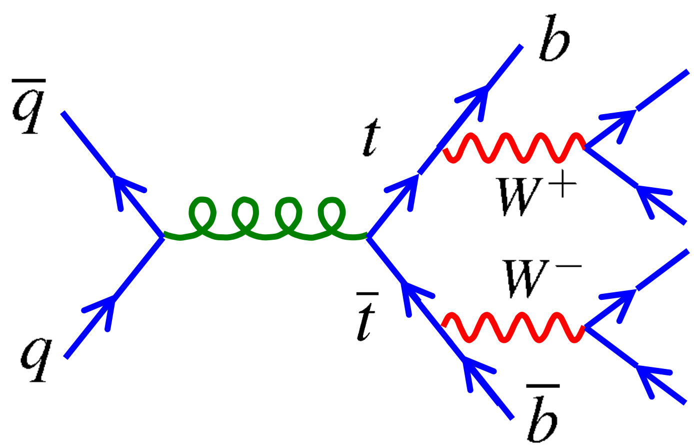
★ Above "discrepancy" due to these virtual loops, i.e. by making very high precision measurements become sensitive to the masses of particles inside the virtual loops !

The Top Quark

★ **From virtual loop corrections and precise LEP data can predict the top quark mass:**

$$\boxed{m_t^{\mathrm{loop}} = 173 \pm 11\,\mathrm{GeV}}$$

★ **In 1994 top quark observed at the Tevatron proton anti-proton collider at Fermilab – with the predicted mass !**



★ **The top quark almost exclusively decays to a bottom quark since**

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

★ **Complicated final state topologies:**

$$t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to 6 \text{ jets}$$

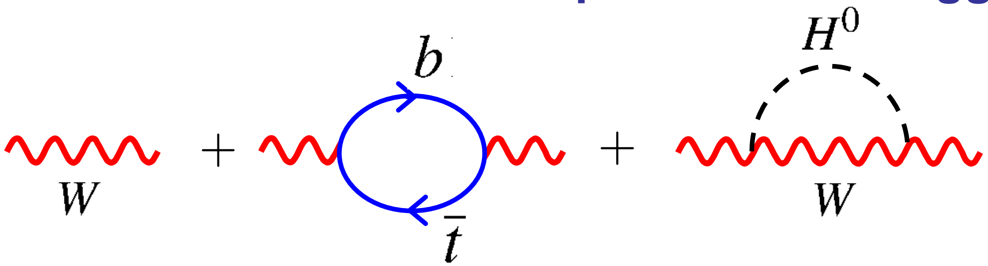
$$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to 4 \text{ jets} + \ell + \nu$$

$$t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to 2 \text{ jets} + 2\ell + 2\nu$$

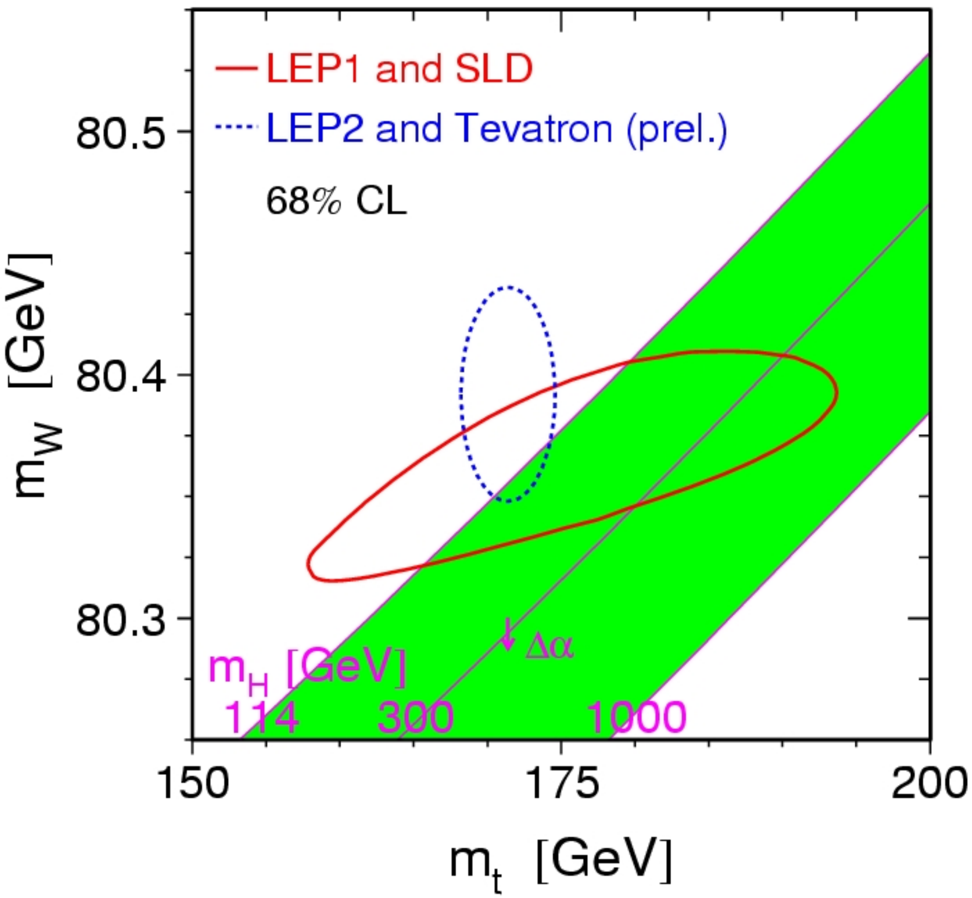
★ **Mass determined by direct reconstruction (see W boson mass)**

$$\boxed{m_t^{\mathrm{meas}} = 174.2 \pm 3.3\,\mathrm{GeV}}$$

★ **But the W mass also depends on the Higgs mass (albeit only logarithmically)**



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



★ **Measurements are sufficiently precise to have some sensitivity to the Higgs mass**

★ **Direct and indirect values of the top and W mass can be compared to prediction for different Higgs mass**

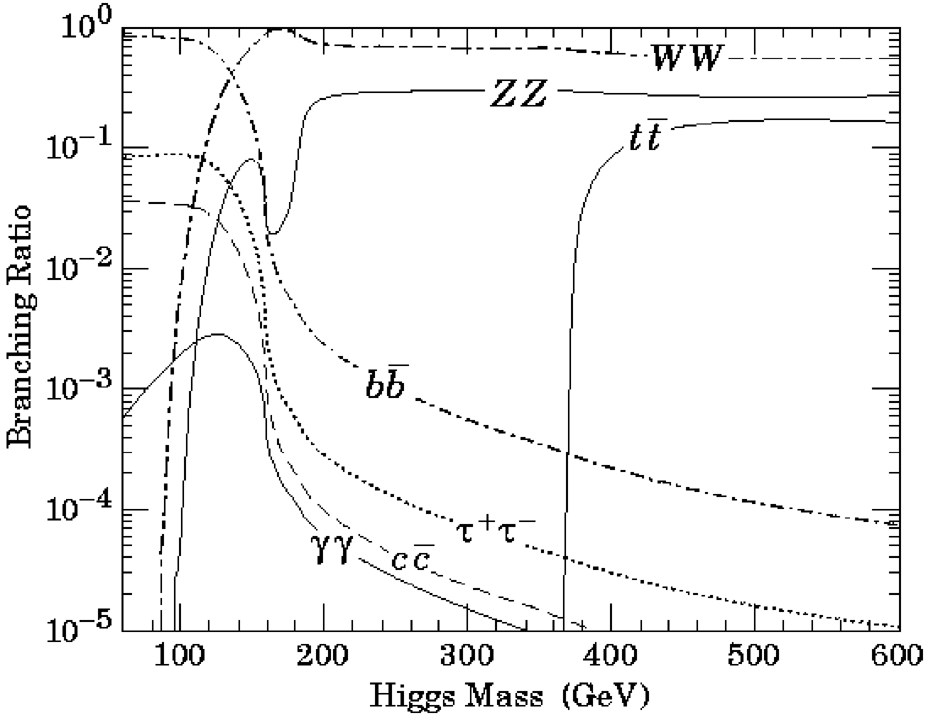
- **Direct:** **W and top masses from direct reconstruction**
- **Indirect:** **from SM interpretation of Z mass, θ_W etc. and**

★ **Data favour a light Higgs:**

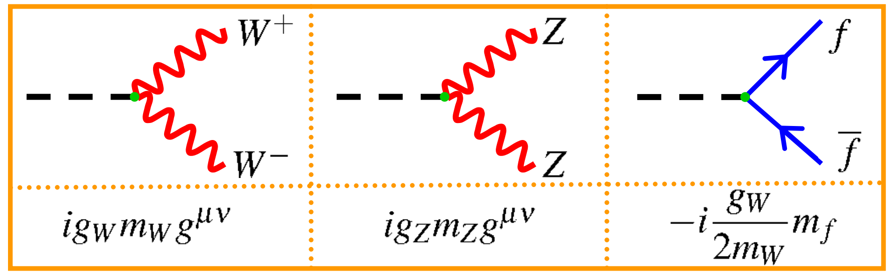
$$\boxed{m_H < 200\,\mathrm{GeV}}$$

Hunting the Higgs

- ★ **The Higgs boson is an essential part of the Standard Model – but does it exist ?**
- ★ **Consider the search at LEP. Need to know how the Higgs decays**



- **Higgs boson couplings proportional to mass**

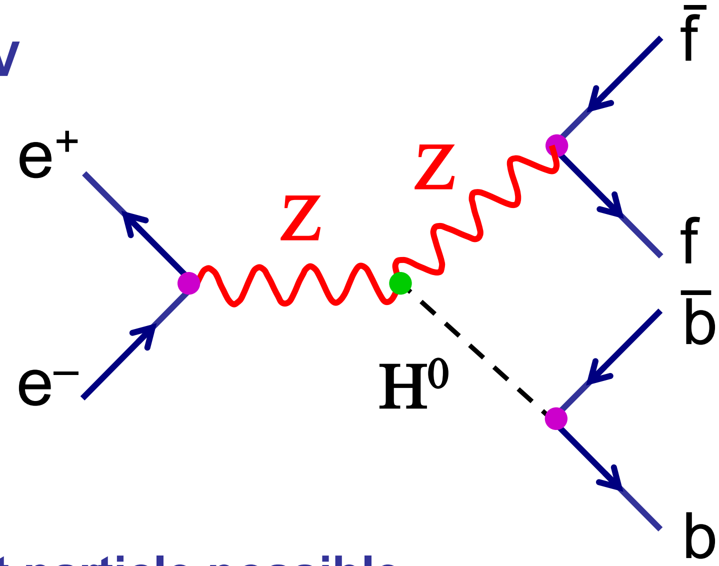


- **Higgs decays predominantly to heaviest particles which are energetically allowed** **(Question 30)**

$m_H < 2m_W$	**mainly** $H^0 \to b\bar{b}$	**+ approx 10%** $H^0 \to \tau^+\tau^-$
$2m_W < m_H < 2m_t$	**almost entirely**	$H^0 \to W^+W^- + H^0 \to ZZ$
$m_H > 2m_t$	**either**	$H^0 \to W^+W^-,\ H^0 \to ZZ,\ H^0 \to t\bar{t}$

A Hint from LEP ?

- ★ LEP operated with a C.o.M. energy upto 207 GeV
- ★ For this energy (assuming the Higgs exists) the main production mechanism would be the "Higgsstrahlung" process

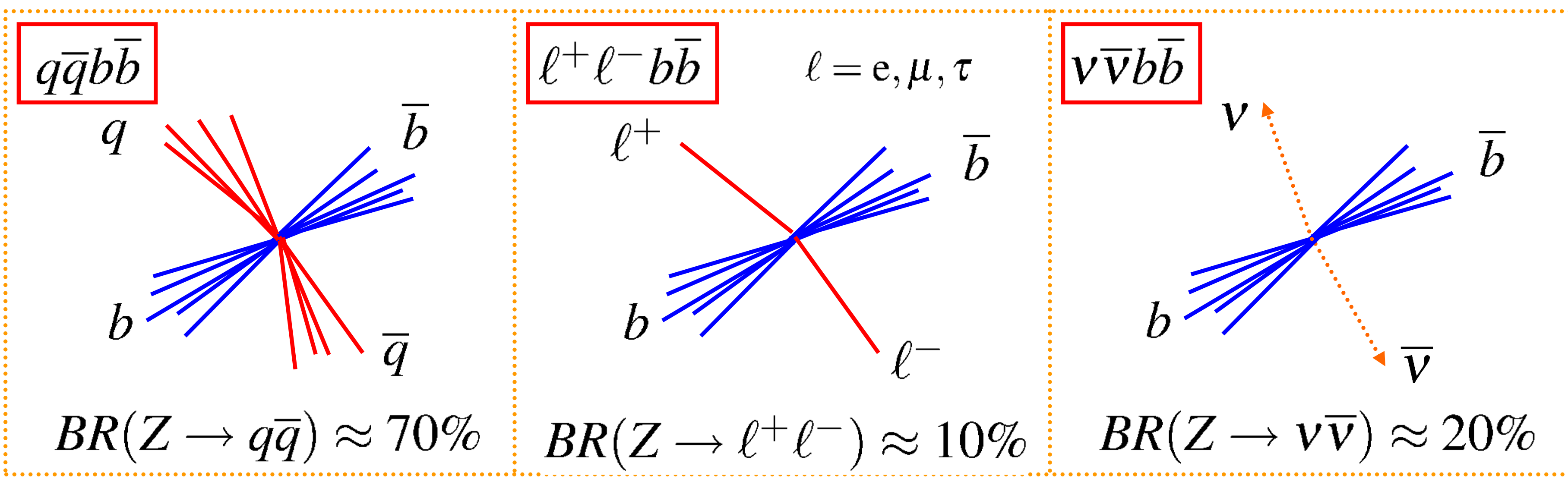


- ★ Need enough energy to make a Z and H; therefore could produce the Higgs boson if

$$m_H < 207\,\mathrm{GeV} - m_Z$$

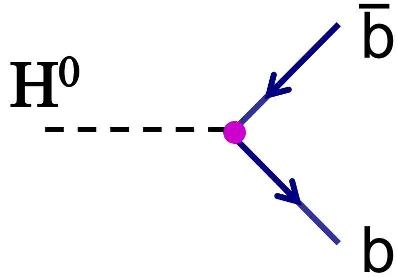
i.e. if $$m_H < 116\,\mathrm{GeV}$$

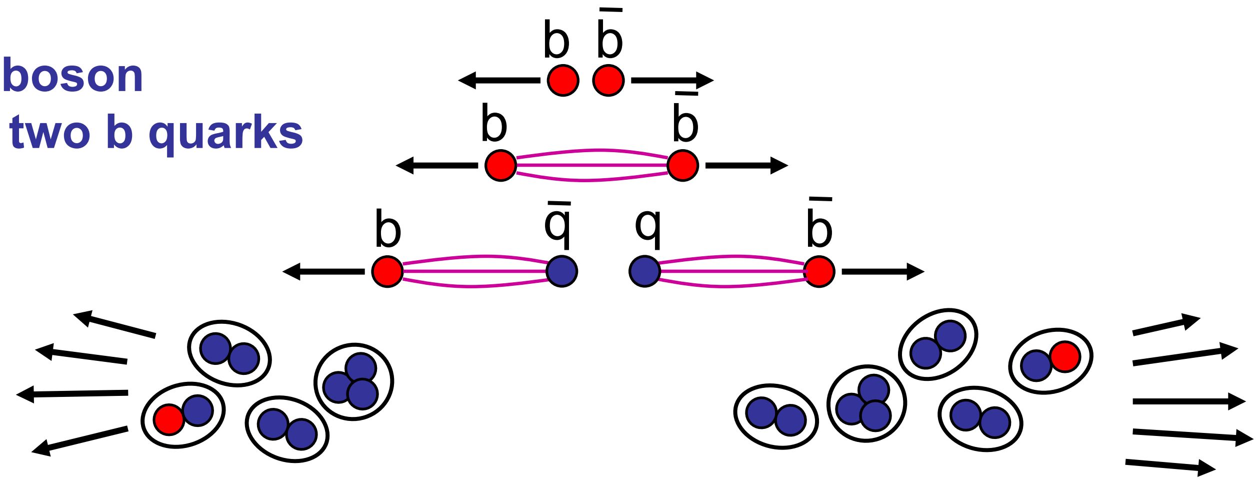
- ★ The Higgs predominantly decays to the heaviest particle possible
- ★ For $m_H < 116\,\mathrm{GeV}$ this is the b-quark (not enough mass to decay to WW/ZZ/tt)



Tagging the Higgs Boson Decays

★ One signature for a Higgs boson decay is the production of two b quarks



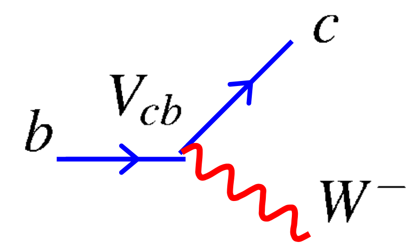


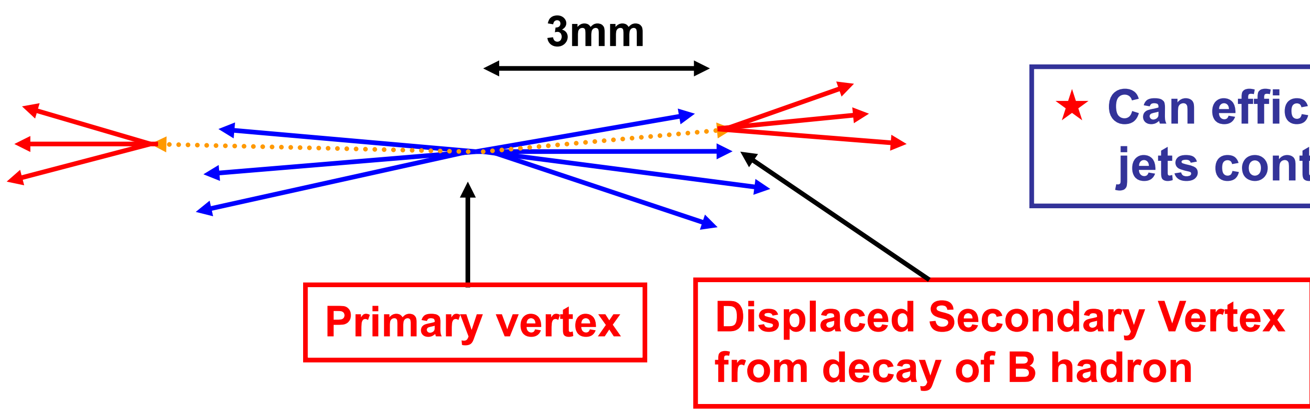
★ Each jet will contain one b-hadron which will decay weakly

★ Because V_{cb} is small $(V_{cb} \approx 0.04)$ hadrons containing b-quarks are relatively long-lived

★ Typical lifetimes of $\tau \sim 1 \times 10^{-12}\,\mathrm{s}$

★ At LEP b-hadrons travel approximately **3mm** before decaying

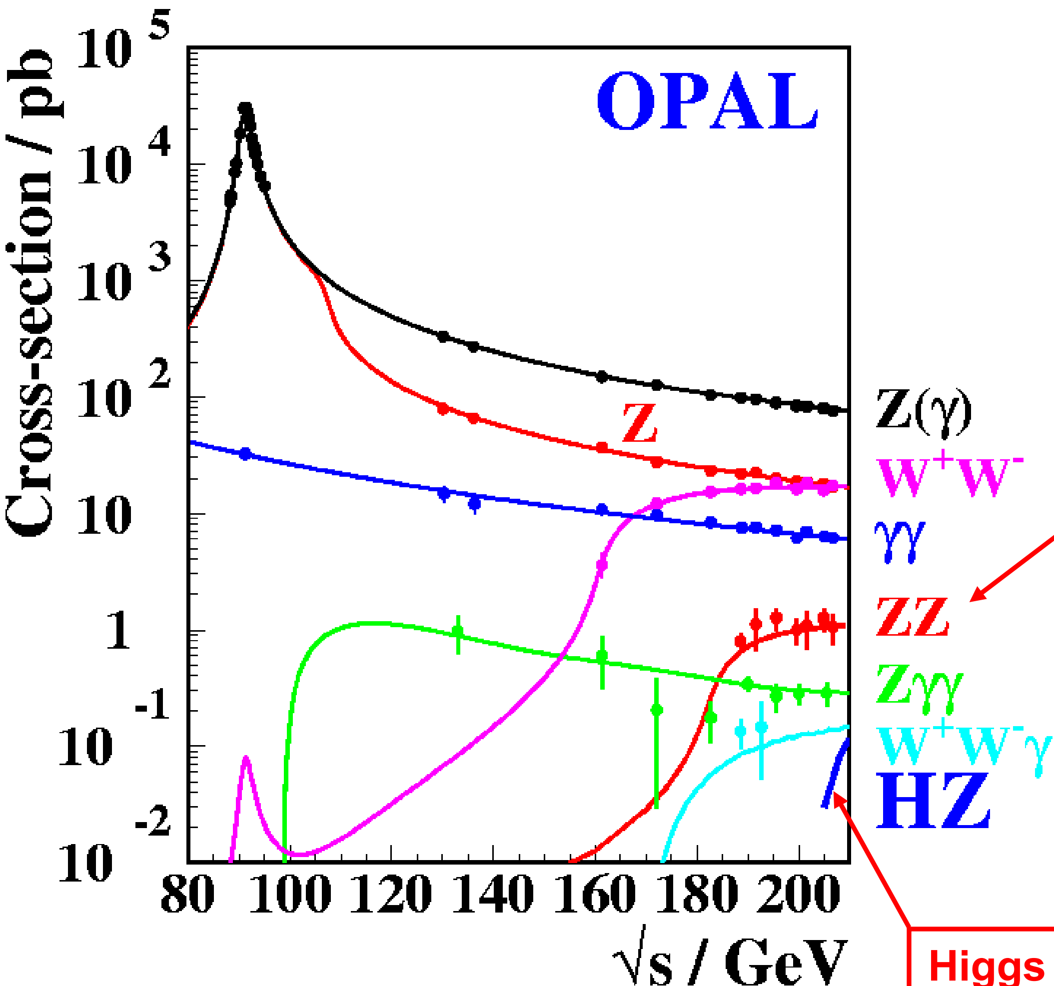




★ Can efficiently identify jets containing b quarks

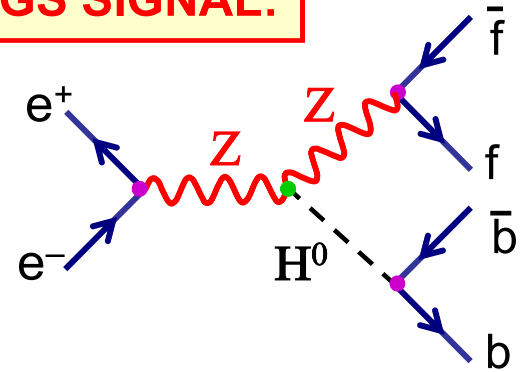
★ Clear experimental signature, but small cross section, e.g. for $m_H \approx 115\,\mathrm{GeV}$ would only produce a few tens of $\mathrm{e}^+\mathrm{e}^- \rightarrow H^0$ events at LEP

★ In addition, there are large “backgrounds”

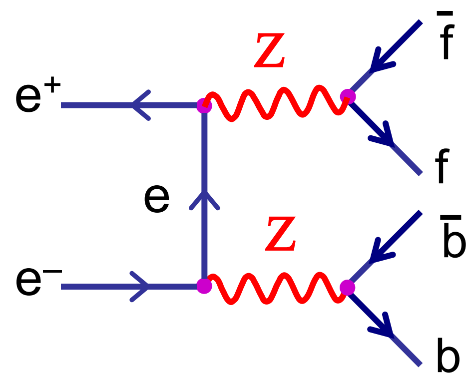


Higgs production cross section (m_H=115 GeV)

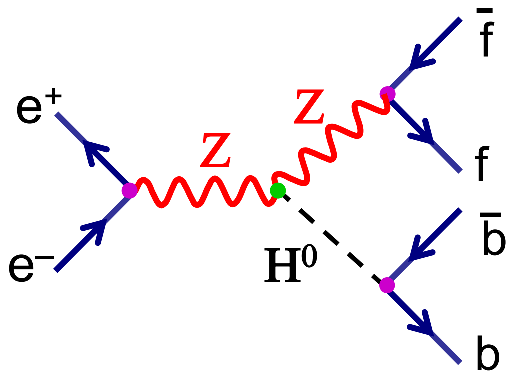
HIGGS SIGNAL:



MAIN BACKGROUND:

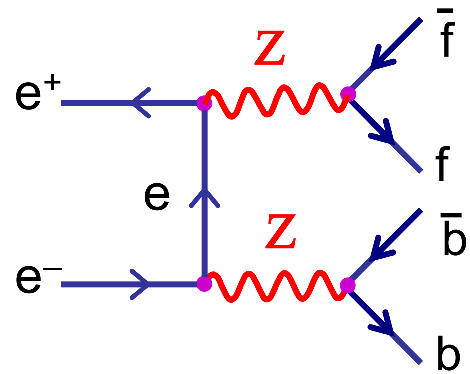


★ The only way to distinguish



$$m_{b\bar{b}} = m_H$$

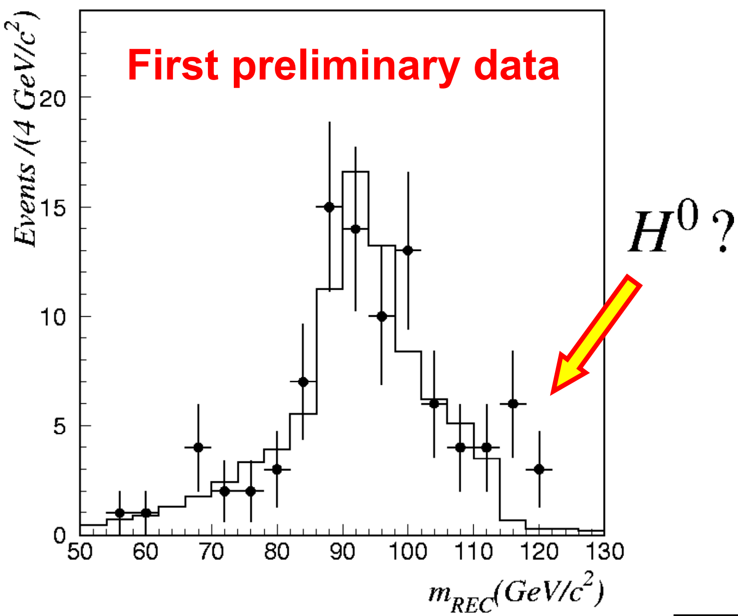
from



$$m_{b\bar{b}} = m_Z$$

is the from the invariant mass of the jets from the boson decays

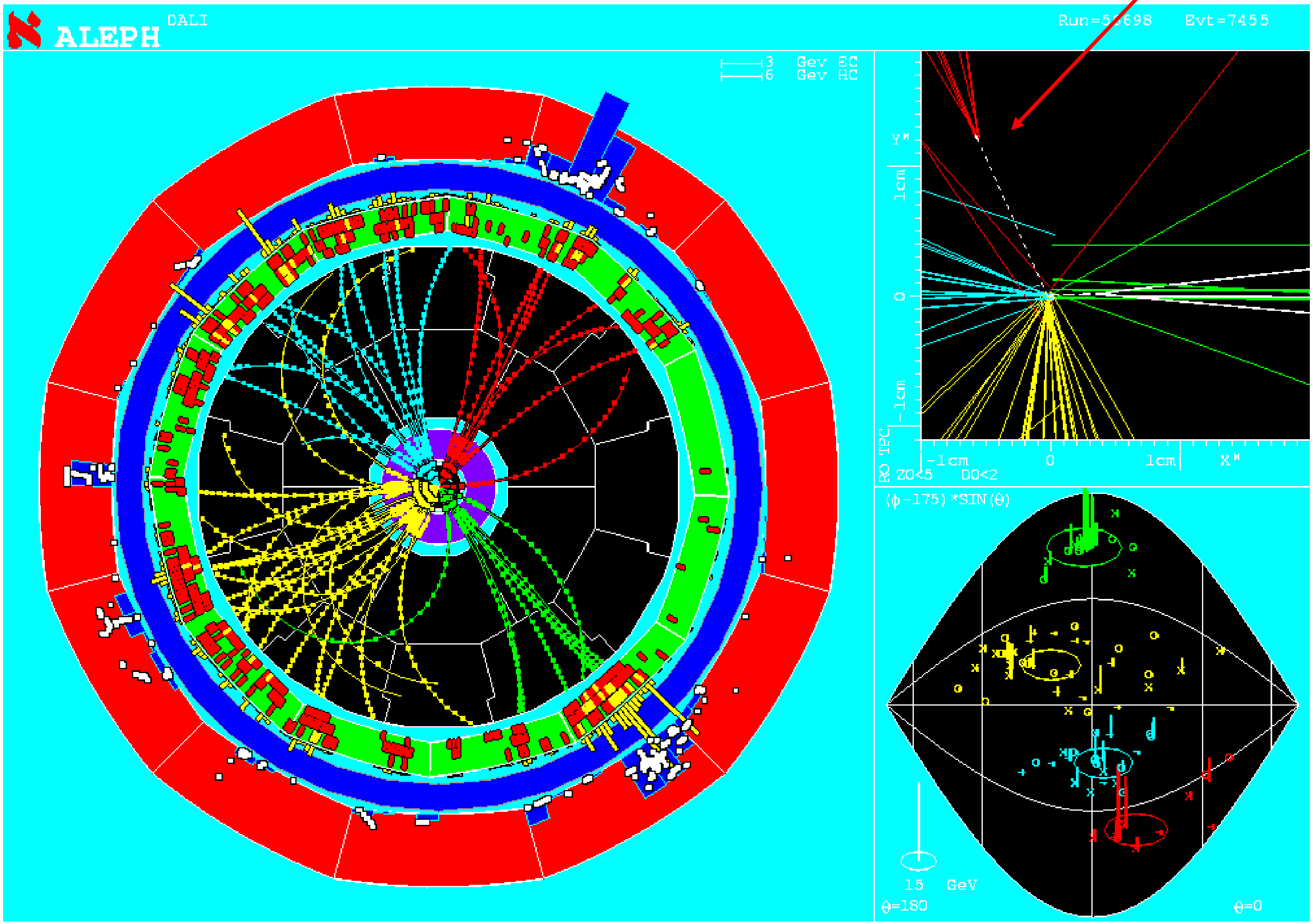
★ In 2000 (the last year of LEP running) the ALEPH experiment reported an excess of events consistent with being a Higgs boson with mass **115 GeV**



- ALEPH found 3 events which were high relative probability of being signal
- L3 found 1 event with high relative probability of being signal
- OPAL and DELPHI found none

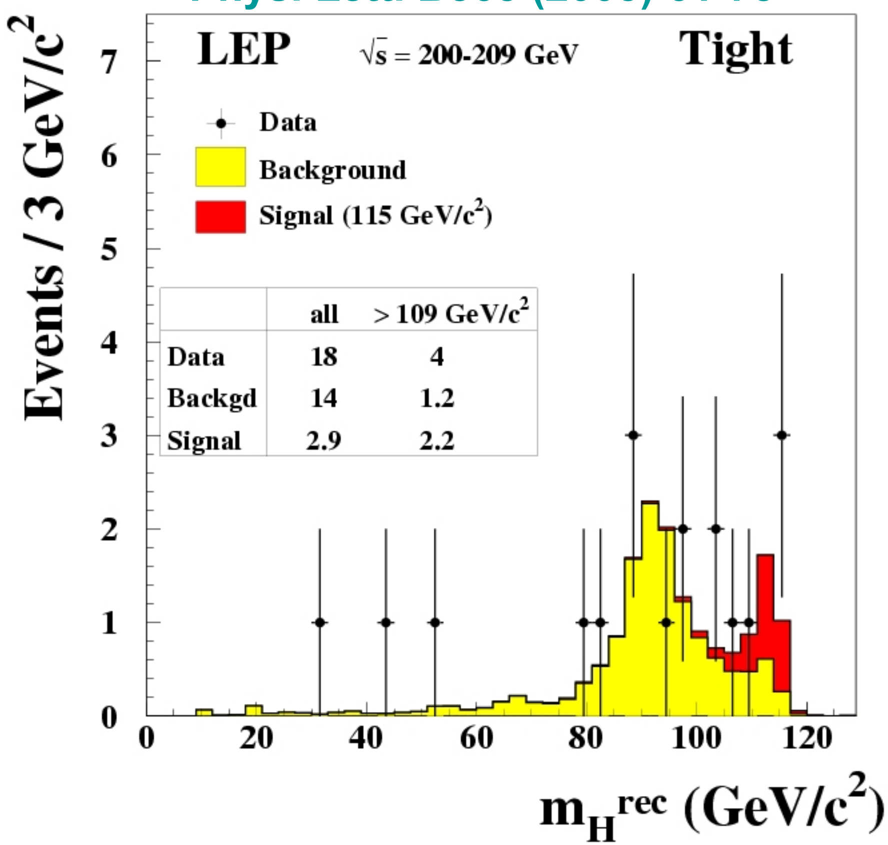
Example event:

Displaced vertex from b-decay



Combined LEP Results

Phys. Lett. B565 (2003) 61-75

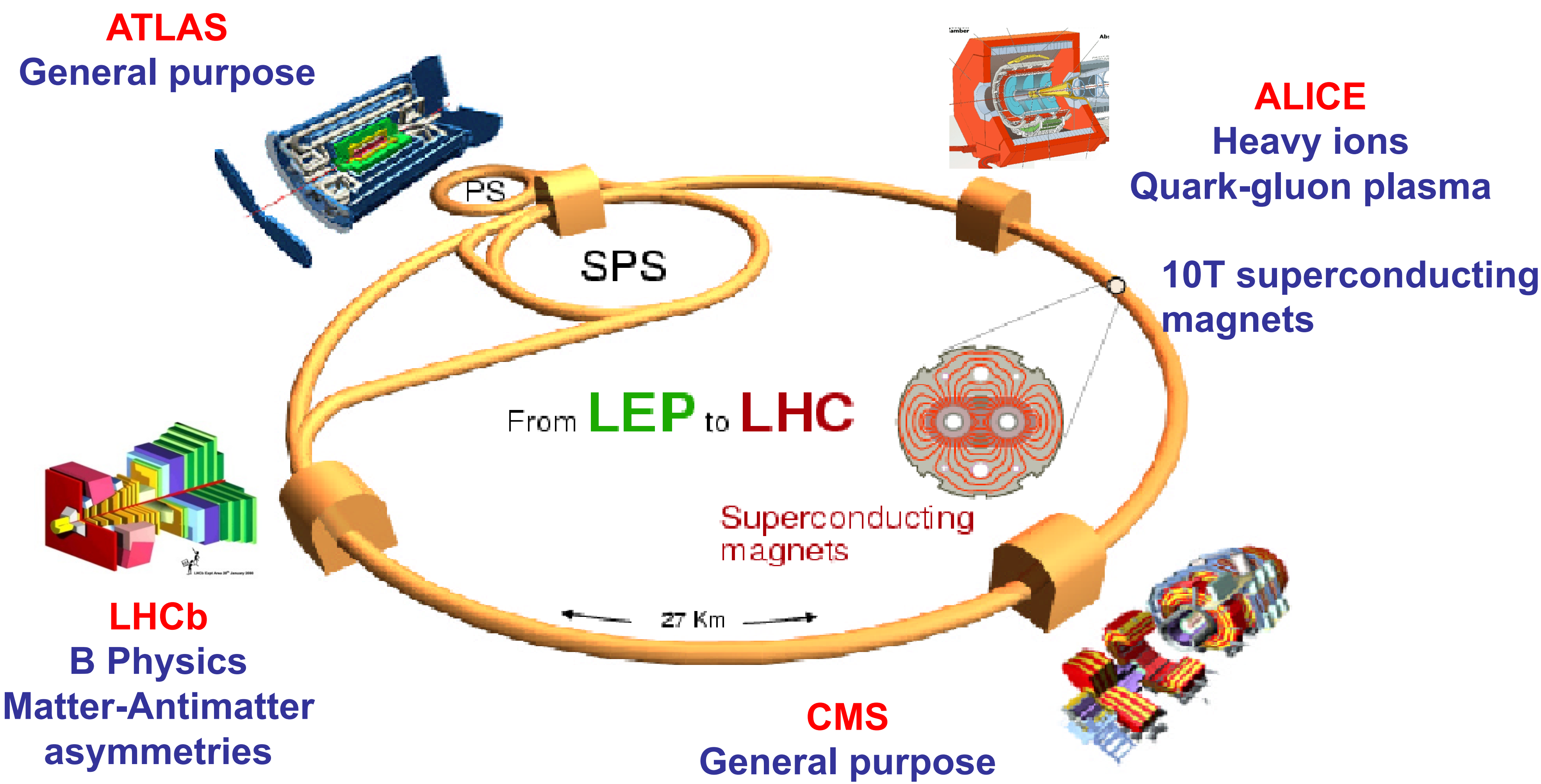


- ★ Final combined LEP results fairly inconclusive
- ★ A hint rather than strong evidence…
- ★ All that can be concluded:

$$\boxed{m_H > 114\,\mathrm{GeV}}$$

The Large Hadron Collider

The LHC is a new proton-proton collider now running in the old LEP tunnel at CERN.

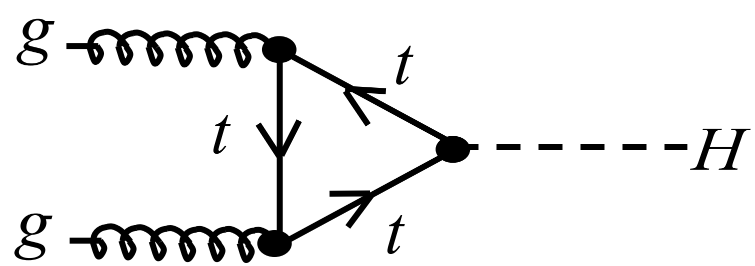


Higgs at Large Hadron Collider

Higgs Production at the LHC

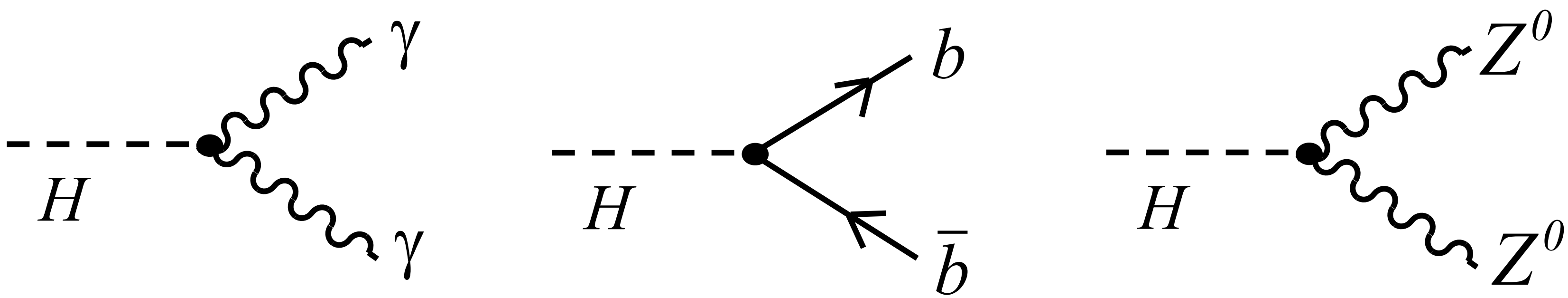
The dominant Higgs production mechanism at the LHC is

"gluon fusion"



Higgs Decay at the LHC

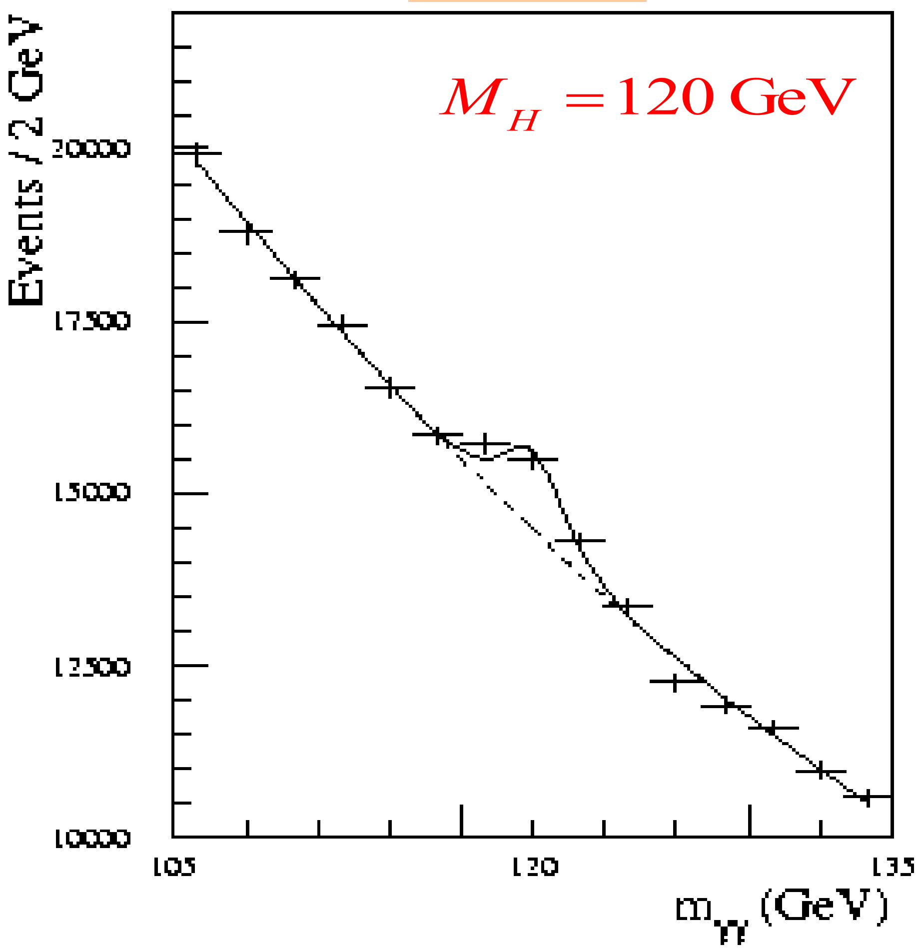
Depending on the mass of the Higgs boson, it will decay in different ways



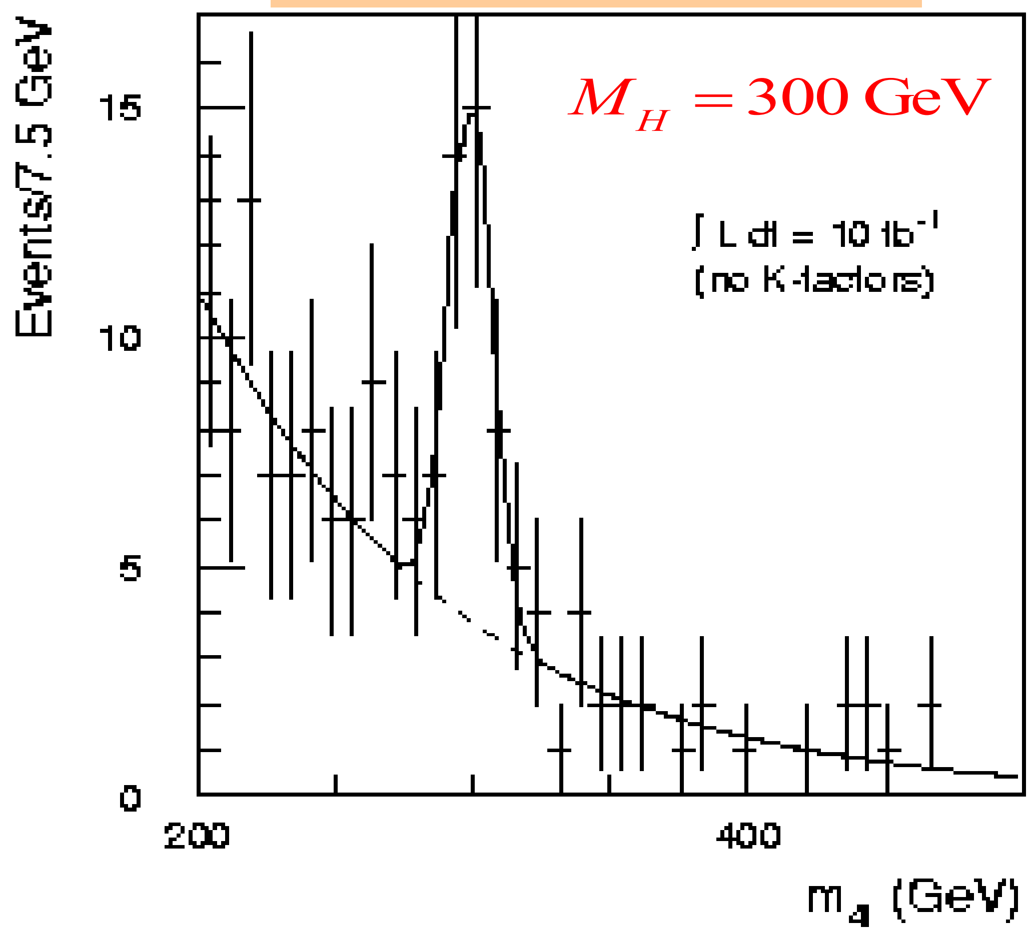
Low Mass **Medium mass** **High mass**

Examples of predicted signals in ATLAS

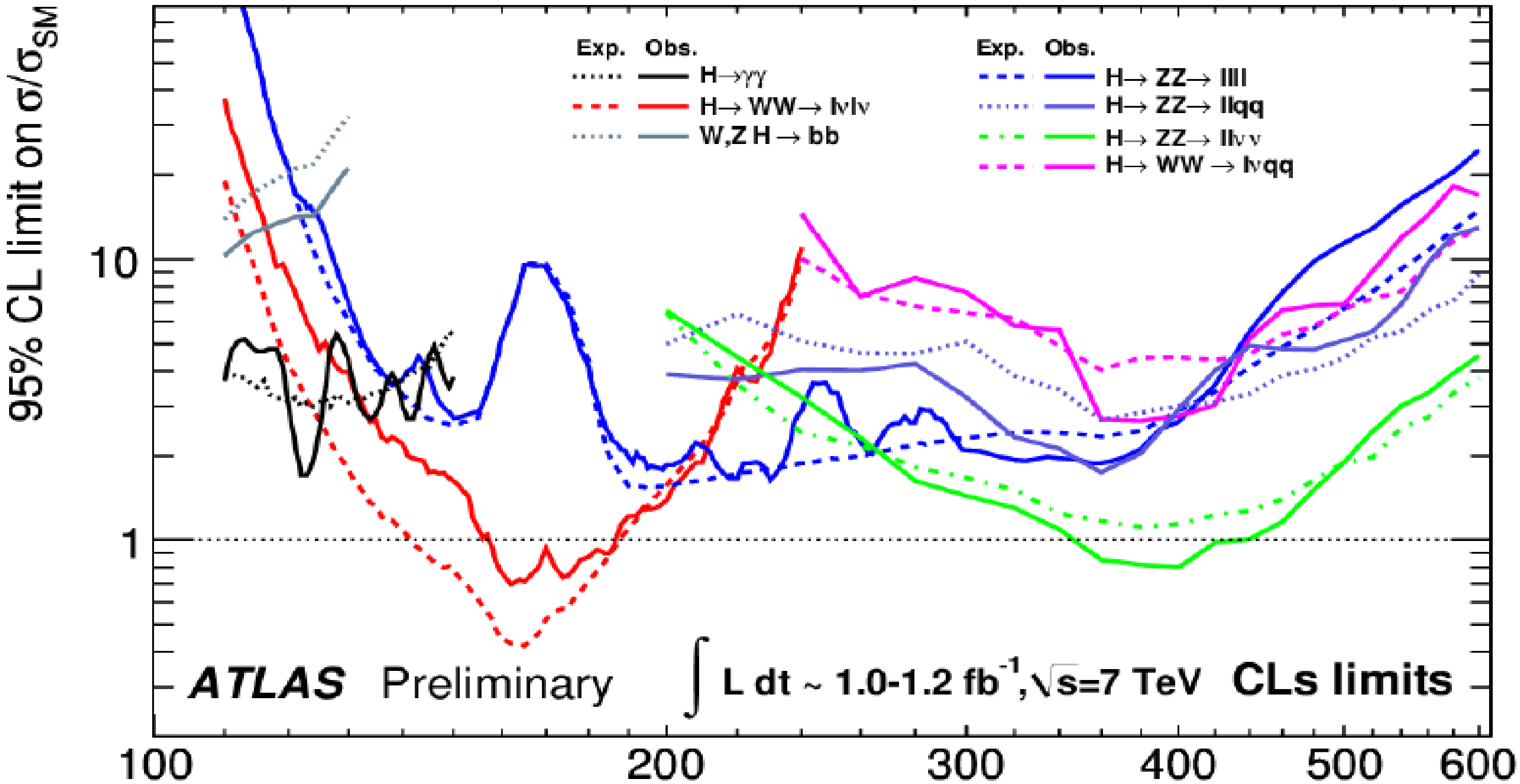
$H \to \gamma\gamma$



$H \to Z^0 Z^0 \to e^+e^-\mu^+\mu^-$

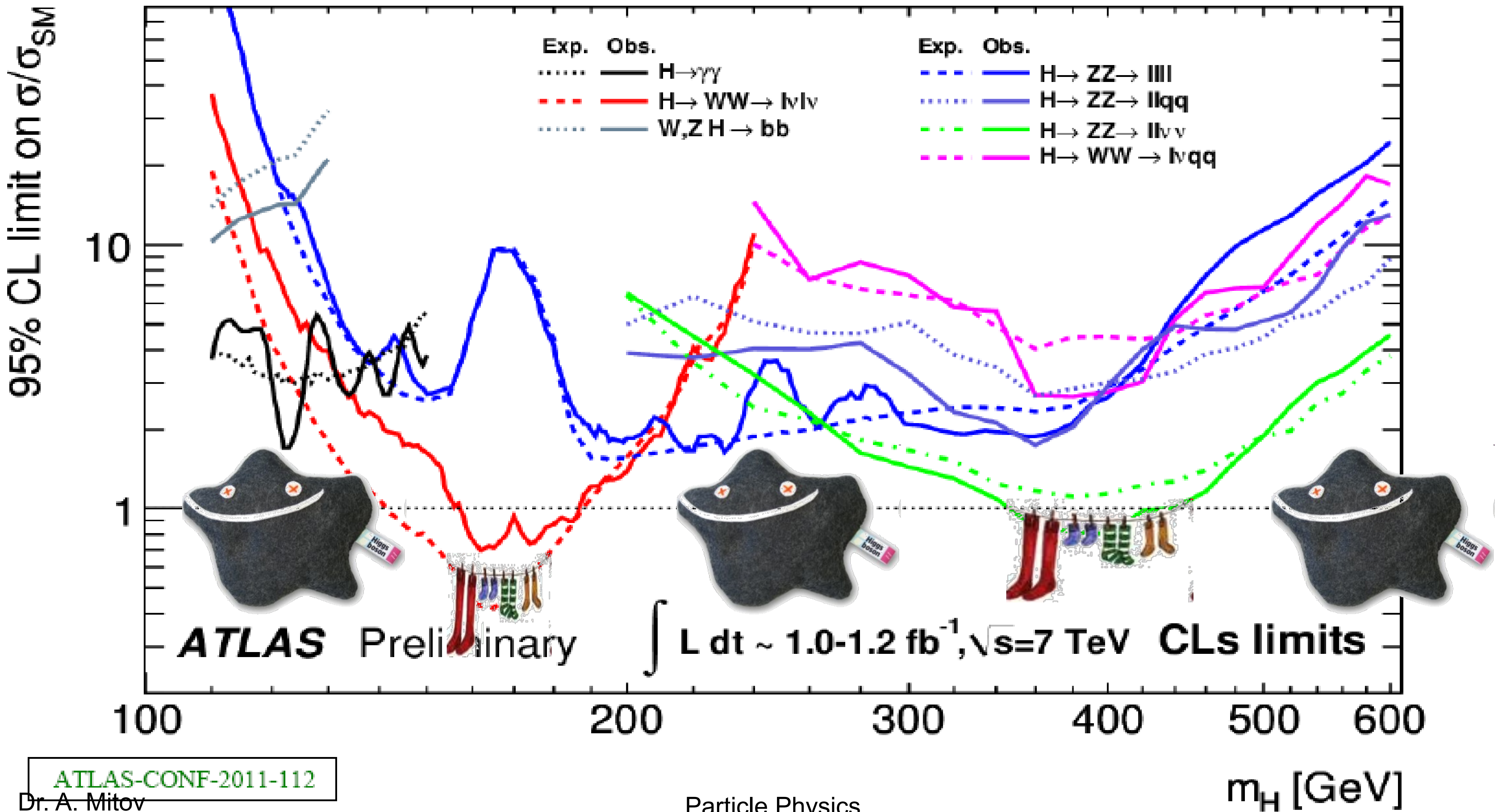


➢ **LHC designed to discover the Higgs boson up to a mass of ~ 1 TeV.**



LHC Higgs data is interpreted in the above plot. For any particular hypothesised Higgs boson mass (shown on the x-axis) the data places (at 95% confidence) an upper bound on the cross section for Higgs-Boson-Like events, in units of "how many would be expected from the Standard Model. In other words, a line level with "10" on the y-axis at mH=125 GeV means "If the Higgs boson has a mass of 125 GeV, then it could have been produced at up to 10 times the rate expected in the Standard Model and could still (just) have gone un-noticed, at 95% confidence".

As data arrives it should lower the curves, unless support from a Higgs boson can prevent curve from passing through dotted line at “1”



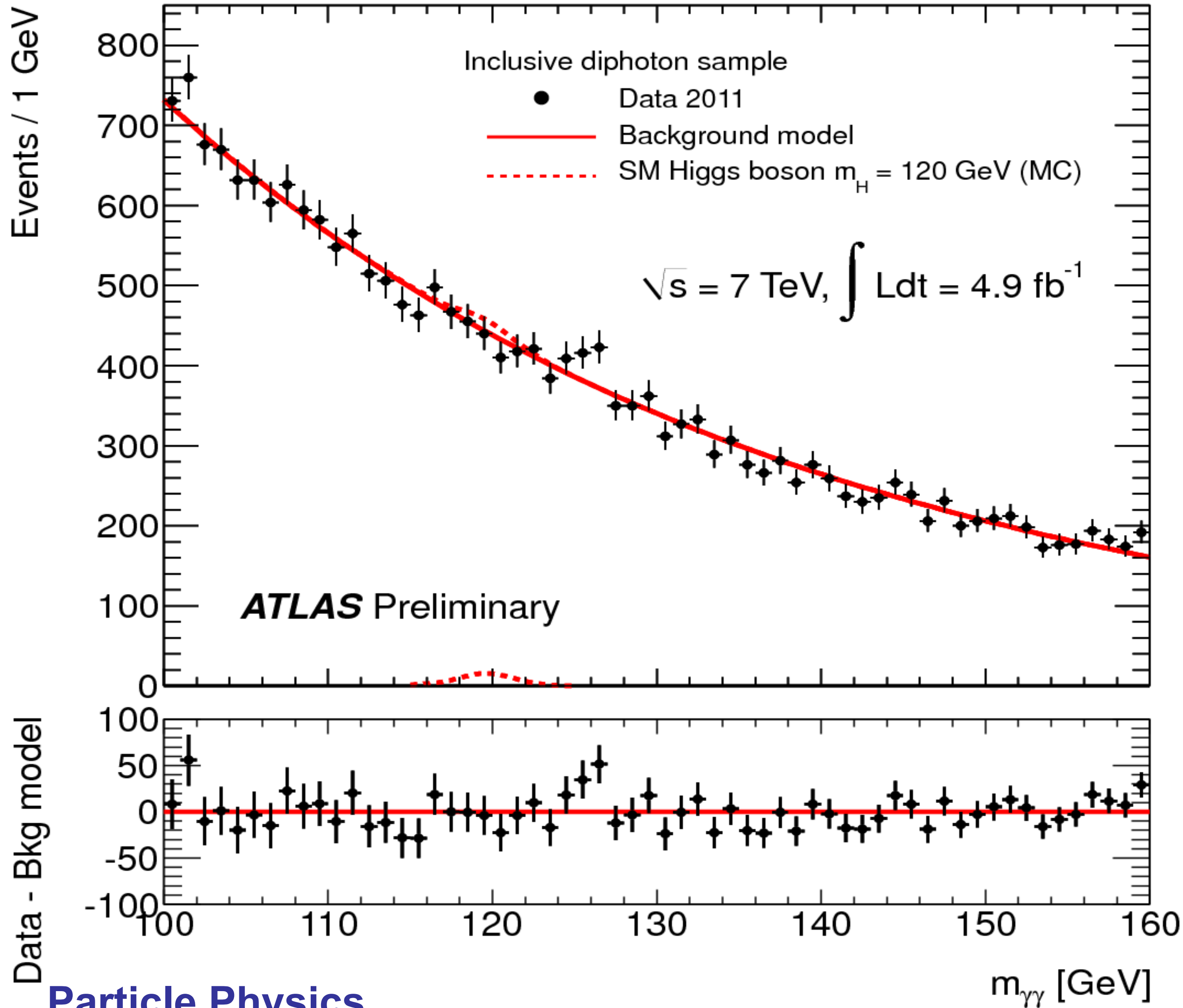
ATLAS-CONF-2011-112

Here is the (unconvincing) data that was shown in Feb 2012

The black blobs are data. The smooth curve is the expected background shape.

The small dotted "bump" indicate how a Higgs signal might change the shape of the distribution if the Higgs boson mass was 120 GeV.

The variable on the x axis is the invariant mass two photons.

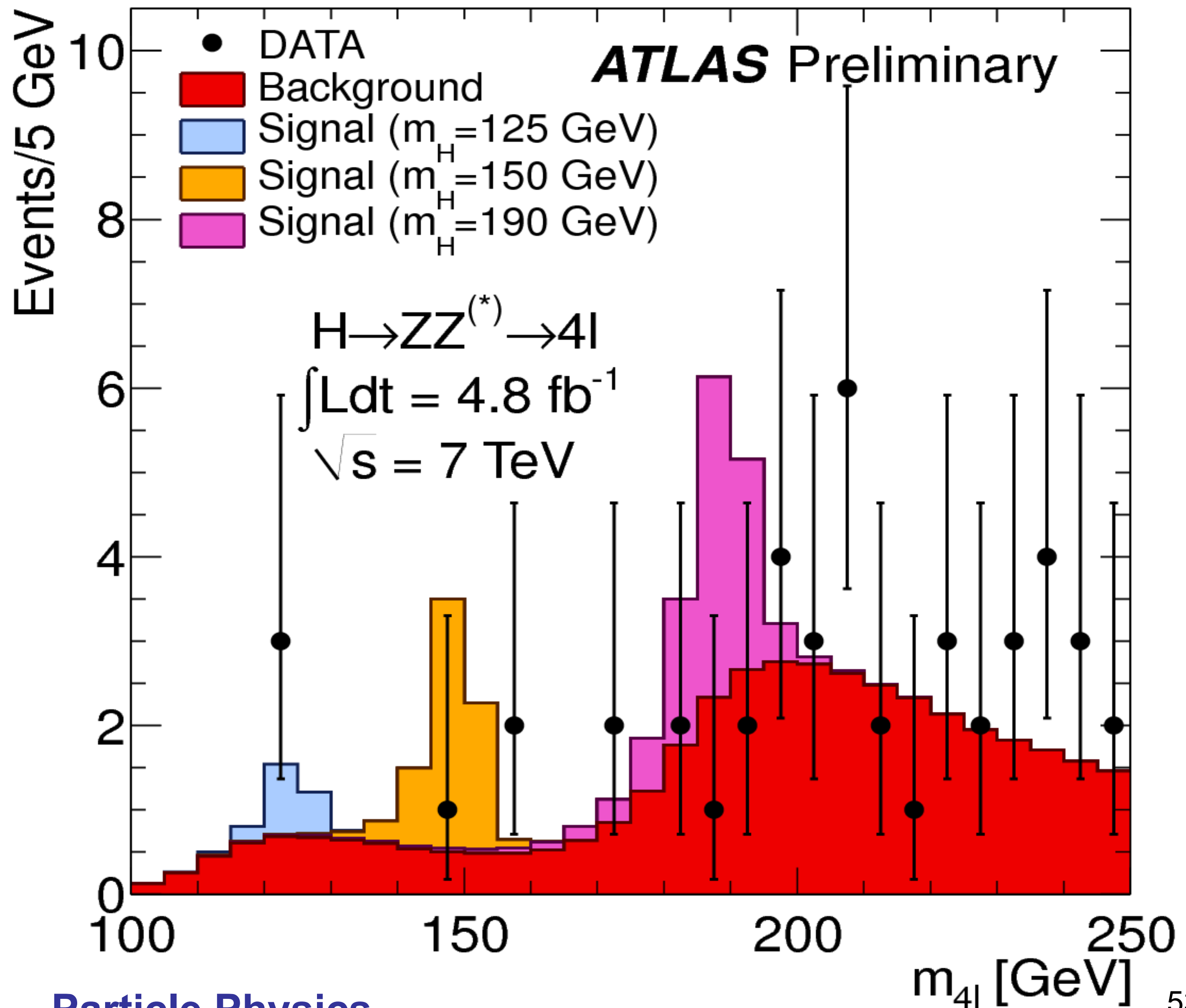


The astonishingly (un?)convincing evidence in the analysis looking for Higgs decays pairs of Z bosons

The black blobs are data.

The three triangular lumps indicate what a Higgs signal might look at (for three different Higgs boson masses).

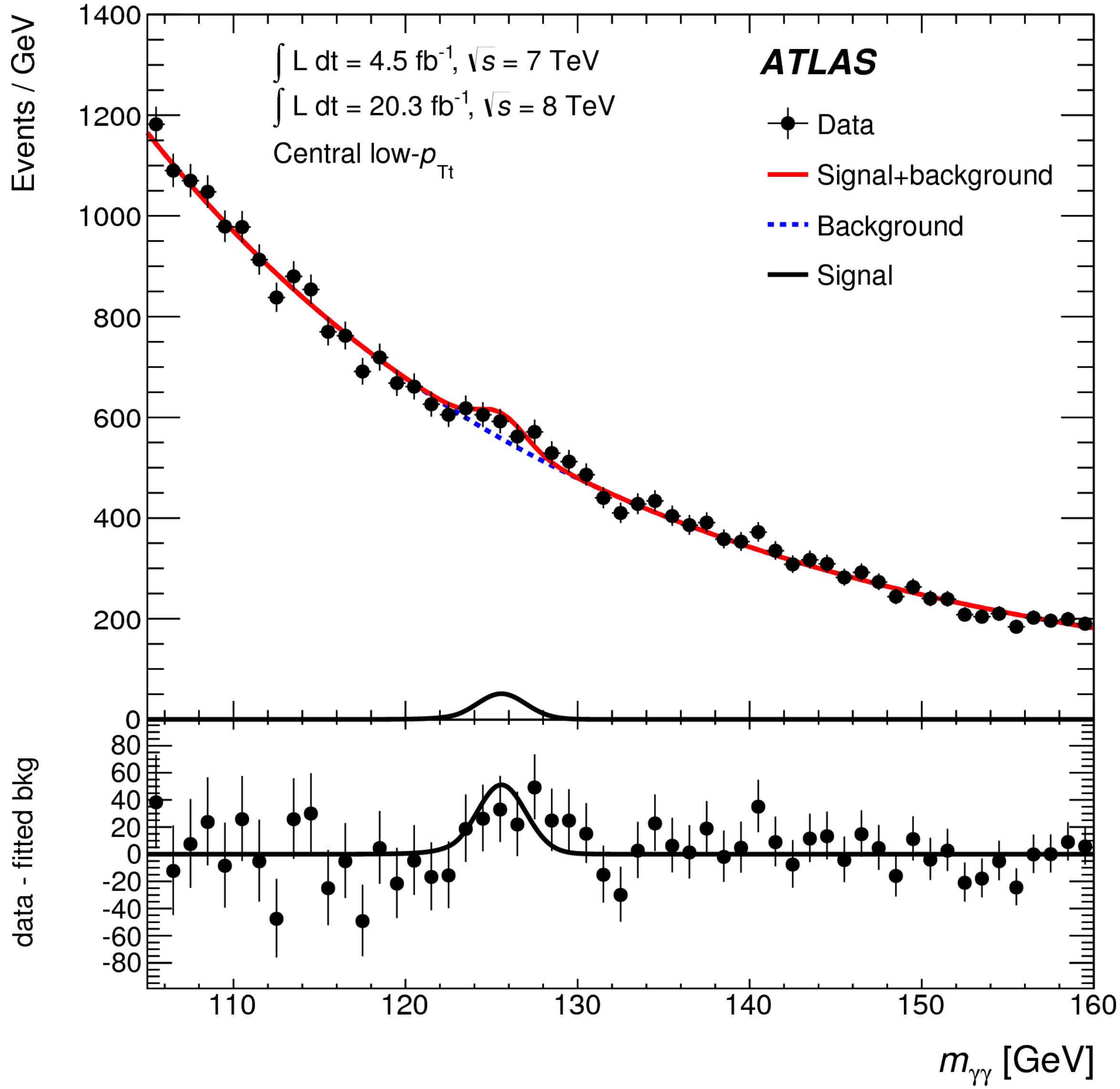
The variable on the x axis is the invariant mass of four leptons which seem to have come from two Z bosons.



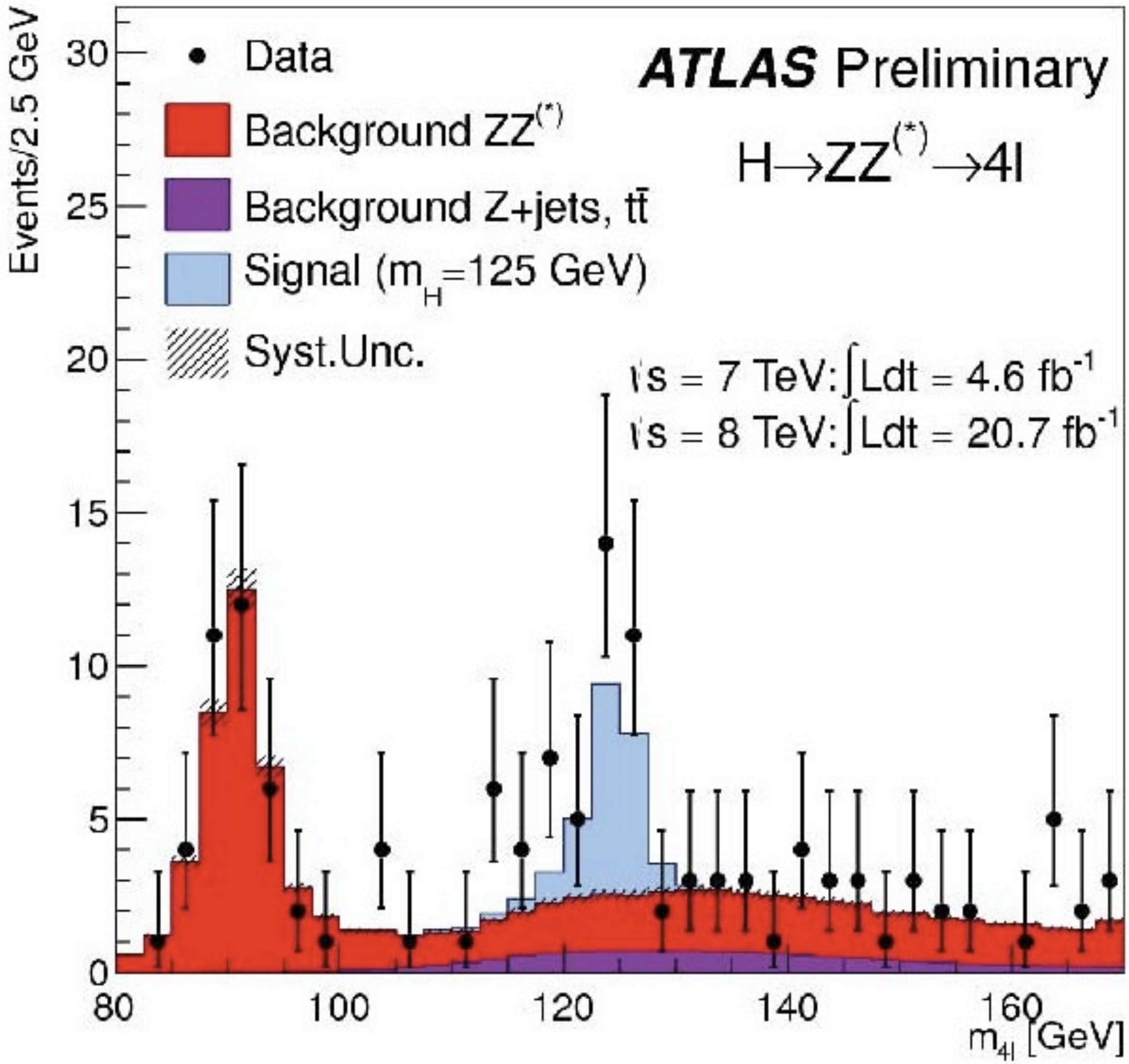
But the Higgs boson is now officially "discovered".

What has the data since then added?

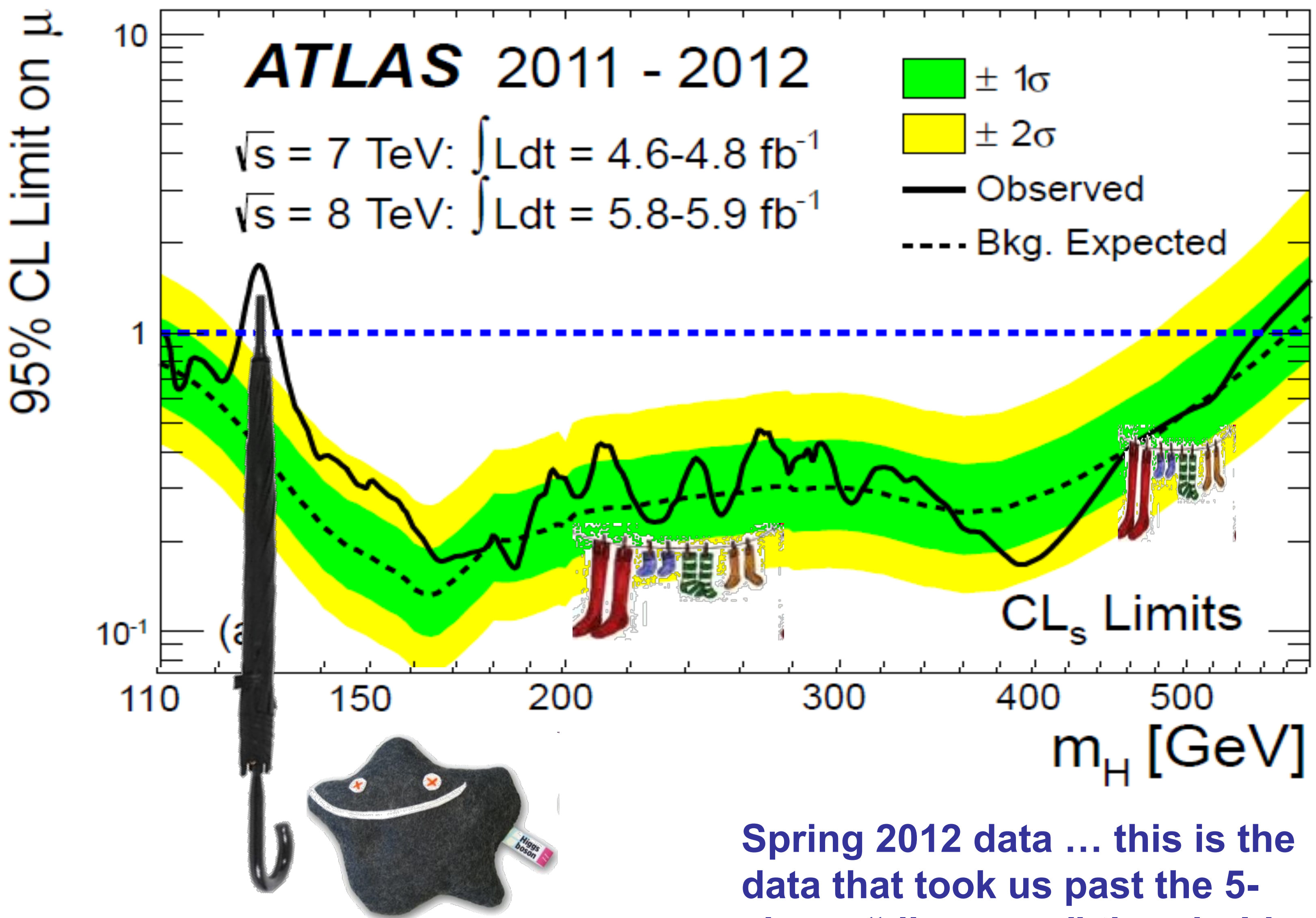
The most recent (2015) public ATLAS data for Higgs turning into two photons



... or turning into four leptons



The discovery plot … $2*10^{-9}$ = probability of fluctuation



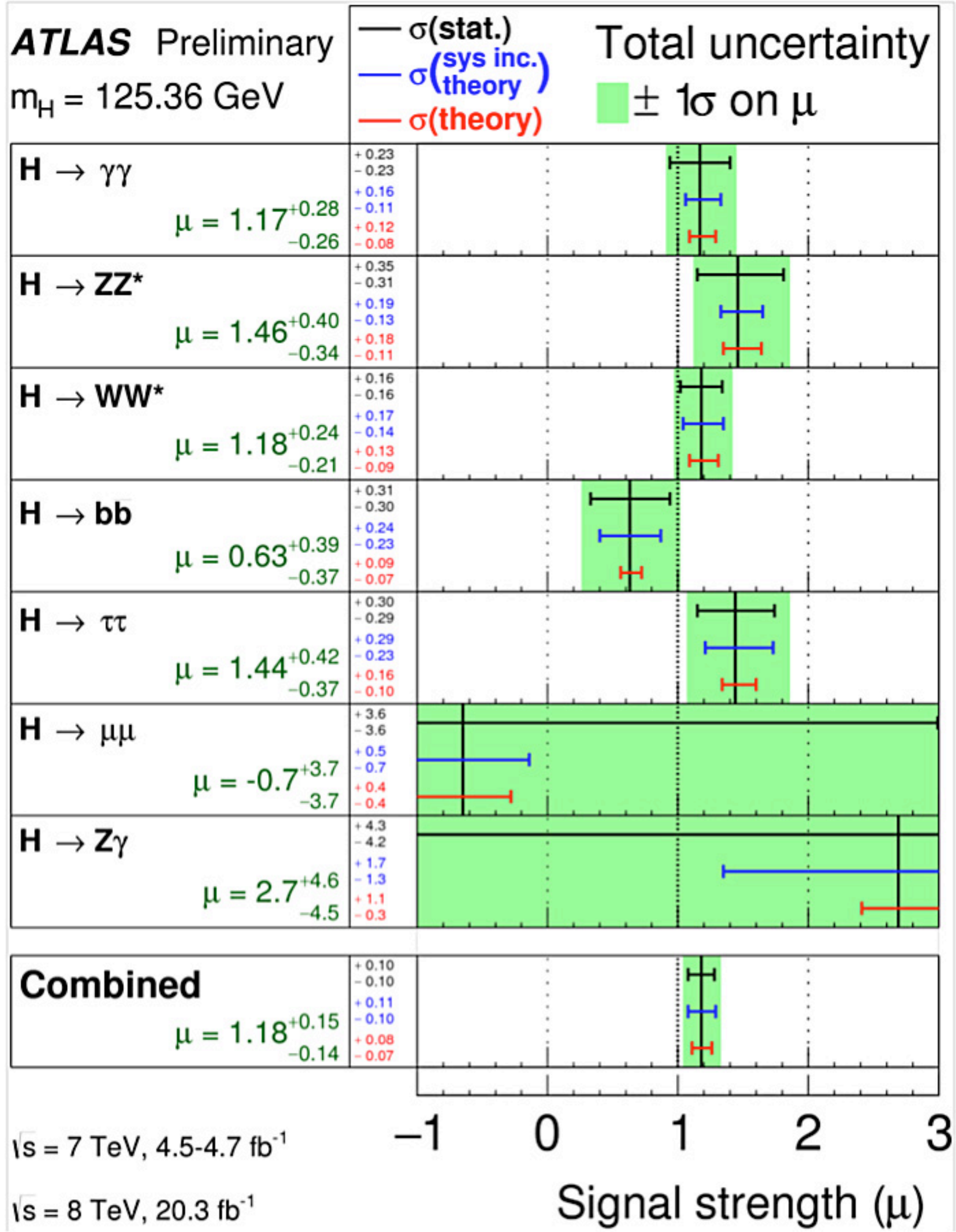
Spring 2012 data … this is the data that took us past the 5-sigma "discovery" threshold

Higgs boson

Now considered to be "discovered". Nobel Prize 2013!

What has been discovered is a bump in the sort of place you'd expect to find a Higgs Boson. In other words, a particle consistent with the Higgs Boson.

To be really sure its "The" Higgs Boson, we are acquiring more information on its spin and couplings (e.g. data shown to the right) . So far everything checks out. The Higgs looks "standard". Nonetheless, other (non-standard) Higgs Bosons could yet be found.

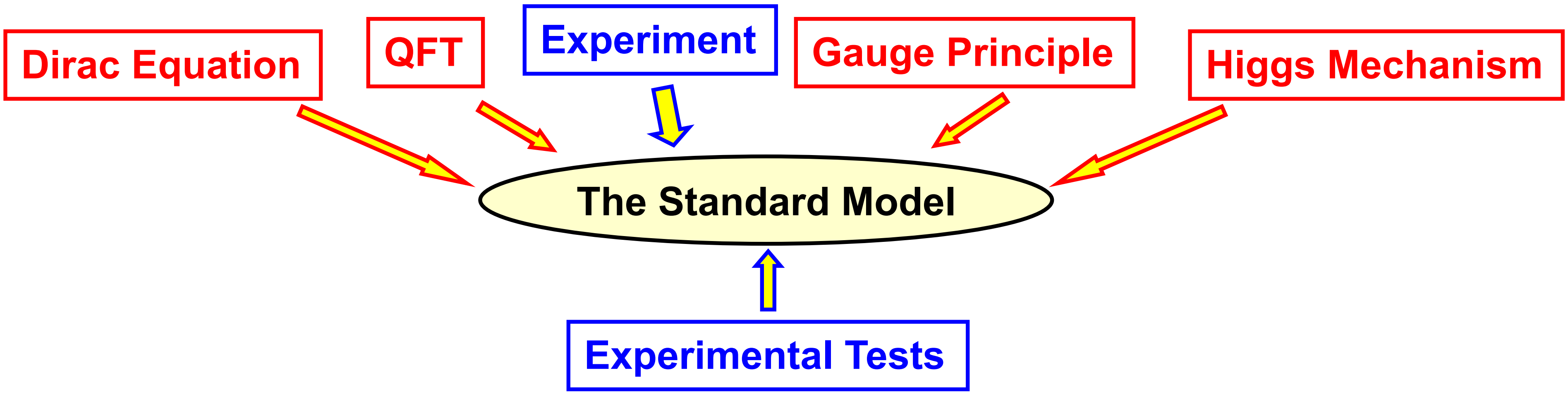


Concluding Remarks

★ In this course (I believe) we have covered almost all aspects of modern particle physics – though in each case we have barely scratched the surface.

★ The Standard Model of Particle Physics is one of the great scientific triumphs of the late 20th century

★ Developed through close interplay of experiment and theory



★ Modern experimental particle physics provides many precise measurements. and the **Standard Model** successfully describes all current data !

★ Despite its great success, we should not forget that it is just a model; a collection of beautiful theoretical ideas cobbled together to fit with experimental data.

★ There are many issues / open questions…

The Standard Model : Problems/Open Questions

- ★ The Standard Model has too many free parameters:

$$m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_e, m_\mu, m_\tau, m_d, m_s, m_b, m_u, m_c, m_t$$

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta \quad + \quad \lambda, A, \rho, \eta \qquad e, G_F, \theta_W, \alpha_S \qquad m_H, \theta_{CP}$$

- ★ Why three generations ?
- ★ Why $SU(3)_c \times SU(2)_L \times U(1)$?
- ★ Unification of the Forces
- ★ Origin of CP violation in early universe ?
- ★ What is Dark Matter ?
- ★ Why is the weak interaction V-A ?
- ★ Why are neutrinos so light ?
- ★ Ultimately need to include gravity

★ **Over the last 25 years particle physics has progressed enormously.**

In the next 10 years we will almost certainly have answers to some of the above questions – maybe not the ones we expect…

The End

Appendix I: Non-relativistic Breit-Wigner

★ For energies close to the peak of the resonance, can write $\sqrt{s} = m_Z + \Delta$

$$s = m_Z^2 + 2m_Z\Delta + \Delta^2 \approx m_Z^2 + 2m_Z\Delta \qquad \text{for} \quad \Delta \ll m_Z$$

so with this approximation

$$\begin{aligned}(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2 \approx (2m_Z\Delta)^2 + m_Z^2\Gamma_Z^2 &= 4m_Z^2(\Delta + \tfrac{1}{4}\Gamma_Z^2) \\ &= 4m_Z^2[(\sqrt{s} - m_Z)^2 + \tfrac{1}{4}\Gamma_Z^2]\end{aligned}$$

★ Giving: $\sigma(e^+e^- \to Z \to f\overline{f}) \approx \dfrac{3\pi}{m_Z^4} \dfrac{s}{(\sqrt{s} - m_Z)^2 + \frac{1}{4}\Gamma_Z^2} \Gamma_e \Gamma_f$

★ Which can be written:

$$\sigma(E) = \frac{g\lambda_e^2}{4\pi} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \frac{1}{4}\Gamma^2} \qquad (3)$$

$\Gamma_i, \; \Gamma_f$: are the partial decay widths of the initial and final states

$E, \; E_0$: are the centre-of-mass energy and the energy of the resonance

$g = \dfrac{(2J_Z+1)}{(2S_e+1)(2S_e+1)}$ is the spin counting factor $g = \dfrac{3}{2\times 2}$

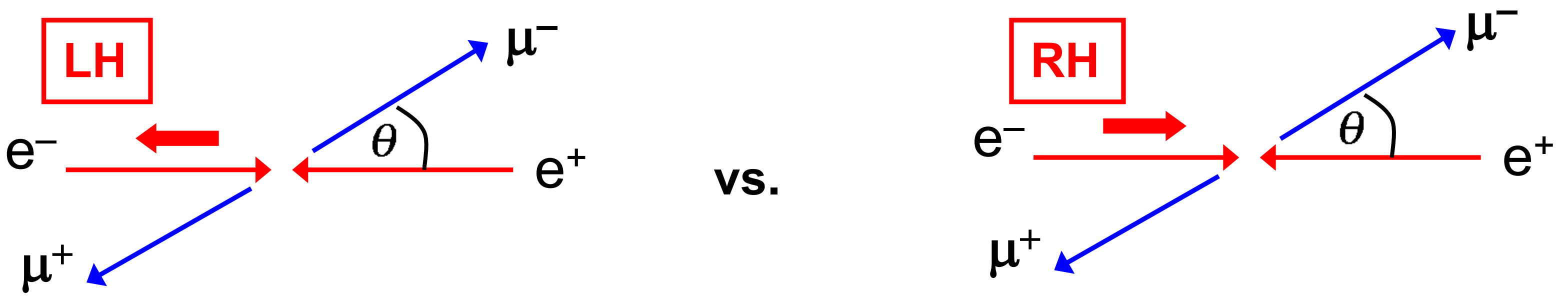
$\lambda_e = \dfrac{2\pi}{E}$: is the Compton wavelength (natural units) in the C.o.M of either initial particle

★ This is the **non-relativistic** form of the **Breit-Wigner** distribution first encountered in the part II particle and nuclear physics course.

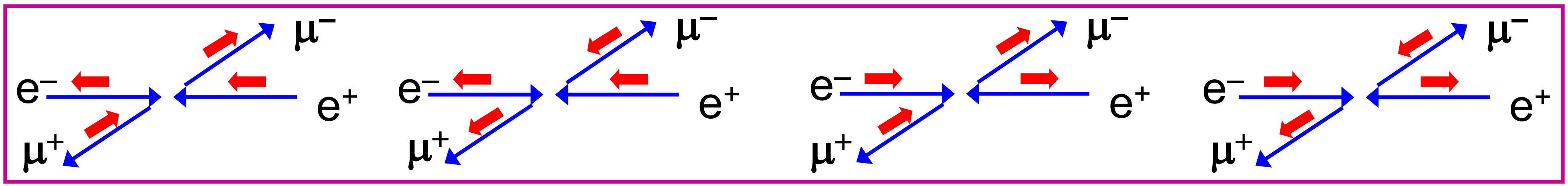
Appendix II: Left-Right Asymmetry, A_{LR}

★ At an e^+e^- linear collider it is possible to produce polarized electron beams e.g. SLC linear collider at SLAC (California), **1989-2000**

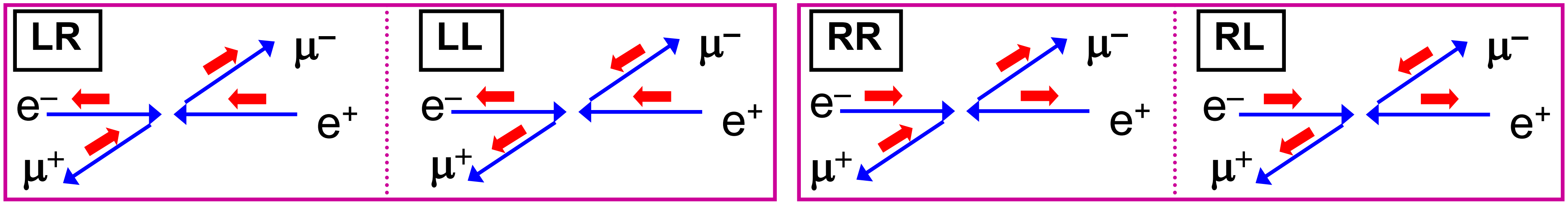
★ Measure cross section for any process for LH and RH electrons separately



- At LEP measure total cross section: sum of 4 helicity combinations:



- At SLC, by choosing the polarization of the electron beam are able to measure cross sections **separately** for LH / RH electrons



★ **Averaging over the two possible polarization states of the positron for a given electron polarization:**

$$\langle|M_L|\rangle^2 = \tfrac{1}{2}(|M_{LL}|^2 + |M_{LR}|^2) \qquad \langle|M_R|\rangle^2 = \tfrac{1}{2}(|M_{RL}|^2 + |M_{RR}|^2)$$

$$\Rightarrow \quad \boxed{\sigma_L = \tfrac{1}{2}(\sigma_{LR} + \sigma_{LL})} \qquad \boxed{\sigma_R = \tfrac{1}{2}(\sigma_{RR} + \sigma_{RL})}$$

★ **Define cross section asymmetry:**

$$\boxed{A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}}$$

★ **Integrating the expressions on page 494 gives:**

$$\sigma_{LL} \propto (c_L^e)^2(c_L^\mu)^2 \quad \sigma_{LR} \propto (c_L^e)^2(c_R^\mu)^2 \quad \sigma_{RL} \propto (c_R^e)^2(c_L^\mu)^2 \quad \sigma_{RR} \propto (c_R^e)^2(c_R^\mu)^2$$

$$\Rightarrow \quad \sigma_L \propto (c_L^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2] \qquad \sigma_R \propto (c_R^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2]$$

$$\boxed{A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e}$$

★ **Hence the Left-Right asymmetry for any cross section depends only on the couplings of the electron**