UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Experimental and Theoretical Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Tuesday 15 January 2013 14.00 to 16.00

MAJOR TOPICS
Paper 140 (Particle Physics)

Answer **two** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains 4 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*
You should use a **separate Answer Book** *for each question.*

STATIONERY REQUIREMENTS
2×20-page answer books
Rough workpad

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Describe the relationship between "the isospin symmetry between the up and down quarks and antiquarks" and "the structure of SU(2) multiplets $\{|I, I_3\rangle\,;\,|I_3| \leq I\}$". Your answer should include a clear description of what a multiplet actually is, and a discussion of the role of ladder operators. Comment on the constraints that isospin symmetry places on the flavour structure of the wavefunctions of light hadrons. [9]

Explain how isospin symmetry extends to the flavour SU(3) symmetry of three quarks (u, d and s), including a discussion of the structure of SU(3) multiplets. [6]

Consider a (fictional) universe, in which particle states ("Bogons") are symmetric under the so-called "Bogus" symmetry. Bogons are indexed by three numbers a, b and c. There are four Bogus ladder operators H_+, H_-, V_+ and V_- which respect the Bogus symmetry. Their action on Bogus states is defined to be:

$$
\begin{aligned}
H_\pm|a, b, c\rangle &= \sqrt{b(b+1) - a(a \pm 1)}\,|a \pm 1, b, c\rangle \\
V_\pm|a, b, c\rangle &= \sqrt{b(b+1) - c(c \pm 1)}\,|a, b, c \pm 1\rangle.
\end{aligned}
$$

You may assume that all Bogus multiplets are connected (i.e. you may assume that it is possible to reach any member of a multiplet from any other member of the same multiplet by application of sufficiently many linear combinations of H_+, H_-, V_+, and V_-) and that every multiplet contains a Bogon represented by $|0, b, 0\rangle$, with b being an integer greater than or equal to one.

Determine the structure of the Bogus multiplets. Specifically:

(i) Determine whether there are any restrictions on a, b and c. Which must be integers? Which need not be? Are any restricted in magnitude? If so by how much? [3]

(ii) Indicate which of the numbers a, b and c label states within a multiplet (as does I_3 in isospin) and which index the multiplets themselves (as does I in isospin). [1]

(iii) Identify the multiplet containing the smallest number of bogons. [2]

(iv) Sketch the multiplet structure of the theory, indicating clearly how many bogons live at any "vertex", and indicating what a general multiplet looks like and how its size relates to any of the parameters a, b and c. [5]

Suppose further that a "Semi-Bogus Mass Formula" predicts that the masses of Bogons satisfy $m(|a, b, c\rangle) = |b - 2| + 3 + |(|a| - |c|)|$. Determine the mass of the lightest Bogus state(s) and how many Bogons share that mass, and indicate where any such state(s) live on your multiplet diagrams. [3]

Can we tell whether the Bogus symmetry is an exact symmetry of the Hamiltonian of the Bogus theory? Explain your reasoning. [1]

2 Top quark pairs are produced in hadron-hadron collisions predominantly via the processes $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$. Draw (in total) three leading-order QCD Feynman diagrams for these processes. [4]

Outline in general terms how the total cross-section for $t\bar{t}$ production can be calculated in terms of integrals over appropriate parton distribution functions, and explain how these distribution functions can be determined experimentally. [6]

The QCD vertex factor associated with the interaction of a quark and a gluon is proportional to $\frac{1}{2}\lambda^a_{ij}$, where $i, j = 1, 2, 3$ (or r,g,b) are colour indices, and λ^a is an SU(3) λ-matrix. Show that the colour factor $C(r\bar{r} \to r\bar{r})$ for the process $q\bar{q} \to t\bar{t}$ is equal to $1/3$, and determine the values of all other non-zero colour factors contributing to the matrix element for that process. [7]

Hence determine the colour factor associated with the $q\bar{q} \to t\bar{t}$ component of the overall cross-section for $t\bar{t}$ production in high-energy hadron-hadron collisions. [3]

A $t\bar{t}$ pair is produced in a proton-proton interaction at the Large Hadron Collider, where two proton beams (beams "1" and "2") each of energy 3.5 TeV collide head-on. The t quark is emitted at a right-angle to the beam direction and has a momentum of 45 GeV. The $\bar{t}$ antiquark is emitted at an angle of 60° to the direction of protons travelling in beam 1. The total momentum of the $t\bar{t}$ system is directed along the direction of beam 1. Calculate the momentum fractions x_1 and x_2 of the partons in beams 1 and 2, respectively, which interacted to produce the $t\bar{t}$ pair. What would be the values of x_1 and x_2 if a $t\bar{t}$ pair with the same four-momentum were to be produced instead in a proton-antiproton collision at the Tevatron, where proton and antiproton beams each of energy 0.98 TeV collide head-on? [*You may neglect the mass of the protons; take the top quark to have a mass of 173 GeV.*] [8]

What is the most likely nature of the partons producing the $t\bar{t}$ pair at the Tevatron and at the LHC? [2]

The standard representation of the SU(3) λ-matrices is:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

3 Write brief notes on **two** of the following topics:

(a) CP-violation in the Standard Model; [15]

(b) electron-proton scattering and measurement of form factors; [15]

(c) helicity, chirality, and the Dirac equation; [15]

(d) Higgs searches and the Higgs boson. [15]

END OF PAPER