UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Tuesday 14 January 2014 14.00 to 16.00

MAJOR TOPICS
Paper 140 (Particle Physics)

Answer **two** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains 4 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*
You should use a **separate Answer Book** *for each question.*

STATIONERY REQUIREMENTS
2 × 20-page answer books
Rough workpad

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Consider the process $e^-_\uparrow e^+_\downarrow \to e^-_\uparrow e^+_\downarrow$, where the electrons have helicity +1 and the positrons have helicity −1. Assume the incident electron and positron move along the $+z$ and $-z$ directions respectively with energy E, that their masses are negligible, and that the outgoing electron has polar angle θ and azimuthal angle $\phi = 0$. Denote the four-momenta of the incoming e^- and e^+ by a^μ and b^μ respectively, and of the outgoing e^- and e^+ by c^μ and d^μ respectively. Make use of the definitions and notation supplied at the end of this question.

(a) Taking M to be the leading order contribution to the matrix element for this process within QED at leading order in the electromagnetic coupling e, draw the diagram(s) which are needed to calculate M. [3]

(b) Use your diagram(s) and your knowledge of the Feynman rules for QED, to write down an expression for $|M|^2$. Leave your answer in terms of spinors and gamma matrices, *etc*. The form of all spinors must be unambiguously specified, including their dependence on the kinematic parameters. [8]

(c) Calculate the four components of the current $j^\mu_{ab} = \bar{\psi}_a \gamma^\mu \psi_b$ in terms of the complex numbers a_1, a_2, b_1 and b_2 (which need not be related to a^μ or b^μ) given that ψ_a and ψ_b are spinors having the particular form

$$\psi_a = \begin{pmatrix} a_1 \\ a_2 \\ a_1 \\ a_2 \end{pmatrix} \quad \text{and} \quad \psi_b = \begin{pmatrix} b_1 \\ b_2 \\ b_1 \\ b_2 \end{pmatrix}.$$

[4]

(d) Assuming that the currents j^μ_{ab} and j^ν_{cd} are both defined in terms of spinors with the form just considered, Show that the product

$$P_{abcd} = j^\mu_{ab} j^\nu_{cd} g_{\mu\nu}$$

can be expressed as

$$P_{abcd} = 8(a_1^* b_1 c_2^* d_2 + a_2^* b_2 c_1^* d_1) - 8(a_1^* b_2 c_2^* d_1 + a_2^* b_1 c_1^* d_2) .$$ [4]

(e) Hence, or otherwise, explicitly evaluate $|M|^2$ in terms of the variables e, E, θ, s, t, u, where the Mandelstam variables: $s = (a^\mu + b^\mu)^2$, $t = (a^\mu - c^\mu)^2$, $u = (a^\mu - d^\mu)^2$, are related by $s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2 \approx 0$ in this case. [4]

Comment on the physical meaning of your result, including the angular distribution. [3]

(f) Re-express your answer for $|M|^2$ just in terms of the coupling, e, and the two Mandelstam variables s and t. Leave your answer in the form

$$|M|^2 = Ae^4(s+t)^B \left(\frac{1}{s} + C\frac{1}{t}\right)^D$$

where A, B, C and D are constants which you should determine. [4]

In the lecture course, the gamma matrix and spinor conventions used were:

$$\gamma^0 = \begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix},$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} \hat{c} \\ e^{i\phi}\hat{s} \\ \alpha\hat{c} \\ \alpha e^{i\phi}\hat{s} \end{pmatrix}, \quad u_\downarrow = \sqrt{E+m}\begin{pmatrix} -\hat{s} \\ e^{i\phi}\hat{c} \\ \alpha\hat{s} \\ -\alpha e^{i\phi}\hat{c} \end{pmatrix},$$

$$v_\uparrow = \sqrt{E+m}\begin{pmatrix} \alpha\hat{s} \\ -\alpha e^{i\phi}\hat{c} \\ -\hat{s} \\ e^{i\phi}\hat{c} \end{pmatrix}, \quad v_\downarrow = \sqrt{E+m}\begin{pmatrix} \alpha\hat{c} \\ \alpha e^{i\phi}\hat{s} \\ \hat{c} \\ e^{i\phi}\hat{s} \end{pmatrix},$$

where $\hat{c} = \cos(\frac{\theta}{2})$, $\hat{s} = \sin(\frac{\theta}{2})$, $\alpha = \frac{|\boldsymbol{p}|}{E+m}$, *and where* θ *and* ϕ *are the usual spherical angles (polar and azimuthal respectively) and* E, $\boldsymbol{p}$ *and* m *are the energy, momentum and mass of the particle (or antiparticle) in question.*

2 (a) The K^0 and $\bar{K}^0$ mesons have quark contents $d\bar{s}$ and $s\bar{d}$ respectively, and belong to a meson nonet with quantum numbers $J^{PC} = 0^{-+}$. Explain how CP eigenstates K_1 and K_2 can be constructed in the $K^0 - \bar{K}^0$ system, and explain how, neglecting CP violation, these can be related to the states K_S and K_L. [5]

Draw Feynman diagrams for the allowed decays of K^0 and $\bar{K}^0$ mesons into the final states $\pi^+ e^- \bar{\nu}_e$ and $\pi^- e^+ \nu_e$. A beam of neutral kaons of momentum 100 GeV is produced in an initial state which is known to consist purely of K^0 mesons. At a point a distance 17.8 m downstream of the production point the decay rate into $\pi^+ e^- \bar{\nu}_e$ is observed to be equal to the decay rate into $\pi^- e^+ \nu_e$. This equality in rates is not seen anywhere closer to the production point. Neglecting the effects of CP violation, derive expressions for the dependence of the $\pi^+ e^- \bar{\nu}_e$ and $\pi^- e^+ \nu_e$ decay rates on the distance along the beam direction, and estimate the difference Δm between the K_S and K_L masses. Show also, neglecting CP violation, that at large distances down the beam line (where the K_S fraction will have died away completely) the decay rates $\Gamma(K_L \to \pi^- e^+ \nu_e)$ and $\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)$ would be expected to be equal. [8]

If a small degree of CP violation is introduced by allowing the K_L meson to be a state of the form

$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(K_2 + \epsilon K_1),$$

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then the expected value of the asymmetry

$$\delta = \frac{\Gamma(\mathrm{K_L} \to \pi^- \mathrm{e}^+ \nu_\mathrm{e}) - \Gamma(\mathrm{K_L} \to \pi^+ \mathrm{e}^- \bar{\nu}_\mathrm{e})}{\Gamma(\mathrm{K_L} \to \pi^- \mathrm{e}^+ \nu_\mathrm{e}) + \Gamma(\mathrm{K_L} \to \pi^+ \mathrm{e}^- \bar{\nu}_\mathrm{e})}$$

could be non-zero. Derive an expression for δ in terms of ϵ. [8]

(b) By the use of graphical outer products acting on general SU(3) multiplets, or otherwise, decompose the product $3 \otimes \bar{3} \otimes 3 \otimes \bar{3}$ of SU(3) colour triplets into a direct sum of other irreducible SU(3) colour multiplets. [*You may wish to start by proving that* $3 \otimes \bar{3} = 8 \oplus 1$, *and then proceed to 'square' this result.*] [7]

Comment on the implications of your result when combined with the colour confinement hypothesis. [2]

3 Write detailed notes on **one** of the following topics:

(a) elastic, inelastic and deep inelastic scattering, and their use in determining the structure of hadrons; [30]

(b) our understanding of the neutrino sector of the Standard Model, from both experimental and theoretical perspectives. [*Do not include neutrino deep inelastic scattering in your answer.*] [30]

END OF PAPER