UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics	NST3PHY
MASTER OF ADVANCED STUDY IN PHYSICS	MAPY
NATURAL SCIENCES TRIPOS: Part III Astrophysics	NST3AS

Tuesday 13 January 2015 14.00 to 16.00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

Answer **two** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains 4 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*
You should use a **separate Answer Book** *for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20-page answer books	Mathematical formulae handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Consider the decay $\pi^+ \to \mu^+\nu_\mu$ where the π^+ has quark content $u\bar{d}$.

(a) Draw a Feynman diagram for this decay. [3]

(b) Draw a diagram showing the momentum and spin directions of the outgoing particles in the centre-of-mass frame, explaining clearly the reasons for your choice of spin state. [3]

(c) The lepton current for the final state can be written as

$$\bar{u}(p_3)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)v(p_4)$$

where p_3 is the four-momentum of the ν_μ and p_4 is that of the μ^+. Forms for the γ-matrices and spinors can be found at the end of the question. Show that the magnitude of the lepton current is proportional to

$$\frac{(E+m-p)\sqrt{p}}{\sqrt{E+m}}$$

where E, m and p are the energy, the mass and the magnitude of the three-momentum of the μ^+. [*You are* **not** *required to compute the components of the current.*] [6]

(d) Given that the two-body decay rate is

$$\Gamma = \frac{p}{8\pi m_\pi^2}\left\langle |M|^2 \right\rangle,$$

where m_π is the mass of the pion, and M is the matrix element for the decay, estimate the ratio

$$\frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)}.$$

The momentum of the muon emitted in the pion decay is 30 MeV while that of electron is 70 MeV. Their respective masses are 105.6 MeV and 0.511 MeV. [6]

(e) Explain how measurements of the decay

$$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^* + e^- + \bar{\nu}_e$$

can be used to determine whether or not the laws of nature are symmetric under parity. To what extent do forward-backward asymmetries measured at LEP test for the presence (or absence) of the same spatial symmetry? [12]

[*In the lecture course, the gamma matrix and spinor conventions used were:*

$$\gamma^0 = \begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3,$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} \hat{c} \\ e^{i\phi}\hat{s} \\ \alpha\hat{c} \\ \alpha e^{i\phi}\hat{s} \end{pmatrix}, \quad u_\downarrow = \sqrt{E+m}\begin{pmatrix} -\hat{s} \\ e^{i\phi}\hat{c} \\ \alpha\hat{s} \\ -\alpha e^{i\phi}\hat{c} \end{pmatrix},$$

$$v_\uparrow = \sqrt{E+m}\begin{pmatrix} \alpha\hat{s} \\ -\alpha e^{i\phi}\hat{c} \\ -\hat{s} \\ e^{i\phi}\hat{c} \end{pmatrix}, \quad v_\downarrow = \sqrt{E+m}\begin{pmatrix} \alpha\hat{c} \\ \alpha e^{i\phi}\hat{s} \\ \hat{c} \\ e^{i\phi}\hat{s} \end{pmatrix},$$

where $\hat{c} = \cos\left(\frac{\theta}{2}\right)$, $\hat{s} = \sin\left(\frac{\theta}{2}\right)$, $\alpha = \dfrac{|\boldsymbol{p}|}{E+m}$, *and where* θ *and* ϕ *are the usual spherical angles (polar and azimuthal respectively) and* E, $\boldsymbol{p}$ *and* m *are the energy, momentum and mass of the particle (or antiparticle) in question.*]

2 Write detailed notes on **one** of the following topics:

(a) CP violation in the neutral Kaon system; [30]

(b) experimental and theoretical aspects of neutrino oscillations. [30]

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3 Consider a process in which two unit-charge, spin-½ particles, treated as point-like, scatter via t-channel photon exchange. Assume that in the laboratory frame the target particle is initially at rest and has mass $M \neq 0$, while the projectile is massless and has energy E_1. Assume that the differential cross-section for (elastic or inelastic) scattering of the projectile into an element of solid angle $\mathrm{d}\Omega$ in the laboratory frame via such a process can be written:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \left(\frac{E_3}{E_1}\right) \left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

where θ is the scattering angle and E_3 the final energy of the projectile, α is the fine structure constant, and q^2 is the Lorentz-invariant quantity $q^\mu q_\mu$ constructed from the four-momentum transfer q^μ. In elastic scattering, the target does not break up.

(a) If $\nu = E_1 - E_3$, show that for elastic scattering: $\nu + \dfrac{q^2}{2M} = 0$. [4]

(b) Prove that the differential cross-section given above may be obtained by integrating the following expression over all values of E_3:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_3\mathrm{d}\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right] \delta\left(\nu + \frac{q^2}{2M}\right).$$

and give a physical interpretation of the constraint provided by the delta-function. [*You may wish to use the result* $\int \delta(g(x))\mathrm{d}x = \int \left|\frac{\mathrm{d}g}{\mathrm{d}x}\right|^{-1} \delta(x - x_0)\mathrm{d}x$, *which is valid when* $g(x)$ *has a single zero at* x_0.] [6]

(c) If the target particle is not point-like, but has internal structure, the differential cross-section can instead be written in the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_3\mathrm{d}\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \left[\frac{F_2(\nu, q^2)}{\nu}\cos^2\left(\frac{\theta}{2}\right) + \frac{2F_1(\nu, q^2)}{M}\sin^2\left(\frac{\theta}{2}\right)\right],$$

in terms of structure functions F_1 and F_2 describing the target. Derive the forms of F_1 and F_2 for a proton of mass M assuming a parton model in which the proton is *defined* to contain just two u-quarks and one d-quark, each being indestructable, each carrying a fraction exactly $x = \frac{1}{3}$ of the four-momentum of the proton, and each having a small mass $m \neq 0$ which you should determine. [8]

(d) Describe two experiments in which electron-hadron differential cross-sections were measured. Some marks will be awarded for description of the apparatus used in each. Explain to what extent the results agreed with the above prediction, and what information on proton structure can be extracted from such experiments. [12]

END OF PAPER