UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics	NST3PHY
MASTER OF ADVANCED STUDY IN PHYSICS	MAPY
NATURAL SCIENCES TRIPOS: Part III Astrophysics	NST3AS

Tuesday 12 January 2016: 14:00 to 16:00

MAJOR TOPICS
1/PP (Particle Physics)

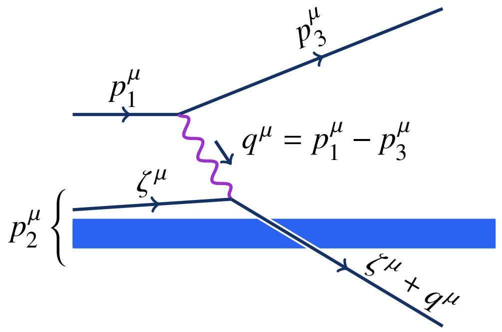
*Answer **two** questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains 3 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

*You should use a **separate Answer Book** for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2×20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 The 'Zarquon' is a hypothetical massive particle composed of indivisible 'Zarks' that have a common, but unknown, mass. Neither the number of Zarks nor the distribution of the magnitudes of the momenta of the Zarks in a Zarquon are known. It is planned to determine the Zark content of the Zarquon by a series of fixed-target deep inelastic scattering experiments in which a beam of electrons is fired at a Zarquon target as shown:



The probe electron has four-momentum p_1^μ when incoming and p_3^μ when outgoing. The Zarquon has initial four-momentum p_2^μ. The struck Zark has momentum ζ^μ before and $\zeta^\mu + q^\mu$ after the interaction. The masses of the Zark and electron are unaffected by their interaction. Assume that in the lab frame the momenta p_1^μ, p_2^μ, p_3^μ and ζ^μ take the form:

$$p_1^\mu = \begin{pmatrix} p \\ p \\ 0 \\ 0 \end{pmatrix}, \qquad p_2^\mu = \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad p_3^\mu = \begin{pmatrix} E \\ E\cos\theta \\ 0 \\ E\sin\theta \end{pmatrix} \quad \text{and} \quad \zeta^\mu = \begin{pmatrix} \sqrt{m^2 + a^2} \\ -a\cos\alpha \\ a\sin\alpha\cos\delta \\ a\sin\alpha\sin\delta \end{pmatrix}$$

where $p > 0$, $M > 0$, $m \geq 0$, $a \geq 0$, $0 \leq \alpha \leq \pi$, $0 \leq \delta < 2\pi$, $0 \leq \theta \leq \pi$ and $E \geq 0$.

(a) Write down the masses attributed above to the electron, the Zark and the Zarquon. [3]

(b) What physical interpretation can be given to the quantities a, α and δ ? [3]

(c) For the process as described, the quantities in $S = \{p, M, m, a, \alpha, \delta, \theta, E\}$ are not all independent. Write down (but do not solve) an equation that, if solved, would fix E in terms of the others. Explain the physical meaning of this constraint. [2]

Let $\Lambda^\mu{}_\nu$ be a tensor that Lorentz boosts a momentum p^μ to another momentum $\tilde{p}^\mu$ according to $\tilde{p}^\mu = \Lambda^\mu{}_\nu p^\nu$. The size and direction of the boost shall be such that if p^μ were initially at rest, then $\tilde{p}^\mu$ would have a speed $\beta > 0$ (in natural units) in the negative x-direction.

(d) Write down the sixteen components of $\Lambda^\mu{}_\nu$ as a 4x4 matrix. [3]

(e) Evaluate A^μ and B^μ in terms of M, m, a, α, δ where

$$A^\mu = \lim_{\beta \to 1} \left(\frac{\Lambda^\mu{}_\nu \zeta^\nu}{\gamma} \right), \qquad B^\mu = \lim_{\beta \to 1} \left(\frac{\Lambda^\mu{}_\nu p_2^\nu}{\gamma} \right) \qquad \text{and} \qquad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$ [4]

(f) Confirm that $A^\mu = \xi B^\mu$ for some ξ and determine its value. Explain what physical meaning this definition gives to ξ . [3]

The 'Bjorken x' observable is *defined* by the equation $x = -q^2/(2p_2.q)$. If x is re-expressed in terms of the independent variables contained within S in the Zarquon model, 'Bjorken x' may be shown to be equal to

$$\frac{\sqrt{m^2+a^2}+a\cos\alpha}{M}\left(1-\frac{a\rho}{p}\right)^{-1} \qquad (\star)$$

where $\rho = \cot\frac{\theta}{2}\sin\alpha\sin\delta + \cos\alpha$. [*You are not asked to show this!*]

(g) By comparing ($\star$) to the expression for ξ found in (f), comment on the approximations that would have to be made by anyone who wished to interpret x as 'the fraction of the momentum of the Zarquon carried by the struck parton, when measured in a frame in which both have infinite momentum'. [5]

(h) Calculate both the Zark mass and the Zarquon's Zark parton distribution function in the following two cases:

(i) That there are four stationary Zarks (and nothing else) within each Zarquon. [2]

(ii) That every Zarquon contains a very large number $N \gg 1$ of Zarks, each of which (in the Zarquon rest frame) has $a = M/N$ and moves around in constantly changing isotropically distributed random directions. You may assume that $p \gg M$. [*Hint: Recall that isotropic distributions are distributed uniformly in the cosines of polar angles but are distributed uniformly in the azimuthal angles themselves.*] [5]

2 Write detailed notes on **one** of the following topics:

(a) Helicity, chirality, and the Dirac equation, **or** [30]

(b) Experimental tests of electroweak unification. [30]

3 Suppose there exists a 'Bogus' universe in which the laws of physics are the same as in ours, except in one respect: quantum chromodynamics in the 'Bogus' universe is based on an $SU(2)$ colour symmetry having only two colours ('red' and 'green') rather than the three colour $SU(3)$ symmetry of our own.

(a) Determine which 'Bogus mesons' and 'Bogus baryons' (or their nearest equivalents) could exist by constructing any important colour, flavour and spin wave-functions. Categorise the expected 'Bogus' hadrons by type (meson/baryon), spin, and the multiplets they inhabit. Compare 'Bogus' hadron structure to that in our own universe, highlighting the main similarities and differences. [*Above you need only consider light quarks types: u, d and s.*] [24]

(b) The change from $SU(3)$ colour to $SU(2)$ colour could affect more than the basic hadron structure considered above. It could have consequences in other areas of particle physics and even further afield. Discuss any such expected differences between the Bogus universe and our own. [6]

END OF PAPER