UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Experimental and Theoretical Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Tuesday 15 January 2013 14.00 to 16.00

MAJOR TOPICS
Paper 140 (Particle Physics)

Draft: 20 September 2013

1 Symmetries and multiplets

(a) Isospin and Flavour

(i) It was hoped that this question would encourage students to recall points such as the following.

- The student should make some attempt to describe a multiplet alone one or any or a combaintion of the following themes:
  * that it is a complete set of states which can be reached from each other by (ladder) operators that rotate or move states of the symmetry
  * that a mutiplet is the maximal set of states able to share some propery under the symmetry in question
  * that all the possible states of a theory may be partitioned into non-overlapping multiplets – and where states *within* a given multiplet cannot be distinguished from each other without breaking the symmetry in question, while states in different multiplets in principle may be distingushed from each other (e.g. by an observable of some kind) even if the symmetry is exact.
- They should convey the impression that they realise that SU(2) multiplets (whether they be flavour, spin, or anything else) can be indexed by a non-negative integer (or half-integer) I, indicating the total spin/isospin/whatever. Elements of such a multiplet are distinguished by a “z-component” (here denominated I_3) which differs from I by an integer amount and satisfies $|I_3| \leq I$ as stated in the question. As such, an element of an SU(2) multiplet might be denoted thus: $|5/2, -1/2>$.
- The isospin symmetry between up and down quarks, associates the quark flavour states $|u>$ and $|d>$ with elements of an SU(2) doublet thus: $|u>= |1/2, 1/2>$, $|d>= |1/2, -1/2>$. The student must give a clear indication that he/she sees an association between these two concepts.
- The rules for analysing the struture of product states (states containing two particles) are well known, and allow us to make associations such as: $|1/2, 1/2> |1/2, 1/2>= |1, 1>$
- Ladder operators allow us to act on both sides of the above to find the isospin structure of compound objects – eg allow us to determine why $|1, 0>$ is $\frac{1}{\sqrt{(2)}}(|1/2, 1/2> |1/2, -1/2> +|1/2, -1/2> |1/2, 1/2>)$ rather than $\frac{1}{\sqrt{(2)}}(|1/2, 1/2> |1/2, -1/2> -|1/2, -1/2> |1/2, 1/2>)$, so long as we know the action of the ladder operators on all of the constituent components.
- $T_+|u>= 0$,$T_+|d>= |u>$, $T_-|u>= |d>$, $T_-|d>= 0$,
- Klebch-Gordon coeffs or ladder op rules of the form given by $H_\pm$ in the Bogon question below, allow explicit computation of (say) isospin-3/2 wave-functions, eg acting H_- repeatedly on $|uuu>$ to give things like

$|3/2, 1/2 >= \frac{1}{\sqrt{3}}(|uud > +|udu > +|duu >)$.

- Orthogonality allows the remaining states (such as the three quark states with isospin $|1/2, 1/2 >$) to be found, and there are found to be two copies of them ... those with mixed symmetry (symm under the exchange 1,2) and those antisymmetric under the exchange 1,2.
- This isospin symmetry thus places constraints on the sort of symmetries wave functions have under flavour. These flavour constraints, when put together with similar constraints that wave functions have under other symmetries (eg colour and spin) and the Fermi-exchange symmetry, determine which hadrons can exist.
- When working with mesons (q+qbar) things work similarly, except that $T_+|\bar{u} >= -|\bar{d} >$,$T_+|\bar{d} >= 0$, $T_-|\bar{u} >= 0$, $T_-|\bar{d} >= -|\bar{u} >$, leading to minus signs in a number of place, eg for mesons: $|1, 0 >= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ in contrast to qq combinations where $|1, 0 >= \frac{1}{\sqrt{2}}(uu + dd)$
- Show some diagrams with u and d spaced out along a line, and/or likewise for $\bar{u}$ and $\bar{d}$ but with these two the other way round.
- Up to two of the nine marks may be awared for the overall quality/consistency of the arguement, if prescribed bullet points have not been hit but the argument still holds and seems deserving.

(ii) Here I want the students to again recall their bookwork, where this time they may wish to use points such as the following:

- Diagram for uds now looks like a triangle.
- This symmetry not as good as the last (s heavier than u,d) but still not too bad.
- Graphical explanation of combination of multiplets by superposition, must make clear general form of SU(3) multiplet so that can explain why 3x$\bar{3}$ = 8+1 and not (say) just “9”.
- Statement that we now have ladder operators that can act in three different directions: (u,d), (u,s), (d,s)
- That the multiplicity of the states in the interior of the multiplets (eg in the centre of the 8) may be determined by looking at (for example) the way that the six ways of promoting the six states on the periphery of the 8 to the centre (by the use of the six ladder operators) lead only to two linearly independent states (which could be taken to be $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ and $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$.
- Show some multiplets in a physical context – eg connect the 8’s to the pseudoscalar mesons, or the 10 to the baryon decuplet.

(b) Bogus Symmetry

This is not bookwork!

(i) 3 marks: (x) a,b,c all integers, (xx) |a|<b and (xxx) |c|<b

(ii) 2 marks for saying *b* indexes multiplets, while *a* and *c* index states within multiplets

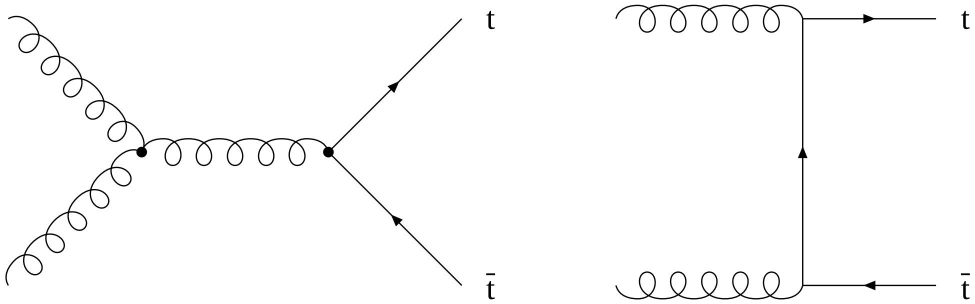
(iii) 1 mark if you said either "it is the $b = 1$ multiplet" or "it is the state containing the nine states $|\{-1, 0, +1\}, 0, \{-1, 0, +1\} >$" or something similar.

(iv) 5 marks total. I want to see some 2d rectangular arrays (1 mark) centred on the origin (1 mark) with it CLEAR that there is only one state at each vertex (1 mark) with the axes labelled "a" and "c" (1 mark) and it clear that there are 2b+1 vertices along each edge (1 mark).

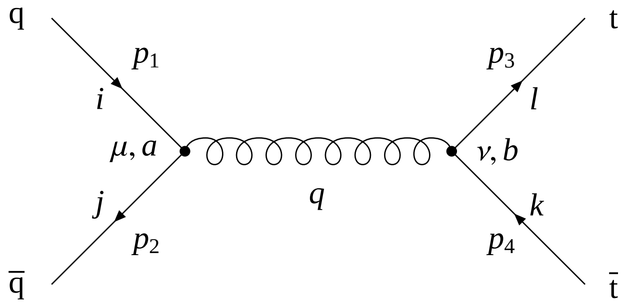
3 marks: The lightest bogons have m=3, realised when b=2 and |a|=|c|. These are the states on the diagnoals of the b=2 multiplet. There are nine such states.

1 mark: The bogus symmetry is broken or inexact, as the states in a multiplet do not all share the same mass.

2 Leading-order Feynman diagrams for gg $\to$ t$\bar{\text{t}}$:



Leading order Feynman diagram for q$\overline{\text{q}}$ $\to$ t$\bar{\text{t}}$:



Answers might get points for including statements about any of (though not limited to) the following:

- summing over final state colour combinations
- averaging over initial state colour combinations
- explaining what the proton pdf for the top quark $t(x)$ actually means (eg a statement relating it to the probability of observing a top-quark qith a momentum frac in the range $[x, x + dx]$
- the need to sum over relevant contributing quark pdf flavours (in this case t, $\bar{\text{t}}$ and g).
- making distinction between sea and valance quarks/pdfs
- emphasising which pdfs are likely to be large in hadrons like the proton and anti-proton.
- writing down an integral or differential cross section related to

$$\sigma(pp \to X) \propto \int_0^1 \int_0^1 \sum_{i,j} pdf_i(x_1) pdf_j(x_2) \sigma(ij \to X) dx_1 dx^2$$

(Note, of the 6 marks, at least 2 require the integral to be written down or described in words.)

Colour factors for $q\bar{q} \to t\bar{t}$: the $q\bar{q}$ and $t\bar{t}$ vertices contribute $\frac{1}{2}\lambda^a_{ij}$ and $\frac{1}{2}\lambda^a_{kl}$ respectively:

$$C(i\bar{j} \to l\bar{k}) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ij}\lambda^a_{kl} \,.$$

For $r\bar{r} \to r\bar{r}$, we have $i = j = k = l = 1$:

$$C(r\bar{r} \to r\bar{r}) = \frac{1}{4}\sum_{a=1}^{8}(\lambda^a_{11})^2 = \frac{1}{4}\left[(\lambda^3_{11})^2 + (\lambda^8_{11})^2\right] = \frac{1}{4}\left(1 + \frac{1}{3}\right) = \frac{1}{3} \,.$$

Similarly:

$$C(g\bar{g} \to g\bar{g}) = \frac{1}{4}\left[(\lambda^3_{22})^2 + (\lambda^8_{22})^2\right] = \frac{1}{4}\left(1 + \frac{1}{3}\right) = \frac{1}{3} \,.$$

$$C(b\bar{b} \to b\bar{b}) = \frac{1}{4}(\lambda^8_{33})^2 = \frac{1}{4}\left(\frac{-2}{\sqrt{3}}\right)^2 = \frac{1}{3} \,.$$

For $r\bar{r} \to g\bar{g}$, we have $i = j = 1$ and $k = l = 2$, so

$$C(r\bar{r} \to g\bar{g}) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{11}\lambda^a_{22} = \frac{1}{4}\left(\lambda^3_{11}\lambda^3_{22} + \lambda^8_{11}\lambda^8_{22}\right) = \frac{1}{4}\left(1 \cdot -1 + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}\right) = -\frac{1}{6} \,.$$

For $r\bar{g} \to r\bar{g}$, we have $i = 1$, $j = 2$ and $k = 2, l = 1$, so

$$C(r\bar{g} \to r\bar{g}) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{12}\lambda^a_{21} = \frac{1}{4}\left(\lambda^1_{12}\lambda^1_{21} + \lambda^2_{12}\lambda^2_{21}\right) = \frac{1}{4}\left(1 \cdot 1 + i \cdot -i\right) = \frac{1}{2} \,.$$

In summary, the allowed colour factors contributing to the matrix element M_{fi} are

$$C(r\bar{r} \to r\bar{r}) = C(g\bar{g} \to g\bar{g}) = C(b\bar{b} \to b\bar{b}) = \frac{1}{3}$$

$$C(r\bar{g} \to r\bar{g}) = C(r\bar{b} \to r\bar{b}) = C(g\bar{r} \to g\bar{r}) = C(g\bar{b} \to g\bar{b}) = C(b\bar{r} \to b\bar{r}) = C(b\bar{g} \to b\bar{g}) = \frac{1}{2}$$

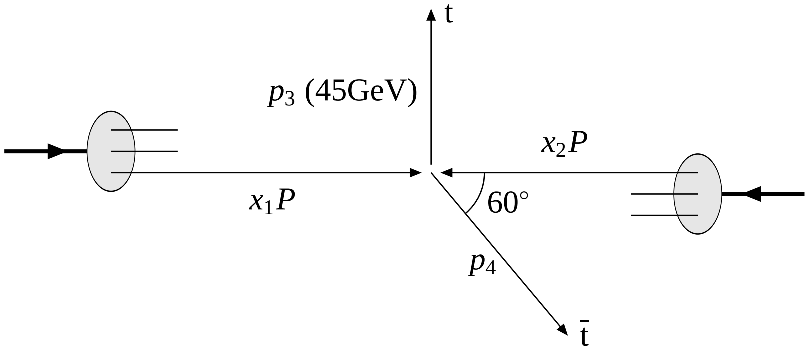
$$C(r\bar{r} \to g\bar{g}) = C(r\bar{r} \to b\bar{b}) = C(g\bar{g} \to r\bar{r}) = C(g\bar{g} \to b\bar{b}) = C(b\bar{b} \to r\bar{r}) = C(b\bar{b} \to g\bar{g}) = -\frac{1}{6}$$

All others are zero (this must be made clear to get full credit!)

In $q\bar{q} \to t\bar{t}$ scattering in high energy hadron-hadron collisions, the initial state q and $\bar{q}$ are not in a well-defined colour state, but rather each is effectively an equal mix (unpolarised mixture) of red, green and blue. The colour factor appearing in the $q\bar{q} \to t\bar{t}$ cross section (which contains $|M_{fi}|^2$) is obtained by summing over all allowed colour configurations for the scattering, and averaging over the possible colours of the initial q and $\bar{q}$ (factor of 1/3 for each):

$$\left\langle |C(q\bar{q} \to t\bar{t})|^2 \right\rangle = \frac{1}{3} \cdot \frac{1}{3}\left[3 \times \left(\frac{1}{3}\right)^2 + 6 \times \left(-\frac{1}{6}\right)^2 + 6 \times \left(\frac{1}{2}\right)^2\right] = \frac{2}{9} \,.$$

Consider the production of a $t\bar{t}$ pair in a hadron-hadron collision, due to the interaction of two partons with momentum fractions x_1 and x_2 :



The t quark has four-momentum (with $p = 45$ GeV)

$$p_3 = (\sqrt{p^2 + m_t^2}, p, 0, 0) = (\sqrt{45^2 + 173^2}, 45, 0, 0) = (178.757, 45, 0, 0) .$$

Since the transverse momentum of the $\bar{t}$ is the same as that of the t, namely 45 GeV, the $\bar{t}$ z-momentum is $45/\sin 60^\circ = 51.96 GeV$. Hence the $\bar{t}$ four-momentum is

$$p_4 = (\sqrt{(p/\sin 60^\circ)^2 + m_t^2}, -p, 0, p\cot 60^\circ) = (180.631, -45, 0, 25.98) ,$$

and the four-momentum of the $t\bar{t}$ system is

$$p_3 + p_4 = (359.388, 0, 0, 25.98) .$$

The incoming partons have 4-momenta $(x_1P, 0, 0, x_1P)$ and $(x_2P, 0, 0, -x_2P)$. Conservation of energy and momentum then gives

$$\begin{aligned}(x_1 + x_2)P &= 359.388 GeV \\ (x_1 - x_2)P &= 25.98 GeV\end{aligned}$$

At the LHC, with beam momenta $P = 3500 GeV$, we have

$$\begin{aligned}x_1 &= (359.388 + 25.98)/(2 \times 3500) = 0.055 \\ x_2 &= (359.388 - 25.98)/(2 \times 3500) = 0.048\end{aligned}$$

At the Tevatron, with beam momenta $P = 980 GeV$, these equations give

$$\begin{aligned}x_1 &= (359.388 + 25.98)/(2 \times 980) = 0.197 \\ x_2 &= (359.388 - 25.98)/(2 \times 980) = 0.170\end{aligned}$$

Measurements of parton distribution functions $q(x)$ and $g(x)$ show that quarks dominate for momentum fractions $x > 0.15$-0.2, and gluons dominate below this. Hence, at the Tevatron, $q\bar{q} \to t\bar{t}$ dominates, while at the LHC, the most likely production mechanism is $gg \to t\bar{t}$.

3 Write brief notes on **two** of the following topics:

(a) CP-violation in the Standard Model; [15]

An answer could include points such as (but need not be limited to) the following:

- Universe is matter dominated - no evidence of regions of anti-matter (lack of annihilation photons at matter−anti-matter boundary)
- To obtain small excess of anti-matter require CP violation at level of $10^9 + 1$ baryons to every 10^9 anti-baryons in early universe.
- CP violation in SM not sufficient to explain baryon dominated universe
- Describe parity.
- Describe charge conjugation.
- Assuming CPT, CP violation implies violation of T.
- In SM two place where CP arises: PMNS matrix and CKM matrix.
- CKM and PMNS matrices are unitary.
- For three generations can have a complex phase which gives CP violation
- Not possible for two generations.
- CP violation observed in kaon system
- Describe main features of CP violation in kaons
  CP eigenstates
  CP even decays to $\pi\pi$ and CP odd decays to $\pi\pi\pi$
  CP states roughly correspond to KS and KL
  At long distance have pure KL beam
  But KL observed to decay to $\pi\pi$ at level of 0.1 %
  explained by CP violation in mixing
  CP violation enters in box diagrams because $V_{ij} \neq V_{ij}^*$
- Describe main features of CP violation in neutrinos. Largely same as in kaons, except:
  Oscillating states have (essentially) infinite lifetime and so oscillate without decreasing amplitude
  Not yet observed, but not ruled out.
  PMNS not CKM
  PMNS much less diagonal than CKM

(b) electron-proton scattering and measurement of form factors; [15]

An answer could include (but not need be limited to) points such as the following:

- Elastic - proton remains intact
  * Virtual photon interacts with proton as a whole (i.e. coherently)
  * Only one independent variable - scattering angle fully determines kinematics, i.e. ($x = 1$)

•Inelastic - proton is broken up

* Two independent variable - scattering angle and scattered (or recoil) energy fully determines kinematics, i.e. cannot assume ($x == 1$)

* Photon sees parton rather than whole proton

•define x.

•Charge distribution has FT relationship to form factor - at least in elastic scattering

•In general, eg high energy, form factors parametrise ignorance in lorentz invariant way. For example, via:

* $\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,q^2)}{x} + y^2 F_1(x,Q^2)\right]$

* F_2 part is electromagnetic, F_1 pure magnetic (spin-spin)

* F_2 flat in Q^2 suggestive of point like structure of quarks and is called "Bjorken Scaling"

* $F_2(x) = 2xF_1(x)$ "Callan-Gross relation" confirms spin-1/2.

* Strong experimental evidence for above thus leads to strong support for quark parton model.

* Experimental method for measuring F_1 or F_2 is to obtain differential cross sections ad several different scattering angles and incoming electron beam energies, to fit above formula

•Could give description of evolution of $\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,q^2)}{x} + y^2 F_1(x,Q^2)\right]$ via other precursor forms (Rutherford Scattering and Mott Scattering purely electric, Rosenbluth with proton recoil at relativistic energies etc)

•Discussion of experimental measurement at low energy

•Explain how in an experiment one can DETERMINE whether the scattering was or wasn't elastic.

•Describe physical experiments/machines/detectors that have performed such measurments

•Due to form factor elastic scattering cross-section falls away rapidly with q^2.

(c) helicity, chirality, and the Dirac equation. [15]

An answer could include (but not need be limited to) points such as the following:

•The Dirac equation in its "common" form: $(i\gamma^\mu\partial_\mu - m)\psi = 0$

•The Dirac equation in Dirac's form: $\boldsymbol{\alpha}.\boldsymbol{p} + \beta m) = i\frac{\partial\psi}{\partial t}$

•Statement of or evident recognition of fact that $\boldsymbol{\alpha}.\boldsymbol{p} + \beta m)$ is Diracs free Hamiltonian.

•Derivation of necessity of four-component spinors based on desire for 1st order equation. As part if that process, would expect that properties of α and β matrices would be determined:

$\alpha_x^2 = 1$

$\alpha_y^2 = 1$
$\alpha_z^2 = 1$
$\beta^2 = 1$
$\alpha_i\beta + \beta\alpha_i = 0$
$\alpha_i\alpha_j + \alpha_j\alpha_i = 0$ if $i \neq j$

- and the need for them to be hermition to ensure that the hamiltonian stay hermition be also established.
- Bonus if all that is derived well, rather than merely stated.
- Establish connection between gammas and alphas $\gamma^0 = \beta$, $\gamma^i = \beta\alpha_i$.
- Establish that solutions of the Dirac equation such as $\psi = u(E, p)e^{i(p.r-Et)}$ are allowed, so long as the components of u satisfy some constraints (see next bullet point)
- Note that the constraints u must satisfy can be expressed as the Dirac equation in "momentum" form: $(\gamma^\mu p_\mu - m)u = 0$.
- Note that this leads to four possible linearly independent plane wave solutions for any fixed momentum vector p and mass m of which two end up representing particles, and two anti-particles.
- Comment on relationship between anti-particles and negative energy solns.
- Statement that solns of Dirac equation have (spin-half) intrinsic angular momentum.
- Dirac particle predicion that parity of particles and anti-particles is opposite
- Note that the two "particle" (or for that matter anti-particle) solutions can be thought of in many ways, eg:
  "Just linearly-indep solns, withot interpretation", "states of different parity", "states of different helicity", "states of different chirality" etc.
- Describe, derive or define the charge conjugation operator $\hat{C}$
  bonus: demonstrate clearly that student understands that definition of $\hat{C}$ is inseparable from the concept of interaction (eg as evidenced by change in sign of e ...)
- Describe Helicity operator as that taking component of spin in direction of motion, eg: $\boldsymbol{\Sigma}.\boldsymbol{p}/|\boldsymbol{p}|$ and note that this is conserved with the motion
- Describe Chirality projection operators: $P_R = \frac{1}{2}(1 + \gamma^5)$ and $P_L = \frac{1}{2}(1 - \gamma^5)$ and how they allow decomposition of spinors into R and L parts via $\psi = \frac{1}{2}(1 + \gamma^5)\psi + \frac{1}{2}(1 - \gamma^5)\psi$
- Note that chiral and helicity states are in general different but become closer and closer to each other in the high momentum or low mass limits.
- Note that chirality is not typically conserved with motion (except in the above limiting cases).
- Note that the vector and axial-vector parts of all the guage interactions can be thought of primarily coupling to states of definite chirality or definite mixtures of chirality, whereas it is usually more helpful to consider

propagation of particles over long distances in the helicity basis as it is conserved with the motion

- Illustrate with examples, such as decay of charged pion into lepton and neutrino, with neutrino decay favoured over electron (despite reduction in phase spce) as helicity conservation and chiral interaction are pulling in different ways.

(d) Higgs searches and the Higgs boson [15]

- Higgs decay possibilities are strong function of mass of higgs, tending to want to go to heaviest kinematically accessible particle, hence searches are very specific to mass range, and can have narrow region of applicability. List some of them, eg:
  * h->gamma gamma (VIA TOP LOOP!)
  * h->$b\bar{b}$
  * h->WW
  * h->ZZ
- Indirect constraints (100-1000 GeV) from accurate top quark and w-mass measurements (two marks here, as have to give some indication of how/why higgs mass should affect w mass..., and mention log)
- Direct LEP search constraints (m_H>114 GeV) from production with Z-boson
- need for b-jet tagging at LEP to distinguish tiny H->bb signal from under gigantic Z+jet backgrounds
- LHC was designed with indirect higgs mass bound(s) in mind, to discover Higgs with mass anywhere from 100 to 1000 GeV.
- LHC has discovered particle consitent with higgs at 125 GeV ish.
- Still need to see that has right spin and coplings to all other particles before can be really “really” convinced that it is “the” Higgs boson, but seems likely.
- LHC bumps seen principally in gamma gamma and in H->ZZ->4 lepton modes, in two different experiments.
- Combined probabilty of bg fluctuation $< 10^{-9}$
- Other higgs bosons could still be found at less than SM predicted cross sections.
- The Higgs boson is electrically neutral but carries weak hypercharge of 1/2
- Does not directly couple to photons (as they are massless)
- Distinguish higgs field from higgs boson
- note how masses for fermions enter as yukawa couplings
- note how spontaneous symmetry breaking gives mass to weak bosons.

END OF PAPER