UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS
NATURAL SCIENCES TRIPOS: Part III Astrophysics

Xxxxxday XX January 2016: XX:XX to XX:XX

MAJOR TOPICS
Paper X/PP (Particle Physics)

Answer ***two*** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains XX sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

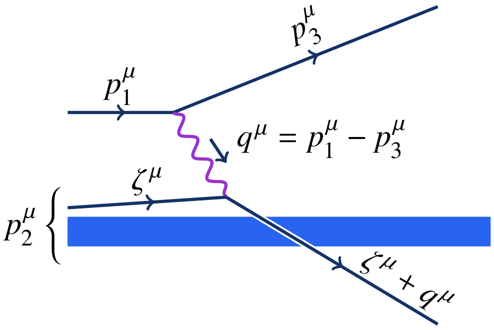
You should use a ***separate Answer Book*** *for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2x20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

V7.4

1 The ‘Zarquon’ is a hypothetical massive particle composed of indivisible ‘Zarks’ that have a common, but unknown, mass. Neither the number of Zarks nor the distribution of the magnitudes of the momenta of the Zarks in a Zarquon are known. It is planned to determine the Zark content of the Zarquon by a series of fixed-target deep inelastic scattering experiments in which a beam of electrons is fired at a Zarquon target as shown:



The probe electron has four-momentum p_1^μ when incoming and p_3^μ when outgoing. The Zarquon has initial four-momentum p_2^μ. The struck Zark has momentum ζ^μ before and $\zeta^\mu + q^\mu$ after the interaction. The masses of the Zark and electron are unaffected by their interaction. Assume that in the lab frame the momenta p_1^μ, p_2^μ, p_3^μ and ζ^μ take the form:

$$p_1^\mu = \begin{pmatrix} p \\ p \\ 0 \\ 0 \end{pmatrix}, \qquad p_2^\mu = \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad p_3^\mu = \begin{pmatrix} E \\ E\cos\theta \\ 0 \\ E\sin\theta \end{pmatrix} \quad \text{and} \quad \zeta^\mu = \begin{pmatrix} \sqrt{m^2+a^2} \\ -a\cos\alpha \\ a\sin\alpha\cos\delta \\ a\sin\alpha\sin\delta \end{pmatrix}$$

where $p > 0$, $M > 0$, $m \geq 0$, $a \geq 0$, $0 \leq \alpha \leq \pi$, $0 \leq \delta < 2\pi$, $0 \leq \theta \leq \pi$ and $E \geq 0$.

(a) Write down the masses attributed above to the electron, the Zark and the Zarquon. [3]

BOOKWORK[Testing recall of $p^\mu p_\mu = E^2 - p^2 = m^2$.]

Electron mass $= p_1^2 = 0$. Zark mass $= \zeta^2 = m$. Zarquon mass $= p_2^2 = M$.

(b) What physical interpretation can be given to the quantities a, α and δ ? [3]

BOOKWORK[Testing recall of meaning of spherical polar co-ordinate definitions, and location of momentum in last three components of a four-vector.]

a is the magnitude of the three momentum of the struck Zark (1 marks), in the Lab Frame (1 mark) , prior to its being struck. α and δ are the polar and azimutal angles for that same momentum, referenced with respect to the negative x-axis. (1 mark)

(c) For the process as described, the quantities in $S = \{p, M, m, a, \alpha, \delta, \theta, E\}$ are not all independent. Write down (but do not solve) an equation that, if solved, would fix E in terms of the others. Explain the physical meaning of this constraint. [2]

The question says that the masses of the Zark and the probe electron are not changed by their being struck. This is self-evident for the latter, as both p_1^μ and p_3^μ have zero invariant mass. The invariant mass of $\zeta^\mu + q^\mu$ is not, however, as parametrised, necessarily still equal to its value before being struck (m). The contraint that could be solved therefore is (in words) 'mass of Zark after being struck = 0'. Algebraically this could be written as:

$$(\zeta + q)^2 = m^2$$

or, eliminating q to gain explicit E dependence could be written

$$(\zeta + p_1 - p_3)^2 = m^2.$$

Full marks could be obtained by saying the above in words and writing down the equation above in some form.

What follows is just additional information for interest only. It is not expected or needed that the candidates do any of the following.

Multiplying out the square, and noting that $\zeta^2 = m^2$ and $p_1^2 = p_3^2 = 0$, the constraint could also be written as

$$\zeta.p_1 - \zeta.p_3 - p_1.p_3 = 0.$$

This latter form is obviously linear in p_3 and so may be solved for E. Since $p_3^\mu = Ek^\mu$ where

$$k^\mu = \begin{pmatrix} 1 \\ \cos\theta \\ 0 \\ \sin\theta \end{pmatrix}$$

we have evidently that:

$$\zeta.p_1 = Ek.(\zeta - p_1)$$

and hence

$$E = \frac{\zeta.p_1}{k.(\zeta + p_1)}.$$

Putting in the components gives

$$E = \frac{p\left(\sqrt{m^2 + a^2} + a\cos\alpha\right)}{\sqrt{m^2 + a^2} + a\cos\theta\cos\alpha - a\sin\theta\sin\alpha\sin\delta + p - p\cos\theta}.$$

Let $\Lambda^\mu{}_\nu$ be a tensor that Lorentz boosts a momentum p^μ to another momentum $\tilde{p}^\mu$ according to $\tilde{p}^\mu = \Lambda^\mu{}_\nu p^\nu$. The size and direction of the boost shall be such that if p^μ were initially at rest, then $\tilde{p}^\mu$ would have a speed $\beta > 0$ (in natural units) in the negative x-direction.

(d) Write down the sixteen components of $\Lambda^\mu{}_\nu$ as a 4x4 matrix. [3]

BOOKWORK[Testing recall of relativistic boost formula.]

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Two marks for top left 2x2, one mark for bottom right 2x2. Marks off for any errors.

(e) Evaluate A^μ and B^μ in terms of M, m, a, α, δ where

$$A^\mu = \lim_{\beta\to 1}\left(\frac{\Lambda^\mu{}_\nu \zeta^\nu}{\gamma}\right), \qquad B^\mu = \lim_{\beta\to 1}\left(\frac{\Lambda^\mu{}_\nu p_2^\nu}{\gamma}\right) \qquad \text{and} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}.$$ [4]

$$A^\mu = \begin{pmatrix} \sqrt{m^2+a^2} + a\cos\alpha \\ -\sqrt{m^2+a^2} - a\cos\alpha \\ 0 \\ 0 \end{pmatrix} \quad \text{(2 marks)}, \qquad \text{and} \qquad B^\mu = \begin{pmatrix} M \\ -M \\ 0 \\ 0 \end{pmatrix} \quad \text{(2 marks)}.$$

(f) Confirm that $A^\mu = \xi B^\mu$ for some ξ and determine its value. Explain what physical meaning this definition gives to ξ . [3]

BOOKWORK[Half of this (the ‘explain what physical meaning’ part) is bookwork. The lectures told attendees that ξ, as defined above (though in words rather than in equations) is the fraction of the momentum of the proton caried by the constituent quark in the infinite momentum frame. This should be recall.]

$$\xi = \frac{\sqrt{m^2+a^2} + a\cos\alpha}{M} \qquad \text{(1 mark)}.$$

ξ, as defined in the question, is the fraction of the momentum of the Zarquon carried by the Zark (1 mark), as measured in the so-called ‘infinite momentum frame’ (1 mark).

The ‘Bjorken x’ observable is *defined* by the equation $x = -q^2/(2p_2.q)$. If x is re-expressed in terms of the independent variables contained within S in the Zarquon model, ‘Bjorken x’ may be shown to be equal to

$$\frac{\sqrt{m^2+a^2} + a\cos\alpha}{M}\left(1 - \frac{a\rho}{p}\right)^{-1} \qquad (\star)$$

where $\rho = \cot\frac{\theta}{2}\sin\alpha\sin\delta + \cos\alpha$. [*You are not asked to show this!*]

(g) By comparing ($\star$) to the expression for ξ found in (f), comment on the approximations that would have to be made by anyone who wished to interpret x as 'the fraction of the momentum of the Zarquon carried by the struck parton, when measured in a frame in which both have infinite momentum'. [5]

The candidates should see that, in terms of the ξ they derived, we have

$$x = \xi \left(1 - \frac{a\rho}{p}\right)^{-1}$$

whereas the usual interpretation of x given tin the lectures and books is that

$$x = \xi.$$

These two results differ by the $(1 - \frac{a\rho}{p})^{-1}$ term. The simpler usual interpretation, however, was derived (in lectures and in books) in a manner that neglected transverse momenta (i.e. assumed that $a \sin\alpha \sim 0$, or (more precisely) that $a \sin\alpha \ll p$ and $a \ll p$. What we have done is just calculate the corrections to the usual picture induced by considering transverse momenta (i.e. $\alpha \neq 0$ and $a > 0$). If we look at the form ($\star$) in the limit $a \sin\alpha \to 0$ (i.e. in the limit of no transverse moementum) it becomes

$$x = \xi \left(1 - \frac{a\cos\alpha}{p}\right)^{-1}$$

and so in the limit that $a \ll p$ we recover the usual $x = \xi$. Note that $a \ll p$ is a common reality in deep inelastic scattering experiments since it follows from $M \ll p$. In short, we see that the **common interpretation** $x = \xi$ can really be seen as the leading term in an expansion of x as a function of $a\rho/p$. The continued presence of the p (albeit in an a/p term) shows that the pdf is still dependent to some degree on the way in which it is measured. This said, in a scenario in which $a/p \ll 1$ (which is implied by $M/p \ll 1$) mean that it is well defined at high enough probe energy.

(h) Calculate both the Zark mass and the Zarquon's Zark parton distribution function in the following two cases:

(i) That there are four stationary Zarks (and nothing else) within each Zarquon. [2]

(ii) That every Zarquon contains a very large number $N \gg 1$ of Zarks, each of which (in the Zarquon rest frame) has $a = M/N$ and moves around in constantly changing isotropically distributed random directions. You may assume that $p \gg M$. [*Hint: Recall that isotropic distributions are distributed uniformly in the cosines of polar angles but are distributed uniformly in the azimuthal angles themselves.*] [5]

(i) When there are ~~two~~ [FOUR] Zarks and they are stationary, each must have $a = 0$ and have mass $m = M/$~~2~~ [FOUR] or else the Zarquon would not have the correct total mass. Consequently each Zark has $x = \xi = m/M = \frac{1}{2}$ [QUARTER] (from substitution in to (1)). In other words, each Zark would always be observed with momentum fraction $\frac{1}{2}$ [QUARTER]. The Zark pdf $z(x)$ would therefore be $z(x) =$ ~~2~~ [FOUR] $\delta(x - 1/$~~3~~ [FOUR] $)$ with the ~~2~~ [FOUR] coming from the presence of two Zarks and the definition that 'the parton distribution

A

function $u(x)$ is such that $u(x)dx$ is the *number* of u's having x between x and $x + dx$. (Here $\delta(x)$ is the Dirac delta-function, not the angle δ within ζ^μ.)

(ii) In the case of the large number N of idential mass zarks, all moving with $a = M/N$, we know will have to have zero mass ($m = 0$) or else the sum of their energies ($N\sqrt{m^2 + (M/N)^2}$) would exceed M. We are told that each has δ distributed uniformly in $[0, 2\pi)$ and $\cos\alpha$ distributed uniformly in $[-1, 1]$. At any fixed δ, since $\xi = \frac{\sqrt{m^2+a^2}+a\cos\alpha}{M} = \frac{a(1+\cos\alpha)}{M} = (1 + \cos\alpha)/N$, we know that ξ will be uniformly distributed in the range $[0, 2/N]$. Technically we need to look at the distribution of x not ξ, however we are told that $p \gg M > a$ so we can neglect the difference between the two. The shape of the Zarquon's Zark parton distrbutio function is therefore $u(x) = \frac{1}{2}N^2$ if $0 \le x \le 2/N$ and $u(x) = 0$ otherwise. The normalisation is chosen such that the area of the allowed region is N (N Zarks).

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2 Write detailed notes on **one** of the following topics:

(a) Helicity, chirality, and the Dirac equation, **or** [30]

(b) Experimental tests of electoweak unification. [30]

(a) Helicity, chirality, and the Dirace quation. An answer could include (but not need be limited to) points such as the following:

- •The Dirac equation in its 'common' form: $(i\gamma^\mu\partial_\mu - m)\psi = 0$
- •The Dirac equation in Dirac's form: $\boldsymbol{\alpha}.\boldsymbol{p} + \beta m) = i\frac{\partial d\psi}{\partial dt}$
- •Statement of or evident recognition of fact that $\boldsymbol{\alpha}.p + \beta m$ is Dirac's free Hamiltonian.
- •Derivation of necessity of four-component spinors based on desire for 1st order equation. As part if that process, would expect that properties of α and β matrices would be determined: $\alpha_x^2 = 1$, $\alpha_y^2 = 1$, $\alpha_z^2 = 1$, $\beta^2 = 1$, $\alpha_i\beta + \beta\alpha_i = 1$, $\alpha_i\alpha_j + \alpha_j\alpha_i = 0$ if $i \ne j$,
- •the need for them to be hermitian to ensure that the hamiltonian stay hermitian be also established.
- •Bonus if all that is derived well, rather than merely stated.
- •Establish connection between gammas and alphas $\gamma^0 = \beta$, $\gamma^i = \beta\alpha^i$.
- •Establish that solutions of the Dirac equation such as $\psi = u(E, p)e^{i(p.r-Et)}$ are allowed, so long as the components of u satisfy some constraints (see next bullet point)
- •Note that the constraints u must satisfy can be expressed as the Dirac equation in momentum form: $(\gamma^\mu p_\mu - m)u = 0$.
- •Note that this leads to four possible linearly independent plane wave solutions for any fixed momentum vector p and mass m of which two end up representing particles, and two anti-particles.
- •Comment on relationship between anti-particles and negative energy solns.
- •Statement that solns of Dirac equation have (spin-half) intrinsic angular momentum.
- •Dirac particle predicion that parity of particles and anti-particles is opposite

- •Note that the two ‘particle’ (or for that matter anti-particle) solutions can be thought of in many ways, e.g.: ‘Just linearly-indep solns, withot interpretation’, ‘states of different parity’, ‘states of different helicity’, ‘states of different chirality’ etc.
- •Describe, derive or define the charge conjugation operator $\hat{C}$ bonus: demonstrate clearly that student understands that definition of $\hat{C}$ is inseparable from the concept of interaction (e.g. as evidenced by change in sign of e ...)
- •Describe Helicity operator as that taking component of spin in direction of motion, eg: $\Sigma.p/|p|$ and note that this is conserved with the motion
- •Describe Chirality projection operators: $P_R = \frac{1}{2}(1+\gamma_5)$ and $P_L = \frac{1}{2}(1-\gamma_5)$ 22 and how they allow decomposition of spinors into R and L parts via $\psi = \frac{1}{2}(1+\gamma_5)\psi + \frac{1}{2}(1-\gamma_5)\psi$
- •Note that chiral and helicity states are in general different but become closer and closer to each other in the high momentum or low mass limits.
- •Note that chirality is not typically conserved with motion (except in the above limiting cases).
- •Note that the vector and axial-vector parts of all the guage interactions can be thought of primarily coupling to states of definite chirality or definite mixtures of chirality, whereas it is usually more helpful to consider propagation of particles over long distances in the helicity basis as it is conserved with the motion
- •Illustrate with examples, such as decay of charged pion into lepton and neutrino, with neutrino decay favoured over electron (despite reduction in phase spce) as helicity conservation and chiral interaction are pulling in different ways.

(b) Experimental tests of electoweak unification. Answers should include a brief discussion of relations between EW paramrters, measurement of the W and Z masses + measurements of weak mixing angle from asymmetries, and the discovery of the higgs boson. Marks for:

- •mentioning EW sector of SM fixed by three parameters but constrained by more than tree experiments
- •relation between higgs mass to top mass
- •Z mass from peak of Breit-Wigner resonance distortion due to ISR biases due to tidal distortions and TGV
- •W mass from direct reconstruction
- •can’t use cross section as not resonant so mention decays and what is measured
- •Mixing angle from asymmetries relation to parity violation relation to couplings
- •Forward backward asymmetry diagram
- •Knew top mass before discovered
- •Higgs mass prediction
- •Higgs discovery

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3 Suppose there exists a ‘Bogus’ universe in which the laws of physics are the same as in ours, except in one respect: quantum chromodynamics in the ‘Bogus’ universe is

based on an $SU(2)$ colour symmetry having only two colours ('red' and 'green') rather than the three colour $SU(3)$ symmetry of our own.

(a) Determine which 'Bogus mesons' and 'Bogus baryons' (or their nearest equivalents) could exist by constructing any important colour, flavour and spin wave-functions. Categorise the expected 'Bogus' hadrons by type (meson/baryon), spin, and the multiplets they inhabit. Compare 'Bogus' hadron structure to that in our own universe, highlighting the main similarities and differences. [*Above you need only consider light quarks types: u, d and s.*] [24]

(b) The change from $SU(3)$ colour to $SU(2)$ colour could affect more than the basic hadron structure considered above. It could have consequences in other areas of particle physics and even further afield. Discuss any such expected differences between the Bogus universe and our own. [6]

This is a question that tries to test candidates understanding of the arguments made in their notes in the meson and baryon part of the lecture course. Mere photographic recall of the lecture notes (without understanding) will not help the candidate, but a candidate who is able to recall the kinds of things said, and can make reasonable educated guesses about how to adapt them from $SU(3)$ colour to $SU(2)$ colour will be rewarded based on the clarity, completeness, and level of understanding of the $SU(3)$ colour theory demonstrated in the nature of their answer(s). Since some parts of $SU(3)$ QCD are not fully understood (e.g. colour confinement, and much of the non-perturbative part of the theory) it would be possible for two equally good candidates to come up with mutually incompatible answers that could both be, on physical grounds, plausible. In that sense there cannot be any 'model' answer, and for this reason marking will always give credit where it is due, even if there is not conformity to the suggested form of the answer below.
BOOKWORK[The bookwork components of this question consist of all the places where the candidate can legitimately bring in a description of the processes used to derive/describe hadron structure in our own three-colour universe to motivate a generalisation to the two-colour case. There are many such places.]

(a) A key fact the candidate should bring to the table here is that the SU(2) colour theory will require a

$$\frac{1}{\sqrt{2}}(r\bar{r} + g\bar{g})$$

equivalent of the SU(3)

$$\frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$$

colour-anticolour singlet thereby permitting mesons to exist for most of the same reasons they can in the real universe. A poor answer would omit this altogether. A medium answer would mention it without proof merely appealing to its plausibility and connection to colour confinement hypothesis. A good answer might demonstrate that this really is a singlet by consideration of the action of properly defined ladder operators on it, etc. It might even go on to question whether the colour confinement hypothesis would still be important in the bogus universe.

Answers will hopefully reproduce the potential spin wavefunctions of the 'real' mesons, noting those in the 'bogus' universe could be identical.

A good answer would hopefully re-capitulate the flavour part of the notes (that coveres the meson nonets) noting that, as in ‘real’-space, the bogus universe allows any spin combinations with any flavour combinations since the lack of any identical fermions in the mesons leads no need to have antisymmetry of the overall wavefunction.

The spetra of excited mesonic states would presumably differ in the real universe from that in the bogus, as the different colout potential would space excitations differently.

(b) Here the key fact is that the three colour singlet of $SU(3)$

$$\frac{1}{\sqrt{6}}(rgb - rbg + gbr - brb + brg - bgr)$$

is replaced in the bogus universe by the

$$\frac{1}{\sqrt{2}}(rg - gr)$$

two-colour singlet of $SU(2)$, meaning that the colour confinement hypothesis (if still needed!) would permit two-quark baryons and forbid three-quark baryons. Again, a poor answer would neglect to mention this at all. A medium answer would just state it. A good answer would argue the case clearly.

The disappearance of one colour would not change the approximate (u,d)-isopin SU(2) flavou or (u,d,s)-isospin SU(3) flavour symmetries available to nature – but the need for only two quark states would require us now to consider only the $3 \otimes 3 = 6 \oplus \bar{3}$ not the $3 \otimes 3 \otimes 3 = 10 + 8 + 8 + 1$ version of before. A good answer would work out that the 6 is symmetric in the two quark flavours, while the $\bar{3}$ is antisymmetric.

What flavour/spin/colour combinations would be allowed? Assuming the lowest angular momentum states would have $L = 0$ making them even parity, and given that the colour singlet is already antisymmetric, we’d need flavour x spin to be symmetric. We would need to combine the 6 with a symmetric $S = 1$ spin-triplet, or the antisymmetric $\bar{3}$ with an antisymmetric $S = 0$ spin-singlet.

The bogus (u,d,s)-baryons would therefore be expected to come in a $S = 1$ hextet of and a $S = 0$ triplet of di-quark states.

Note that the charges of these bogus baryons would be non-integer: the lightest three (uu, ud, dd) having charges $\frac{4}{3}$, $\frac{1}{3}$ and $-\frac{2}{3}$ respectively.

(c) There are many potential things that could be mentioned here – no doubt candidates will come up with valid ideas that I have not forseen and will be rewarded accordingly even if they are not on this list:

- Mesons play very little role in the day-to-day life of organisms on present-day earth, as they can usually decay (via $q\bar{q}$ annihilation) to other things, and so life on earth is based on the more stable bosons. Changes to the mesonic structure the measons might be expected to be less important in the current universe, though presumably they would make considerable differences to some parts of the big-bang/cosmological models around the transition from radiation to matter domination.
- The change in baryon structure, however (removal of the proton!!) would have very profound implications for chemistry. With the lightest baryons now being fractionally

charged, atoms as we know them would cease to exist. Indeed the whole periodic table is based on assembling elements from two nucleon types (proton and neutron) and would have to change to a system based on three nucleons ... so elements would be in trouble too.

- The bogus universe would only have $2^2 - 1 = 3$ gluons, not the $3^2 - 1 = 9$ in the real universe.
- The rate of running of a_s will change due to fewer gluons/colours.
- The linear term in effective colour potential between two quarks would probably be different (less?) as a result of fewer quarks, possibly making jets less jetty.
- The possiblilty of $q\bar{q}q\bar{q}$ states would be present in both Bogus and Real universes. But whereas the real universe forbids $qqqq$ and allows $qqqq\bar{q}$ states, the Bogus would allow $qqqq$ and forbid $qqqq\bar{q}$ due to the change in which contains a colour singleton.
- Colour factors would change leading to, say, some hadron-hadron cross sections to get enhanced or reduced.
- A good answer that has not already considered this point in an earlier part (a) or (b) might andvance some ideas on why/whether the colour confinement hypothesis would hold for SU(2)-based colour.

Another examiner has suggested that the question would benefit from proving a finer-grained breakdown of marks than the (a) [24], (b) [6] suggestion given. After some consideration, I have come to the conclusion that would like to resist such an alteration, as to do so (i) begins to artifically constrain the order that a candidate might find it most convenient to attack the problem (I suspect there are many ways the problem could be approached), and (ii) it starts to remove some parts of the problem altogether: for example, I see a significant part of the problem is actually identifitying that there even are Bogus baryons at all. The bogus measons' existence is not too difficult to show as they are almost the same as those in our own universe. However the bogus baryons no longer have three quarks – and there is some effort in finding that. If the question starts to break things down into a tiny mark here and a tiny one there, it is in great danger of becoming (in my view) overly prescritive and gives the game away of how much of each thing there is to find. The question is (I hope) clear in its lists of what is required to be delivered. As to the weights given to different sections within that – though I can guess its form at present (i.e. about 12 marks for Baryons and 12 for Mesons), it would always need to be adapted in the light of what the candidates write: mark-schemes are only ever indicative. My experience of marking to the required final distributions is that it is always necessary to gauge what candidates make of a question, to discover what interesting ways they have of answering it, whether they found the question hard or easy, and then identify where in that spectrum of answers creativity and insight can be rewarded to different degrees. I believe that a looser mark structure here (in contrast to the highly prescriptive one in question 1) is the right match for what is, by its nature, a relatively free-form question.
