UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS
NATURAL SCIENCES TRIPOS: Part III Astrophysics

Tuesday 17 January 2017: 14:00 to 16:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

Answer ***two*** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains four sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You should use a ***separate Answer Book*** *for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2x20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

V7.4

A

1 The Mandelstam variables s, t and u for $2 \to 2$ scattering processes are defined as $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$, where p_1 and p_2 are incoming four-momenta and p_3 and p_4 are outgoing four-momenta. Within this question you may neglect the masses of all incoming and outgoing particles.

(a) Show that s, t and u are not independent by considering $s + t + u$. [2]

BOOKWORK[One of the example sheet questions required students to show that

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2,$$

which is a statement that was also made in Lecture 1 and printed on Handout 1, so this question (which asks for less since all masses may be neglected) is bookwork. One mark for simply stating that $s + t + u = 0$ (even if given without proof) and another for giving some clear indication that it follows from momentum conservation .... perhaps by writing

$$s + t + u = 2p_1.p_2 - 2p_1.p_3 - 2p_1.p_4 = 2p_1.(p_2 - p_3 - p_4) = 2p_1.(-p_1) = -2m_1^2 = 0.$$

]

(b) Find s, t and u in terms of p and θ, where p is the magnitude of the three-momentum of p_1 in the centre-of-mass frame, and θ is the angle between the spatial parts of p_1 and p_3 in that same frame. [4]

BOOKWORK[

$$s = ((p, 0, 0, p) + (p, 0, 0, -p))^2 = 4p^2 \qquad \textbf{(one mark)} \tag{1}$$

$$\begin{aligned}
t &= ((p, 0, 0, p) - (p, p\sin\theta, 0, p\cos\theta))^2 && (2)\\
&= (0, -p\sin\theta, 0, p(1-\cos\theta))^2 && (3)\\
&= -p^2\sin^2\theta - p^2(1-\cos\theta)^2 && (4)\\
&= p^2(-\sin^2\theta - 1 - \cos^2\theta) + 2p^2\cos\theta && (5)\\
&= -2p^2(1-\cos\theta) \qquad \textbf{(two marks)} && (6)
\end{aligned}$$

$$\begin{aligned}
u &= ((p, 0, 0, p) - (p, -p\sin\theta, 0, -p\cos\theta))^2 && (7)\\
&= (0, p\sin\theta, 0, p(1+\cos\theta))^2 && (8)\\
&= -p^2\sin^2\theta - p^2(1+\cos\theta)^2 && (9)\\
&= p^2(-\sin^2\theta - 1 - cos^2\theta) - 2p^2\cos\theta && (10)\\
&= -2p^2(\cos\theta + 1) \qquad \textbf{(two marks).} && (11)
\end{aligned}$$

Note this means that $t^2 = \frac{1}{4}(1-\cos\theta)^2 s^2$ and $u^2 = \frac{1}{4}(1+\cos\theta)^2 s^2$.]

Ten scattering processes (numbered 0 to 9) and six quantities (denoted A to F) are listed in the following tables.

0	$e^-\mu^- \to e^-\mu^-$	5	$e^-\mu^+ \to e^+\mu^-$
1	$e^-e^+ \to \mu^-\mu^+$	6	$e^-e^- \to \mu^-\mu^-$
2	$e^-e^- \to e^-e^-$	7	$e^-_R\mu^-_R \to e^-\mu^-$
3	$e^-e^+ \to e^-e^+$	8	$e^-_R\mu^-_L \to e^-\mu^-$
4	$e^-\mu^+ \to e^-\mu^+$	9	$e^-_R e^+_R \to \mu^-\mu^+$

A	$\frac{u^2}{t^2}$
B	$\frac{s^2+u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2+t^2}{u^2}$
C	$\frac{t^2+u^2}{s^2}$
D	$\frac{s^2+u^2}{t^2}$
E	$\frac{s^2}{t^2}$
F	$\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}$

Each of the quantities A to F represents the square of the modulus of the tree-level QED spin-averaged matrix element $\left\langle |M|^2 \right\rangle$ of one or more of the processes 0 to 9, after omission of any overall factors that do not depend on s, t or u. For each process the particle names are given in the order corresponding to momentum labels $p_1 p_2 \to p_3 p_4$.

(c) Determine which (if any) of the processes in 0 to 9 are 'allowed' at tree-level in QED, and which (if any) are correspondingly 'not allowed'. [1]

(d) For each 'allowed' process in 0 to 9, in turn:

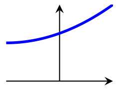
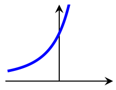
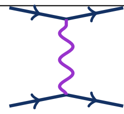
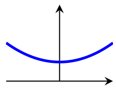
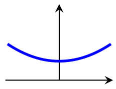
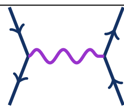
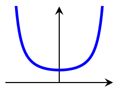
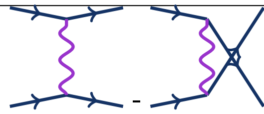
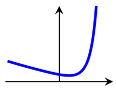
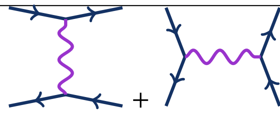
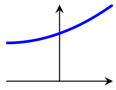
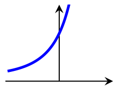
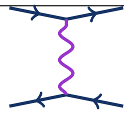
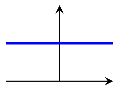
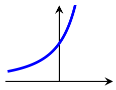
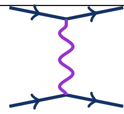
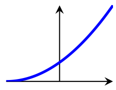
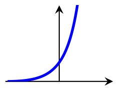
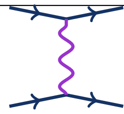
(i) draw all the tree-level QED Feynman diagrams for that process;

(ii) identify the quantity in A to F which represents the s, t and u dependence of $\left\langle |M|^2 \right\rangle$ for that process, explaining your reasoning; and

(iii) qualitatively sketch the $\cos\theta$ dependence of $\left\langle |M|^2 \right\rangle$.

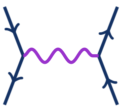
[*Hint: Answers to part (ii) need not contain lengthy mathematical proofs or deriavations from first-principles where simpler arguments can be found.*] [23]

The main parts of the answer are as summarised in the following table:

	process		$\propto \left\langle \lvert M\rvert^2 \right\rangle$	numerator sketch (info only)	whole sketch $[-1, 1]$	diagram
0	$e^-\mu^- \to e^-\mu^-$	D	$\frac{s^2+u^2}{t^2}$			
1	$e^-e^+ \to \mu^-\mu^+$	C	$\frac{t^2+u^2}{s^2}$			
2	$e^-e^- \to e^-e^-$	B	$\frac{s^2+u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2+t^2}{u^2}$			
3	$e^-e^+ \to e^-e^+$	F	$\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}$			
4	$e^-\mu^+ \to e^-\mu^+$	D	$\frac{s^2+u^2}{t^2}$			
5	$e^-\mu^+ \to e^+\mu^-$	not allowed	0			no tree level diagram
6	$e^-e^- \to \mu^-\mu^-$	not allowed	0			no tree level diagram
7	$e^-_R\mu^-_R \to e^-\mu^-$	E	$\frac{s^2}{t^2}$			
8	$e^-_R\mu^-_L \to e^-\mu^-$	A	$\frac{u^2}{t^2}$			
9	$e^-_Re^+_R \to \mu^-\mu^+$	not allowed	0			no tree level diagram

The last two non-zero Feynman diagrams implicitly contain only one spin combination each ($RR \to RR$ and $RL \to RL$ respectively) while all others non-zero diagrams implicity average over four incoming spin combinations, in terms of which the outgoing spins are fixed by the rule that the helicity arrows (not drawn) pass through each QED vertex without changing direcection (i.e. one helicity arrow into and one out of each vertex, or vice versa). The assignments of quantity labels to process labels may be done as follows:

1. The three 'impossible' processes can be immediately identified by either the lack of a valid lepton flavour conserving Feynman diagram (5:$e^-\mu^+ \to e^+\mu^-$ and 6:$e^-e^- \to \mu^-\mu^-$) or lack of helicity conservation (9:$e^-_Re^+_R \to \mu^-\mu^+$). These are all therefore not allowed with a zero matrix element having no releveant dependence on s, t or u.

2. There is only one s-channel Feynman diagram ( for 1:$e^-e^+ \to \mu^-\mu^+$), and

A

only one quantity in the list having an s^2 in the denominator (C: $\frac{t^2+u^2}{s^2}$). The wording of the question informs us that every listed quantity corresponds to at least one of the listed processes, so these two must be one and the same. Furthermore, most students should recall that this is the first diagram they computed in lectures. They saw computed the angular dependence of the numerator (proportional to $(1+\cos\theta)^2 + (1-\cos\theta)^2$) not only using spinors and a sum of modulus squares of Lorentz dot products of electron and muon currents, but also by considering the angular dependence of the initial state. In the case of the latter, it was noted that thie initial state was either $|1,+1\rangle$ or $|1,-1\rangle$ and so either full backward (or respectively full forward) scattering of the electron would be impossible and indicated a $(1 \pm \cos\theta)^2$ dependence in the numerator. These recollections can be compared to th values for t and u just found in the earlier part of this question, to reconfirm that the numerator supplied agrees with those facts.

3. There are four pure t-channel Feynam diagrams (4: $e^-\mu^+ \to e^-\mu^+$, 0: $e^-\mu^- \to e^-\mu^-$, 7: $e^-_R\mu^-_R \to e^-\mu^-$ and 8: $e^-_R\mu^-_L \to e^-\mu^-$) and three pure t-channel quantities (D: $\frac{s^2+u^2}{t^2}$, E: $\frac{s^2}{t^2}$ and A: $\frac{u^2}{t^2}$). Given that every listed quantity corresponds to at least one of the listed processes, this means that two of these quantities are used once and the other is used twice. On close inspection it should become readily apparent that the two processes with specified initial spins (7: $e^-_R\mu^-_R \to e^-\mu^-$ and 8: $e^-_R\mu^-_L \to e^-\mu^-$) are just two of the four fixed-helicity initial states that need to be considered when looking at 0: $e^-\mu^- \to e^-\mu^-$. The first is a mixture of $|1,0\rangle$ and $|0,0\rangle$ without any preferred z-component of spin, and so must have an isotropic numerator of which s^2 is the only possiblity. The second is a $|1,1\rangle$ state which (by helicity conservation in each vertex will prefer to forward scatter (and cannot fully backward scatter), as discussed in the lectures, and so must have a $(1+\cos\theta)^2$ numerator which we have already seen is proportional to t^2. The same would be true for the other two of the four fixed-helicity states not considered. Therefore it can be seen that $\frac{s^2}{t^2}$ is $e^-_R\mu^-_R \to e^-\mu^-$, $\frac{u^2}{t^2}$ is $e^-_R\mu^-_L \to e^-\mu^-$, and their sum $\frac{s^2+u^2}{t^2}$ must be $e^-\mu^- \to e^-\mu^-$. This leaves $e^-\mu^+ \to e^-\mu^+$. After a little thought it shoud be apparent that the only difference between it and $e^-\mu^- \to e^-\mu^-$ is the sign of the muon. This might matter if we were looking at the Weak interaction which threats the L and R parts of particles and antiparticles differently, but the students have seen in this course that the QED vertex does not treat particles and anti-particles differently. It is true that the muon to anti-muon change will generate a sign change in the matrix element, but this will disappear when it is squared. Accordingly we conclude that $e^-\mu^+ \to e^-\mu^+$ is also $\frac{s^2+u^2}{t^2}$. This could also be seen, as before, from considering angular momentum only.

4. This only leaves 2: $e^-e^- \to e^-e^-$ and 3: $e^-e^+ \to e^-e^+$ to be paired with B: $\frac{s^2+u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2+t^2}{u^2}$ and F: $\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}$ since each must go with one or the other as every 'quantity' must be used (so we are told). Here it is easy to tell which is which again from the propagators alone: Each process has two diagrams. The former has an s- and a u-channel diagram, the latter an s- and a t-channel diagram. Prior to summing over spins, the matrix elements M_1 and M_2 for each separate diagram will be added to make the total matrix element for $M = M_1 + M_2$ for that spin combination, which will itslef then be squared $|M|^2 = |M_1 + M_2|^2 = |M_1|^2 + |M_2|^2 + 2\Re(M_1 M_2^*)$ and then summed over spins. It is clear from these three terms, then, that we should expect a process requireing a mixture of s- and t-channels to have a matrix element built of a sum of (i) a spin-averaged matrix

element squared for the s-channel part, (ii) a similar term for the t-channel part, and (iii) a third term containing a product of an s-channel propagator and a t-channel propatator term (rather than either one on its own squared).

The sketching of the $\cos\theta$ distributions will, for the most part, consist of recalling that $t = -\frac{1}{2}(1 - \cos\theta)s$ and $u = \frac{1}{2}(1 + \cos\theta)s$ which (after squaring and realising that s is effectively a constant) give respectively backward or forward quadratic biasses in $\cos\theta$ to numerators, while s^2 terms give isotropic decay. The puropose of these sketches is not to catch students out, but rather to remind them, before they struggle with part (iii), that they SHOULD be able to predict largely the form of the cos-theta distribution from angular momentum conservation in most cases, and can thereby use this fact to help them answer (iii). Where there are t^2 terms in the denominator, there will inevitably be a divergence in the differential cross section in the forward region, as described in lectures. The hardest plots to draw will be two that come from crossed diagrams, as these have a cross term and (in one case) both forward and backward peaks. Nonetheless, sketching ought to be within the capability of the students given that they dependence of numerator and denominator separately is no worse than quadratic.

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2 Write detailed notes on **one** of the following topics:

(a) Baryon wave functions, **or** [30]

(b) Experiments that provide evidence for neutrino oscillations. [30]

(a) Baryon wave functions, **or** [30]

- •Mention of SU(3) colour
- •Mention of SU(3) flavour
- •Mentioning colour confinement
- •Relating confinement to colour singlet states
- •Constraints on symmetry of flavour and spin based on colour singlet plus Fermi condition relating to exchange of identical fermions (quarks)
- •Mention of Gell-Mann matrices
- •Drawing ‘triangleâĂİ of colour isospin and hypercharge for quarks.
- •Correctly locating quarks on diagram
- •Drawing anti-quark ‘triangle’
- •Colour singlets have I3C = 0 and YC = 0
- •Mention of ladder operators and relation to colour singlets
- •Indicating how to combine representations (multiple marsk)
- •3 x 3=8+1 and why banned
- •3 x 3 x 3=1+8+8+10 and why OK
- •Giving colour wave functions for qq or qqq

- •Giving spin wave functions for qq or qqq
- •Giving flavour wave functions for qq or qqq
- •Colour singlets only exist for qq and qqq
- •mention qqq colour singlet is anti-symmetric
- •mention of overall symmetry of wave-function
- •overall structure and clarity of argument
- •actual proton or neutron wave function given

(b) Experiments that provide evidence for neutrino oscillations. [30]

•This is not a theory question, but a connection should nonetheless be drawn between experiment and theory that establishes why it is worth looking for oscillations at all, and why this sets things like the length scales of the exeriments involved, etc (multiple marks are available here). The above might include a connections that lead to

$-\lambda_{21} 4\pi E/\Delta m^2_{21}$,

$-\Delta_{21} = 1.27\frac{\Delta^2_{21}(\text{eV}^2)L(\text{km})}{E(\text{GeV})}$ i,

$-\lambda_{\text{osc}} = 2.47\frac{E(\text{GeV})}{\Delta^2_m(\text{eV}^2)}$

and similar.

•Neutrino Charge Current interactions

•Neutrino Neutral Current intereactions

•Kinematic Interaction Thresholds and their variation with lepton flavour

•List sources of neutrinos (atmostpheric, solar, reactor, beam, ...)

•Describe neutrino energies for typical sources

•Relationship of thresholds to typical energies and consequences for detection

•Lower kinematic thresholds in neutrino-nucleon rather than neutrino-lepton interactions.

•ATMOS/BEAM $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ above 1 GeV using Water Cherenkov (e.g. Super Kamiokande) or Iron Caloromiters (e.g. MINOS, CDHS)

•Solar E below 20 MeV, ν_e only using Water Cherenkov (e.g. super K) and Radio-Chemical (e.g. Homestake, SAGE, GALLEX)

•Reactor $\bar{\nu}_e$ below 5 MeV using Liquid Scintillator (e.g. KamLAND) using Liquid Scintillator (e.g. KamLAND)

•Demonstrating that each technology understoo, i.e.:

•... Cherenkov Light from superluminal electrons and muons

•... minimal neutrino energy for cherenkov set by background radioactivity levels

•... difference between e and μ cherenkov signals

•... doubly magnetic Oxygen nucleus prevents $\nu_e + n \rightarrow e^- + p$

•... Radio-chemical methos use, e.g. $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$ extract chemically and count decays

•... Liquid scintillator (KamLand) low energies = large radioactive backround

- ... look for delayed co-inclidence between photon from neutron capture and light from prompt positron capture $\bar{\nu}_e + p \rightarrow e^+ n$
- Need for near and far detectors
- sampling iron calorimeters
- that MINOS shows $|\Delta m^2_{32}|$ is of order $2.4 * 10^{-3}\mathrm{eV}^2$
- SuperK or Homestake solar defecit shows DATA/StandardSolarModel = 0.45 ± 0.02. "The Solar Neutrino Problem". Uses angle to sun.
- SuperK up down atomspheric flux differencs show muon neutrino disappearance (to tau neutrinos). Sets $\sin\theta_{23} \approx 1/\sqrt{2}$
- Solar from SNO: CC NC and Elastic Scattering allow separate measurements of: (i) electron neutrino flux, (ii) total flux, and (iii) flux of muon and taus combined.
- Have Solar result $\Delta m^2_{\mathrm{solar}} \approx 8x10^{-5}\mathrm{eV}^2$ and $\sin^2 2\theta_{\mathrm{solar}} \approx 0.85$.
- Reactor constraints (Describe Chooz + Daya Bay + Kamland) and describe effect on θ_{13}, Δm^2_{21} and θ_{12}.

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3 In electron-proton scattering, the Lorentz invariant quantities:

$$s = (p_1 + p_2)^2, \quad Q^2 = -q^2 = -(p_1 - p_3)^2, \quad x = \frac{Q^2}{2p_2.q} \quad \text{and} \quad y = \frac{p_2.q}{p_1.p_2},$$

are defined in terms of p_1 and p_2, the four-momenta of the initial-state electron and proton respecively, and p_3, the four-momentum of the scattered electron. Neglecting the Q^2 dependence of the structure functions, F_1^{ep} and F_2^{ep}, the differential cross section for electron-proton deep inelastic scattering can be written as

$$\frac{d^2\sigma^{ep}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x)}{x} + y^2 F_1^{ep}(x)\right].$$

This is a re-hash of 2010 question 2. Though this paper is among the last 16 years of past tripos papers in this course that are available on the course website, it is one of the three years for which no worked solutions were ever published. Given that it is 7 years old, and a nice question, it seems OK to re-use it without much variation.

(a) For the case where the proton is at rest, express s, Q^2, x and y in terms of the proton mass, m_p, the electron scattering angle θ in the lab frame, and the energies of the incoming and scattered electron, E_1 and E_3. [4]

BOOKWORK[These variables were all defined and evaluated in the course handout, so a student could simply drop in values they recall here. Alternatively, they could derive them very quckly using little more than recall of how the Lorentz-invariant dot-product is defined.]

(b) In the parton model, show that x can be interpreted as the fraction of the proton's momentum carried by the struck quark in a frame where the proton has infinite momentum. Explain any assumptions made. [4]

BOOKWORK[Again this answer is largely bookwork as it involves re-capitulating the content of pages 188 ad 189 of the handout, these being the pages that describe the Quark-Parton model and identify x with the momentum fraction of the struck parton in the infinite momentum frame. It will be necessary here to neglect the mass of the struck parton in comparison to the energy of the proton, and to regard the struck parton having negligible transverse momentum.]

(c) The differential cross section for electron-quark scattering can be written as

$$\frac{d\sigma^{eq}}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

where e_q is the charge of the quark. Using the parton model, including contributions from the light quarks (u, d, s) only, show that

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)],$$

where $u(x)$, $d(x)$ and $s(x)$ are the up-, down- and strange-quark parton distribution functions for the proton. Obtain a similar expression for the electron-neutron structure function $F_2^{en}(x)$. [6]

BOOKWORK[This is very close to bookwork. The notes make much use of the Callan-Gross relation $F_2(x) = 2xF_1(x)$. With that in mind, it is a trivial exercise to spot that the difference btween the doubly differential ep scattering formula given in the rubric of the question differs from that given in this local part by the sum of the relevant pdfs each weighted by the charge of the relevant quark squared – as expected. The expression for the neutron should be the same as that of the proton, except that it needs neutron rather than proton structure functions.]

(d) Stating clearly any assumptions made, show that

$$\int_0^1 \frac{[F_2^{ep}(x) - F_2^{en}(x)]}{x}\mathrm{d}x = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]\mathrm{d}x$$

and comment on the consequences of the observed value being 0.24 ± 0.03. [6]

$$\int_0^1 \frac{[F_2^{ep} - F_2^{en}]}{x}\mathrm{d}x = \int_0^1 \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]dx$$

$$- \int_0^1 \frac{4}{9}[u^n(x) + \bar{u}^n(x)] + \frac{1}{9}[d^n(x) + \bar{d}^n(x) + s(x) + \bar{s}(x)]dx \qquad (12)$$

$$= \int_0^1 \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x))]dx$$

$$- \int_0^1 \frac{4}{9}[u^n(x) + \bar{u}^n(x)] + \frac{1}{9}[d^n(x) + \bar{d}^n(x)]dx \qquad \text{(cancelling the } s \text{ terms)} \qquad (13)$$

$$= \int_0^1 \frac{4}{9}[u(x)_v + 2\bar{u}(x)] + \frac{1}{9}[d_v(x) + 2\bar{d}(x))]dx$$

$$- \int_0^1 \frac{4}{9}[u_v^n(x) + 2\bar{u}^n(x)] + \frac{1}{9}[d_v^n(x) + 2\bar{d}^n(x)]dx \qquad (14)$$

$$= \int_0^1 \frac{4}{9}[u(x)_v + 2\bar{u}(x)] + \frac{1}{9}[d_v(x) + 2\bar{d}(x))]dx$$

$$- \int_0^1 \frac{4}{9}[d_v(x) + 2\bar{d}(x)] + \frac{1}{9}[u_v(x) + 2\bar{u}(x)]dx \qquad (15)$$

$$= \int_0^1 \frac{1}{3}[u(x)_v - d_v(x)]\mathrm{d}x + \int_0^1 \frac{2}{3}[\bar{u}(x) - \bar{d}(x)]\mathrm{d}x \qquad \text{(collecting terms)} \qquad (16)$$

$$= \frac{1}{3}(2 - 1) + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]\mathrm{d}x \qquad (17)$$

wherein the first step we have assumed that $s(s)$ and $\bar{s}(x)$ are identical for neutron and proton, and wherein the third step we have split the quark PDFs into valence and sea parts $u(x) = u_v(x) + u_s(x)$, $d(x) = d_v(x) + d_s(x)$ and simultanesouly assumed that the sea u distribution is the same as the $\bar{u}$ disribution etc, i.e. have replaced $u_s(x)$ with $\bar{u}(x)$ etc. In the fourth step we assumed isospin symmetry $u \leftrightarrow d^n$, $d \leftrightarrow u^n$ (both for sea and for valence quarks). In the sixth step we assumed two valence up quarks in the proton ($\int_0^1 u_v(x)\mathrm{d} = 2$) and two valence down quark in the proton ($\int_0^1 d_v(x)\mathrm{d} = 1$).

Since the measured value is significantly less than 1/3, and since the prediction is $1/3$ plus an integral of a difference between the number of sea anti-ups and sea anti-downs, we can interpret this result as telling us that there are more anti-downs in the proton than anti-ups, at least on average (i.e. when integrated over x). This is likely to mean the same thing for the sea-component of the ups and downs too, since they should be created most frequently by gluon splitting.

(e) In the parton model for neutrino-nucleon scattering the structure functions are

$$F_2^{\nu p}(x) = 2x[d(x) + s(x) + \bar{u}(x)] \qquad \text{and} \qquad F_2^{\nu n}(x) = 2x[u(x) + \bar{s}(x) + \bar{d}(x)].$$

Assuming $s(x) = \bar{s}(x)$, obtain an expression for $xs(x)$ in terms of the structure functions for neutrino- and electron-nucleon scattering,

$F_2^{\nu N}(x) = \frac{1}{2}(F_2^{\nu p}(x) + F_2^{\nu n}(x))$ and $F_2^{eN}(x) = \frac{1}{2}(F_2^{ep}(x) + F_2^{en}(x))$. [7]

$$\begin{aligned}
F_2^{\nu N}(x) &= \frac{1}{2}(F_2^{\nu p}(x) + F_2^{\nu n}(x)) && (18)\\
&= \frac{1}{2}(2x[d(x) + s(x) + \bar{u}(x)] + 2x[u(x) + \bar{s}(x) + \bar{d}(x)]) && (19)\\
&= x[d(x) + s(x) + \bar{u}(x) + u(x) + \bar{s}(x) + \bar{d}(x)] && (20)\\
&= x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] + 2xs(x). \qquad \text{(assuming } s(x) = \bar{s}(x)\text{))} && (21)
\end{aligned}$$

$$\begin{aligned}
F_2^{eN}(x) &= \frac{1}{2}(F_2^{ep}(x) + F_2^{en}(x)) && (22)\\
&= \frac{4}{9}x[u(x) + \bar{u}(x)] + \frac{1}{9}x[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]\\
&+ \frac{4}{9}x[u^n(x) + \bar{u}^n(x)] + \frac{1}{9}x[d^n(x) + \bar{d}^n(x) + s(x) + \bar{s}(x)] && (23)\\
&= \frac{4}{9}x[u(x) + \bar{u}(x)] + \frac{1}{9}x[d(x) + \bar{d}(x) + 2s(x)]\\
&+ \frac{4}{9}x[d(x) + \bar{d}(x)] + \frac{1}{9}x[u(x) + \bar{u}(x) + 2s(x)] && (24)\\
&= \frac{5}{9}x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] + \frac{4}{9}xs(x). && (25)
\end{aligned}$$

where again we assumed isospin symmetry $u \leftrightarrow d^n$, $d \leftrightarrow u^n$. Putting these two results together:

$$\begin{aligned}
\frac{5}{9}F_2^{\nu N}(x) - F_2^{eN}(x) &= \left(\frac{10}{9} - \frac{4}{9}\right)xs(x) && (26)\\
&= \frac{2}{3}xs(x) && (27)
\end{aligned}$$

and so

$$\begin{aligned}
xs(x) &= \frac{3}{2}\left(\frac{5}{9}F_2^{\nu N}(x) - F_2^{eN}(x)\right) && (28)\\
&= \frac{5}{6}F_2^{\nu N}(x) - \frac{3}{2}F_2^{eN}(x). && (29)
\end{aligned}$$

(f) Provide possible physical explanations for why $\bar{d}(x) > \bar{u}(x) > \bar{s}(x)$. [3]

The strange quark has by most methods of determination a substantially higher mass than the up or down, and so should be considerably kinematically suppressed in the sea with respect to the other two. It is harder to say why the sea contribution of the d should be higher than that of the u. One cannot rely again on mass since the up and down are very similar in mass and almost massless relative to the proton (having 0.2% and 0.5% of the proton's mass), unlike the strange quark (which has about 10% of the proton mass, a significant fraction - given that only half of the momentum of a proton is typically carried by quarks!). In the lecture course the supplied plot of proton parton distribution functions (with $Q^2 = 10\mathrm{GeV}^2$) indicated the sea d-quark contribution was a 20 to 30% higher than that of the u, noting that this effect was 'not understood – exclusion principle?'. This would be acceptable as an answer if backed up by some additional explanation. E.g. "Whenever a sea $\bar{u}$ is created by a gluon, there must be an associated u. This u would then find itself competing against the valence up quarks in the proton for phase space. Given that there are already two valence up quarks in the proton but only one valence down quark, sea up quarks (being fermions and therefore being subject to Fermi's exclusion principle) might find it harder than down quarks to find states that are not already occupied by other similar quarks. This might disfavour $\bar{u}(x)$ with respect to $\bar{d}(x)$."

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