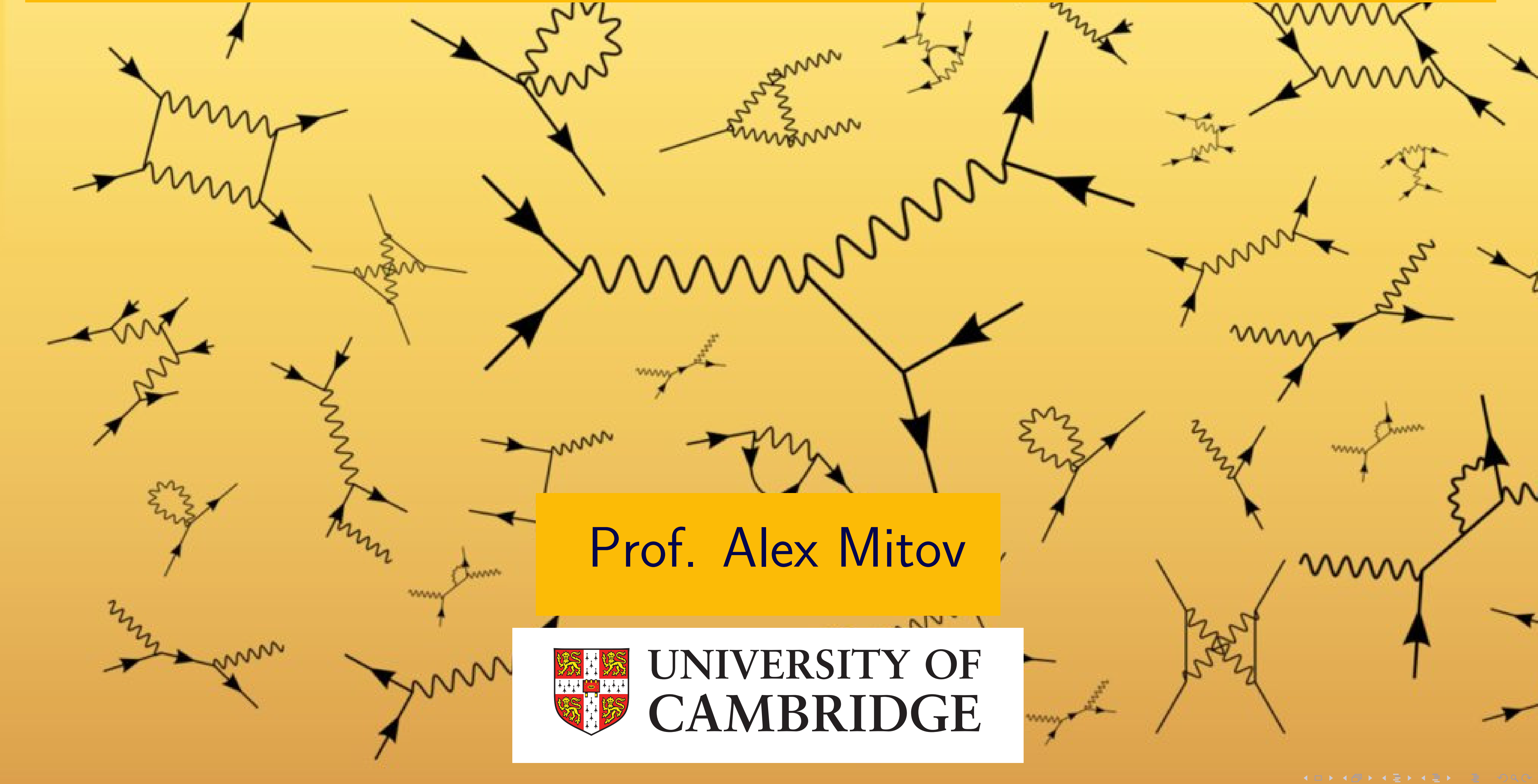
5. Feynman Diagrams

Particle and Nuclear Physics



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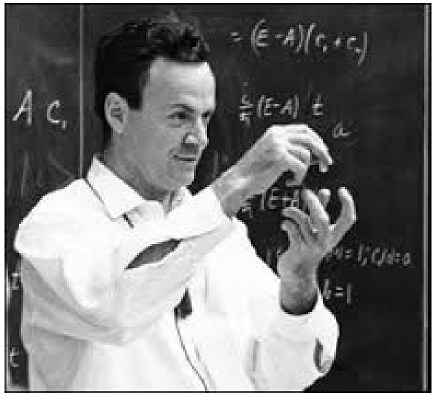
In this section...

- Introduction to Feynman diagrams.
- Anatomy of Feynman diagrams.
- Allowed vertices.
- General rules

Feynman Diagrams

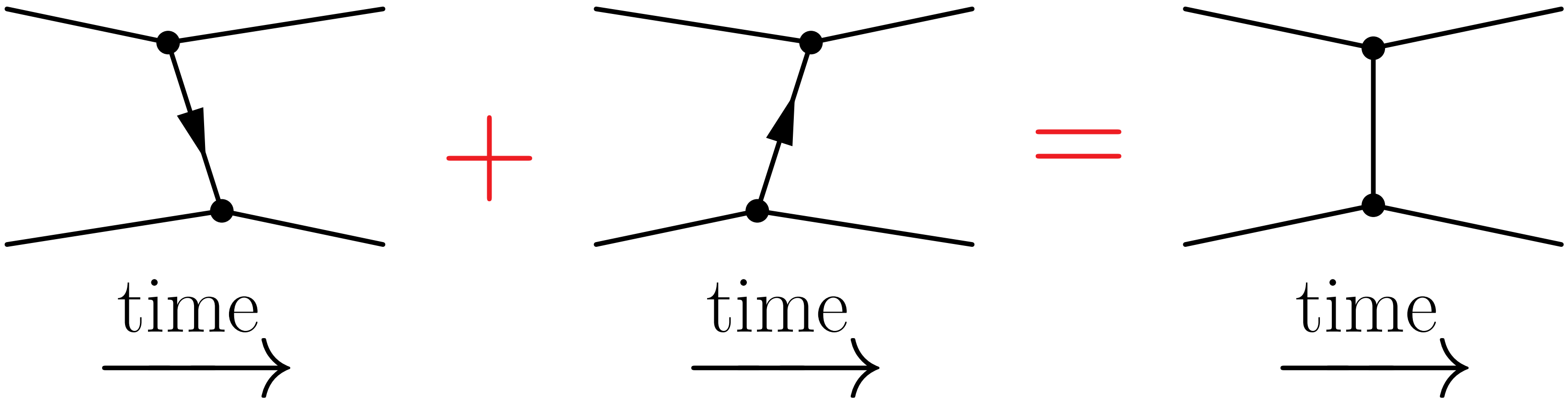


Richard Feynman
1965 Nobel Prize

The results of calculations based on a single process in Time-Ordered Perturbation Theory (sometimes called old-fashioned, OFPT) depend on the reference frame.

The sum of all time orderings is frame independent and provides the basis for our relativistic theory of Quantum Mechanics.

A Feynman diagram represents the sum of **all** time orderings



Feynman Diagrams

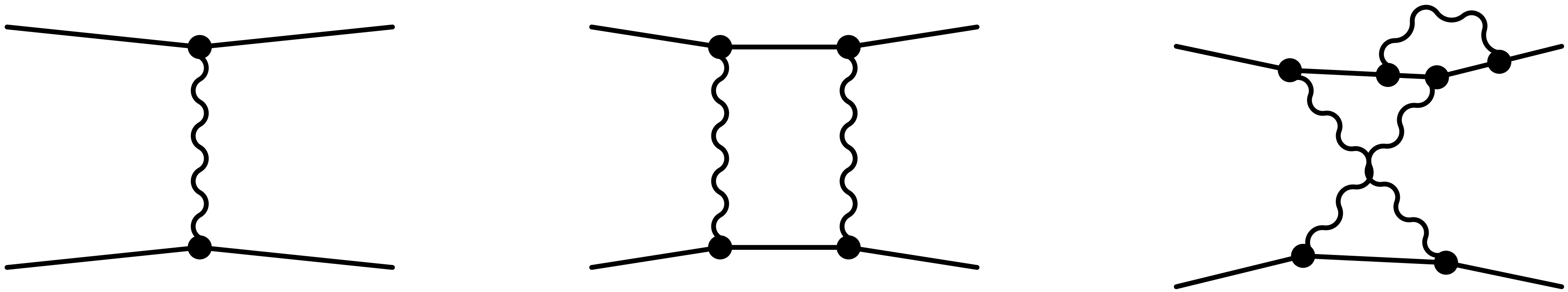
Each Feynman diagram represents a term in the perturbation theory expansion of the matrix element for an interaction.

Normally, a full matrix element contains an infinite number of Feynman diagrams.

Total amplitude $\quad M_{\mathrm{fi}} = M_1 + M_2 + M_3 + \ldots$

Total rate $\quad \Gamma_{\mathrm{fi}} = 2\pi |M_1 + M_2 + M_3 + \ldots|^2 \rho(E) \quad$ Fermi's Golden Rule

But each vertex gives a factor of g, so if g is small (i.e. the perturbation is small) only need the first few. *(Lowest order = fewest vertices possible)*



g^2 $\qquad$ g^4 $\qquad$ g^6

Example: QED $\quad g = e = \sqrt{4\pi\alpha} \sim 0.30, \quad \alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$

Feynman Diagrams

Perturbation Theory

Calculating Matrix Elements from Perturbation Theory from first principles is cumbersome – so we don't usually use it.

- Need to do time-ordered sums of (on mass shell) particles whose production and decay does not conserve energy and momentum.

Feynman Diagrams

Represent the maths of Perturbation Theory with Feynman Diagrams in a very simple way (to arbitrary order, if couplings are small enough). Use them to calculate matrix elements.

- Approx size of matrix element may be estimated from the simplest valid Feynman Diagram for given process.
- Full matrix element requires infinite number of diagrams.
- Now only need one exchanged particle, but it is now off mass shell, however production/decay now conserves energy and momentum.

Anatomy of Feynman Diagrams

Feynman devised a pictorial method for evaluating matrix elements for the interactions between fundamental particles in a few simple rules. We shall use Feynman diagrams extensively throughout this course.

Topological features of Feynman diagrams are straightforwardly associated with terms in the Matrix element

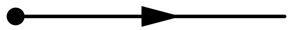
Represent particles (and antiparticles):

Spin 1/2	Quarks and Leptons	
Spin 1	γ, $W^{\pm}$, Z	
	g	

And each interaction point (vertex) with a $\bullet$
Each vertex contributes a factor of the coupling constant, g.

Anatomy of Feynman Diagrams

External lines (visible real particles)

Spin 1/2	Particle		Incoming
			Outgoing
	Antiparticle		Incoming
			Outgoing
Spin 1	Particle		Incoming
			Outgoing

Internal lines (propagators; virtual particles)

Spin 1/2	Particle/antiparticle		Each propagator gives a factor of $\dfrac{1}{q^2 - m^2}$
Spin 1	γ, $W^{\pm}$, Z		
	g		

Vertices

A vertex represents a point of interaction: either EM, weak or strong.

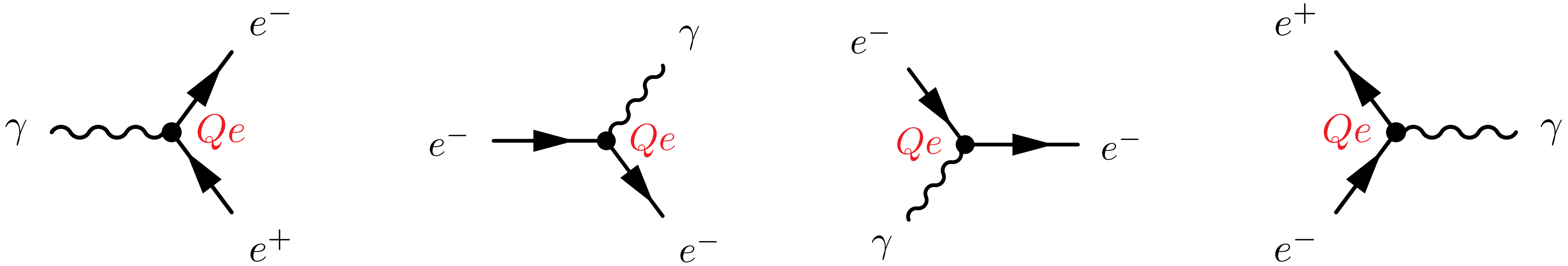
The strength of the interaction is denoted by g

EM interaction: $g = Qe$ (sometimes denoted as $Q\sqrt{\alpha}$, where $\alpha = e^2/4\pi$)

Weak interaction: $g = g_W$

Strong interaction: $g = \sqrt{\alpha_s}$

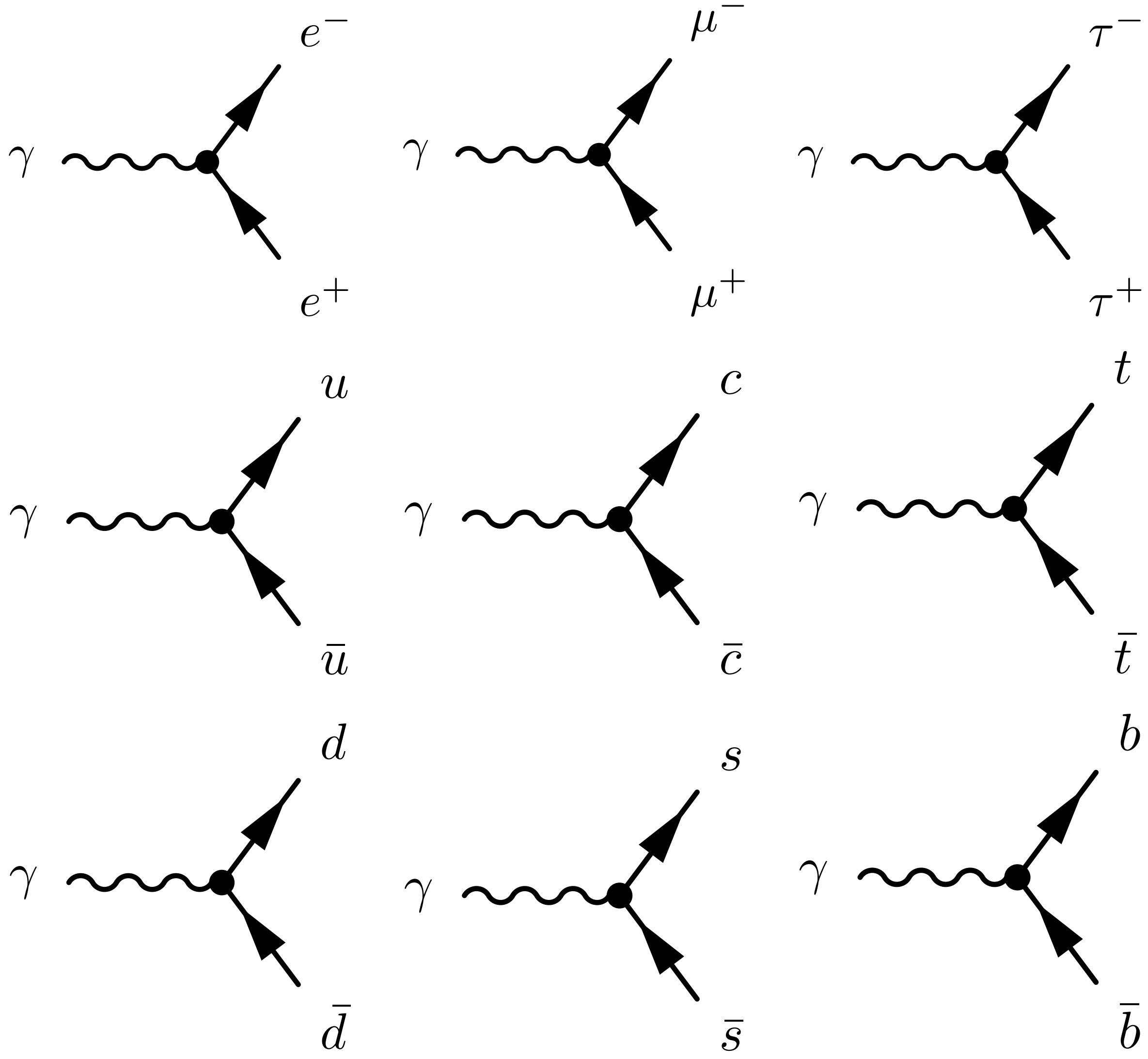
A vertex will have three (in rare cases four) lines attached, e.g.



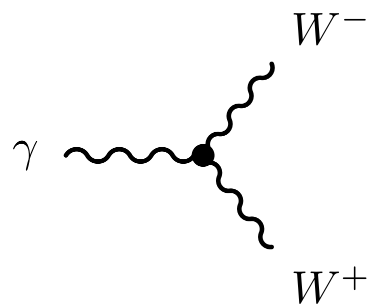
At each vertex, conserve energy, momentum, angular momentum, charge, lepton number ($L_e = +1$ for e^-, ν_e, $= -1$ for $e^+, \bar{\nu}_e$, similar for L_μ, L_τ), baryon number ($B = \frac{1}{3}(n_q - n_{\bar{q}})$), strangeness ($S = -(n_s - n_{\bar{s}})$) & parity – except in weak interactions.

Allowed Vertices *EM*

- must involve a photon γ, and charged particles
- coupling strength Qe Q=charge



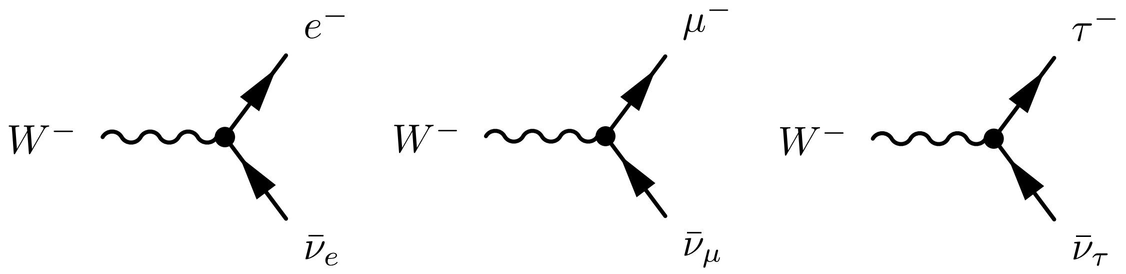
Triple Gauge Vertex

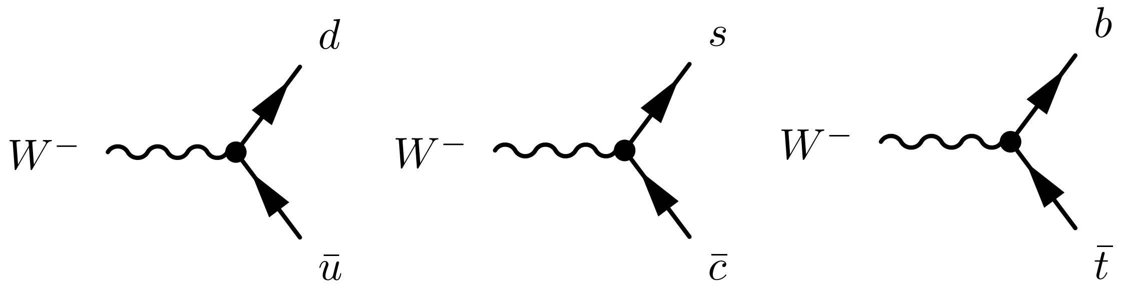


Allowed Vertices *Weak*

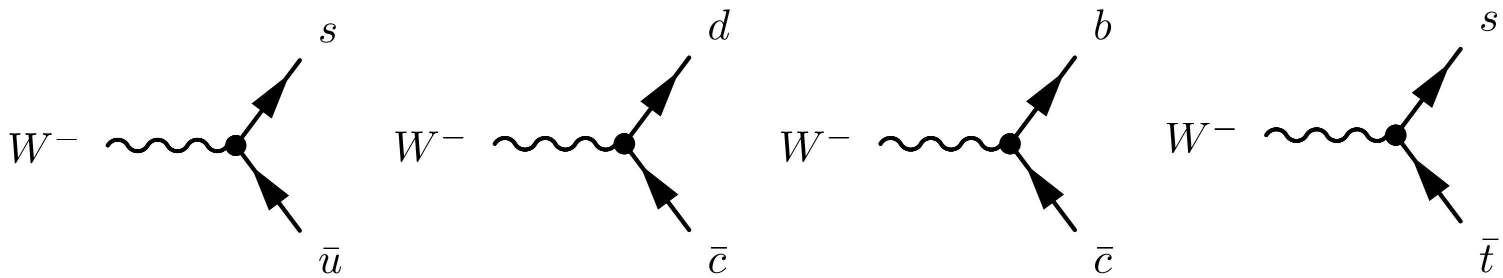
- must involve a gauge vector boson Z or $W^{\pm}$
- coupling strength g_W
- tip: if you see a ν or $\bar{\nu}$, it must be a weak interaction

with $W^{\pm}$





$\Rightarrow$ Same family quarks are **Cabibbo favoured**

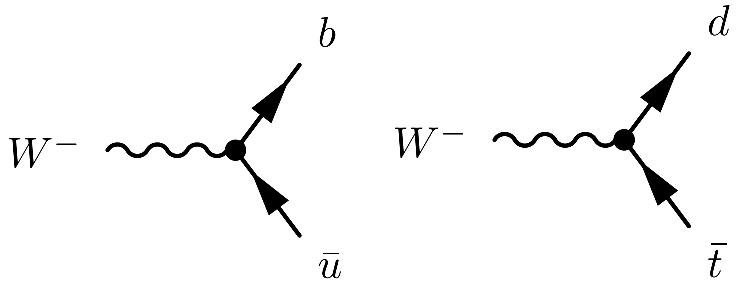


$\Rightarrow$ Cross one family **Cabibbo suppressed**

Allowed Vertices *Weak*

- must involve a gauge vector boson Z or $W^{\pm}$
- coupling strength g_W
- tip: if you see a ν or $\bar{\nu}$, it must be a weak interaction

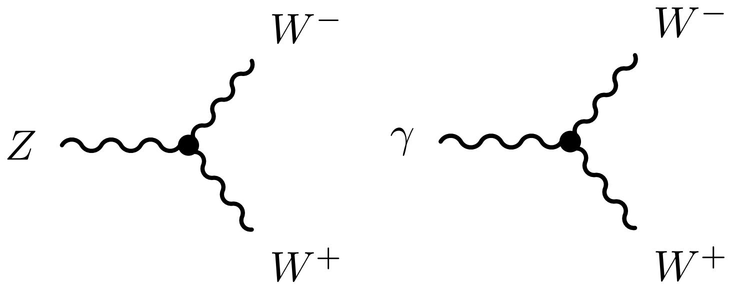
with $W^{\pm}$

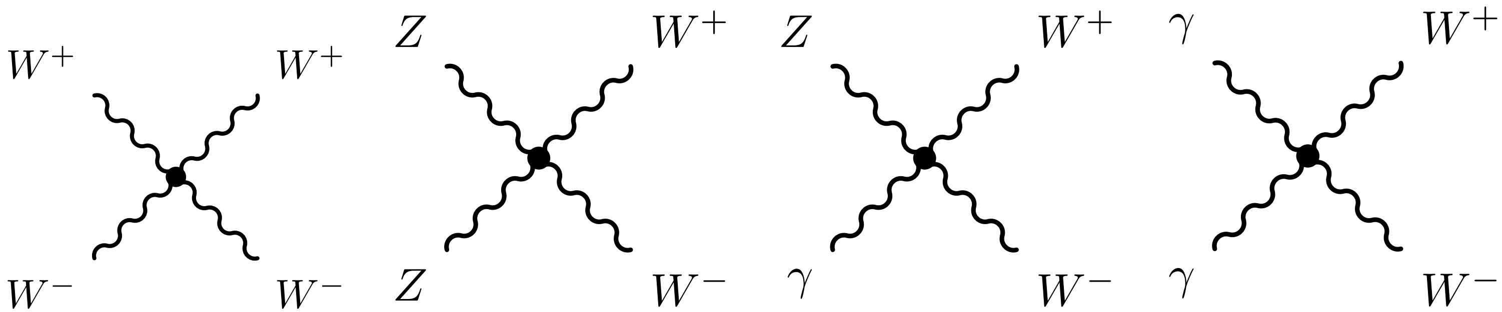


$\Rightarrow$ Cross two families

Doubly Cabibbo suppressed

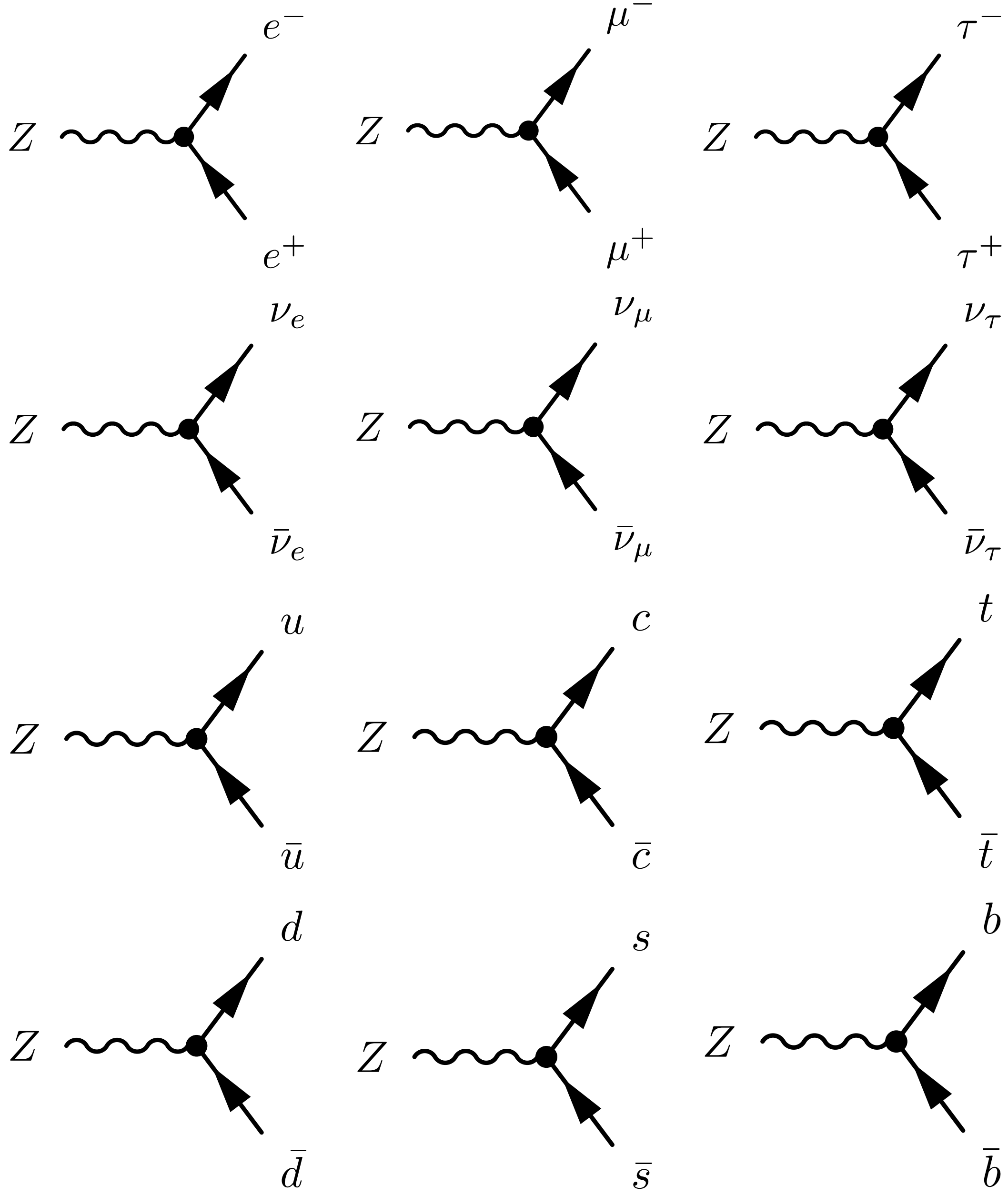
Also, Triple/Four Gauge Vertex



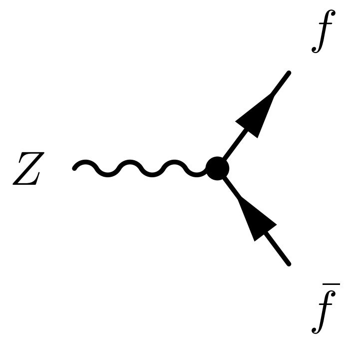


Allowed Vertices *Weak*

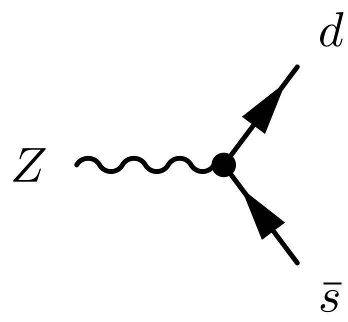
with Z Same as γ diagrams, but also vertices with ν



i.e.

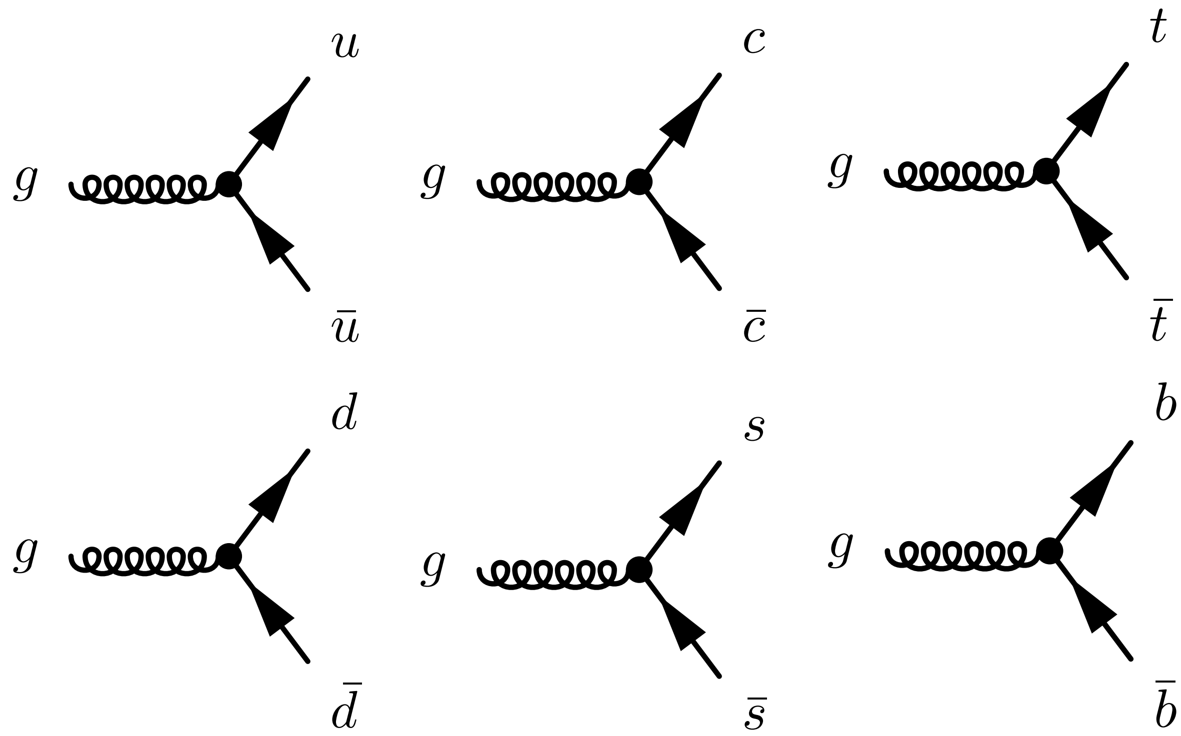


Not Allowed:
Flavour Changing
Neutral Currents (FCNC)

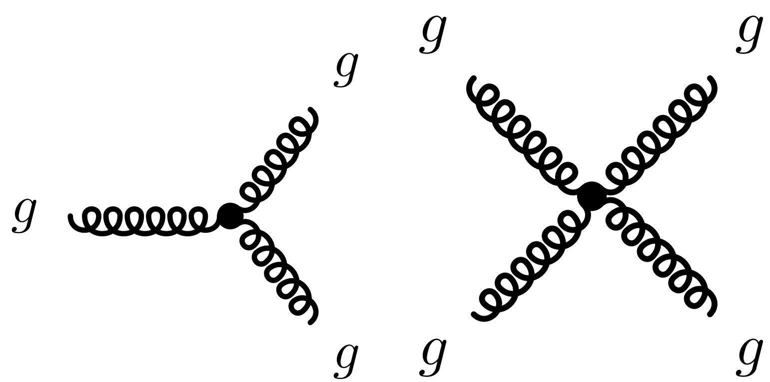


Allowed Vertices *Strong*

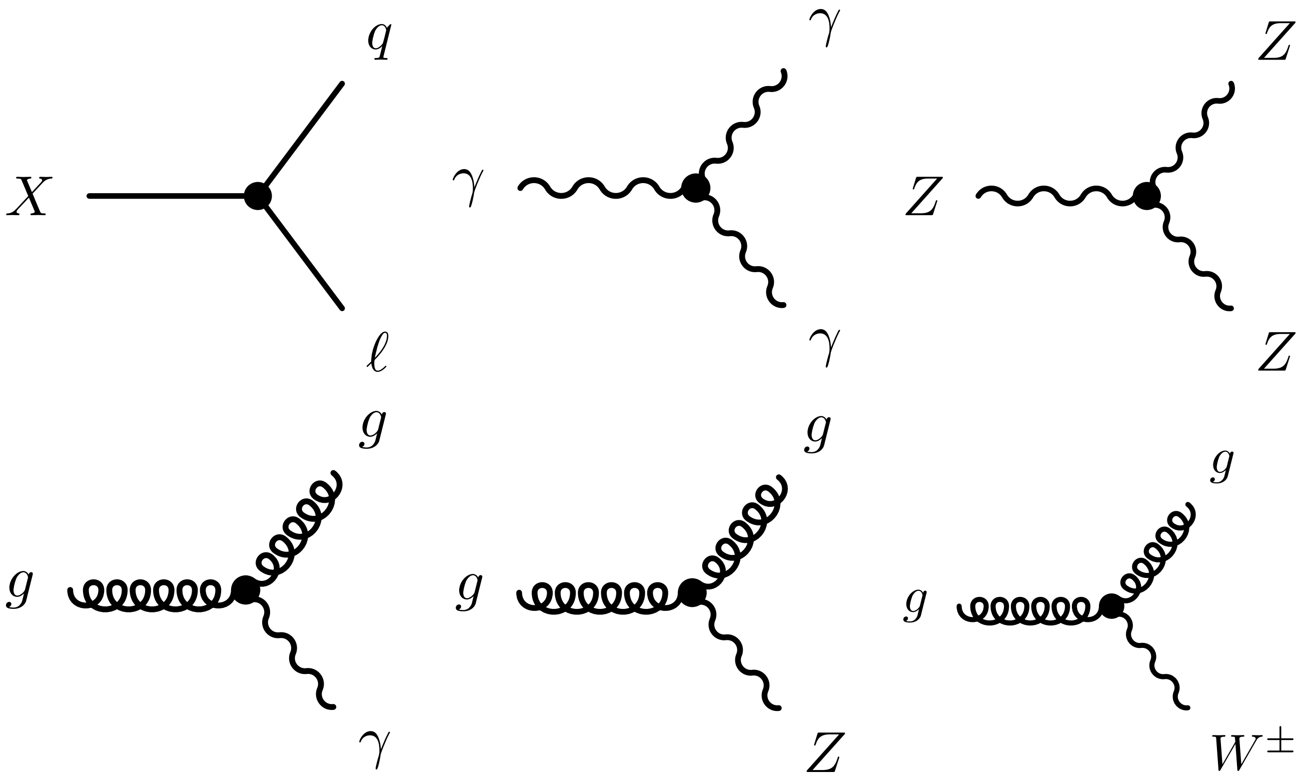
- must involve a gluon g and/or quark q
- coupling strength $\sqrt{\alpha_s}$
- conserve strangeness, charm etc



Also, Triple Gauge Vertex

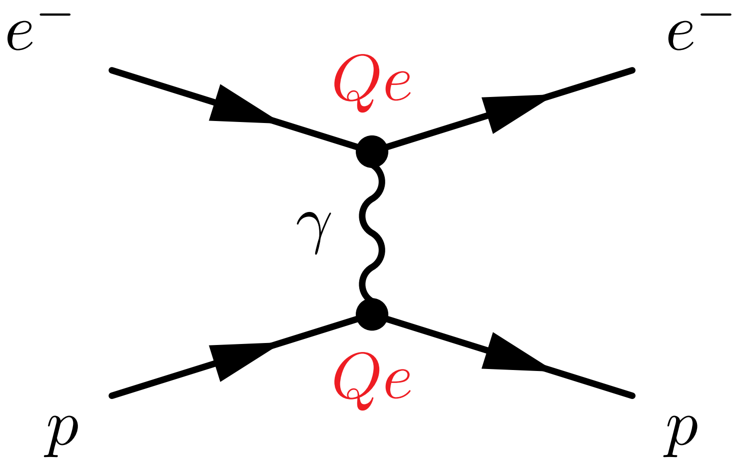


Forbidden Vertices



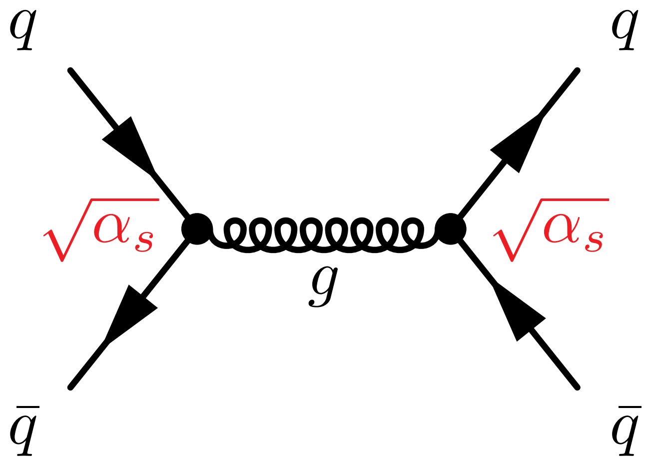
Examples

Electromagnetic



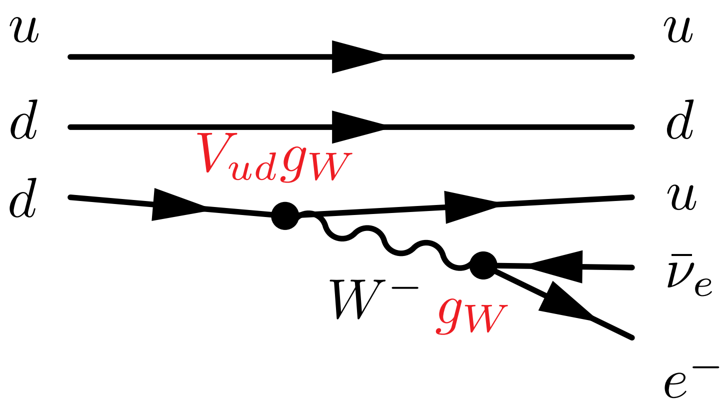
$$M \sim \frac{(e)^2}{q^2}$$

Strong



$$M \sim \frac{(\sqrt{\alpha_s})^2}{q^2}$$

Weak



$$M \sim \frac{V_{ud} g_W^2}{q^2 - m_W^2}$$

Drawing Feynman Diagrams

A Feynman diagram is a pictorial representation of the matrix element describing particle decay or interaction

$$a \rightarrow b + c + \ldots \qquad a + b \rightarrow c + d$$

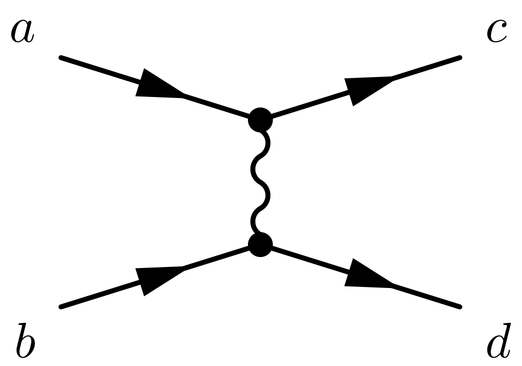
To draw a Feynman diagram and determine whether a process is allowed, follow the five basic steps below:

1. Write down the initial and final state particles and antiparticles and note the quark content of all hadrons.
2. Draw the simplest Feynman diagram using the Standard Model vertices. Bearing in mind:
   - Similar diagrams for particles/antiparticles
   - Never have a vertex connecting a lepton to a quark
   - Only the weak charged current ($W^{\pm}$) vertex changes flavour
     within generations for leptons
     within/between generations for quarks

Drawing Feynman Diagrams *Particle scattering*

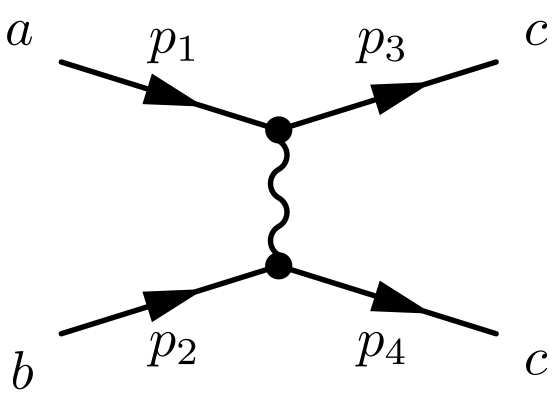
- If all are particles (or all are antiparticles), only scattering diagrams involved e.g. $a + b \to c + d$

Initial State

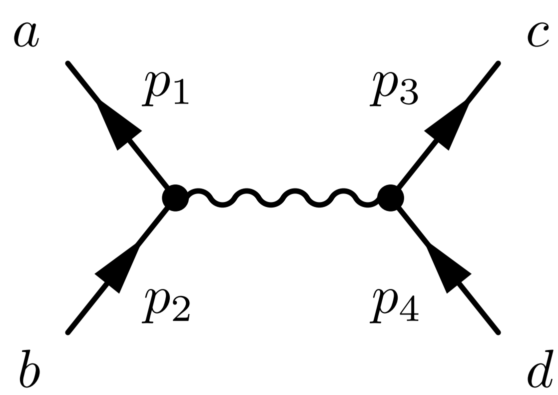


Final State

- If particles and antiparticles, may be able to have scattering and/or annihilation diagrams e.g. $a + b \to c + d$ (Mandelstam variables s, t, u)



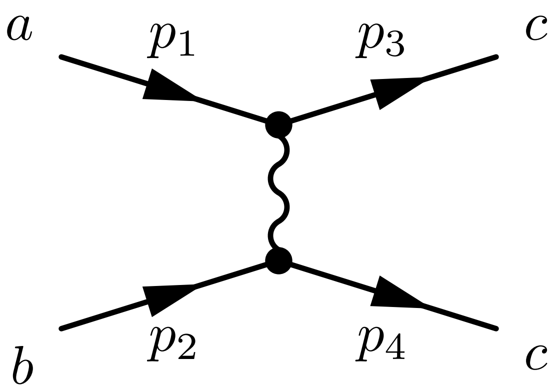
"t-channel",
$q^2 = t = (p_1 - p_3)^2 = (p_2 - p_4)^2$



"s-channel",
$q^2 = s = (p_1 + p_2)^2 = (p_3 + p_4)^2$

If we have identical particles in final state, e.g. $a + b \to c + c$
may not know which particle comes from which vertex.

Two possibilities are separate final Feynman diagrams:



"t-channel",
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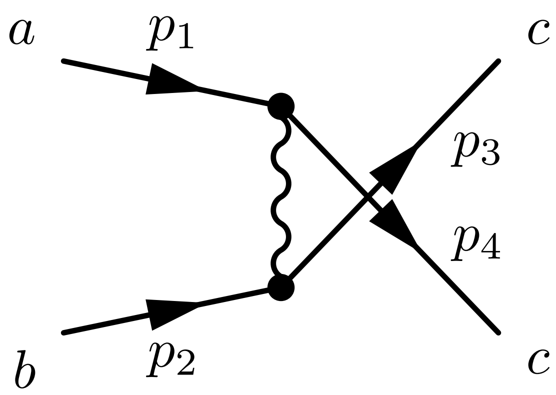

"u-channel",
$q^2 = u = (p_1 - p_4)^2 = (p_2 - p_3)^2$

Crossing not a vertex

Drawing Feynman Diagrams

Being able to draw a Feynman diagram is a necessary, but not a sufficient condition for the process to occur. Also need to check:

3. Check that the whole system conserves
   - Energy, momentum (trivially satisfied for interactions, so long as sufficient KE in initial state. May forbid decays)
   - Charge
   - Angular momentum
4. Parity
   - Conserved in EM/Strong interaction
   - Can be violated in the Weak interaction
5. Check symmetry for identical particles in the final state
   - Bosons $\psi(1,2) = +\psi(2,1)$
   - Fermions $\psi(1,2) = -\psi(2,1)$

Finally, a process will occur via the Strong, EM and Weak interaction (in that order of preference) if steps 1 – 5 are satisfied.

Summary

- Feynman diagrams are a core part of the course.
  Make sure you can draw them!
- Feynman diagrams are a sum over time orderings.
- Associate topological features of the diagrams with terms in matrix elements.
- Vertices $\leftrightarrow$ coupling strength between particles and field quanta
- Propagator for each internal line (off-mass shell, virtual particles)
- Conservation of quantum numbers at each vertex

Problem Sheet: q.11

Up next...
Section 6: QED