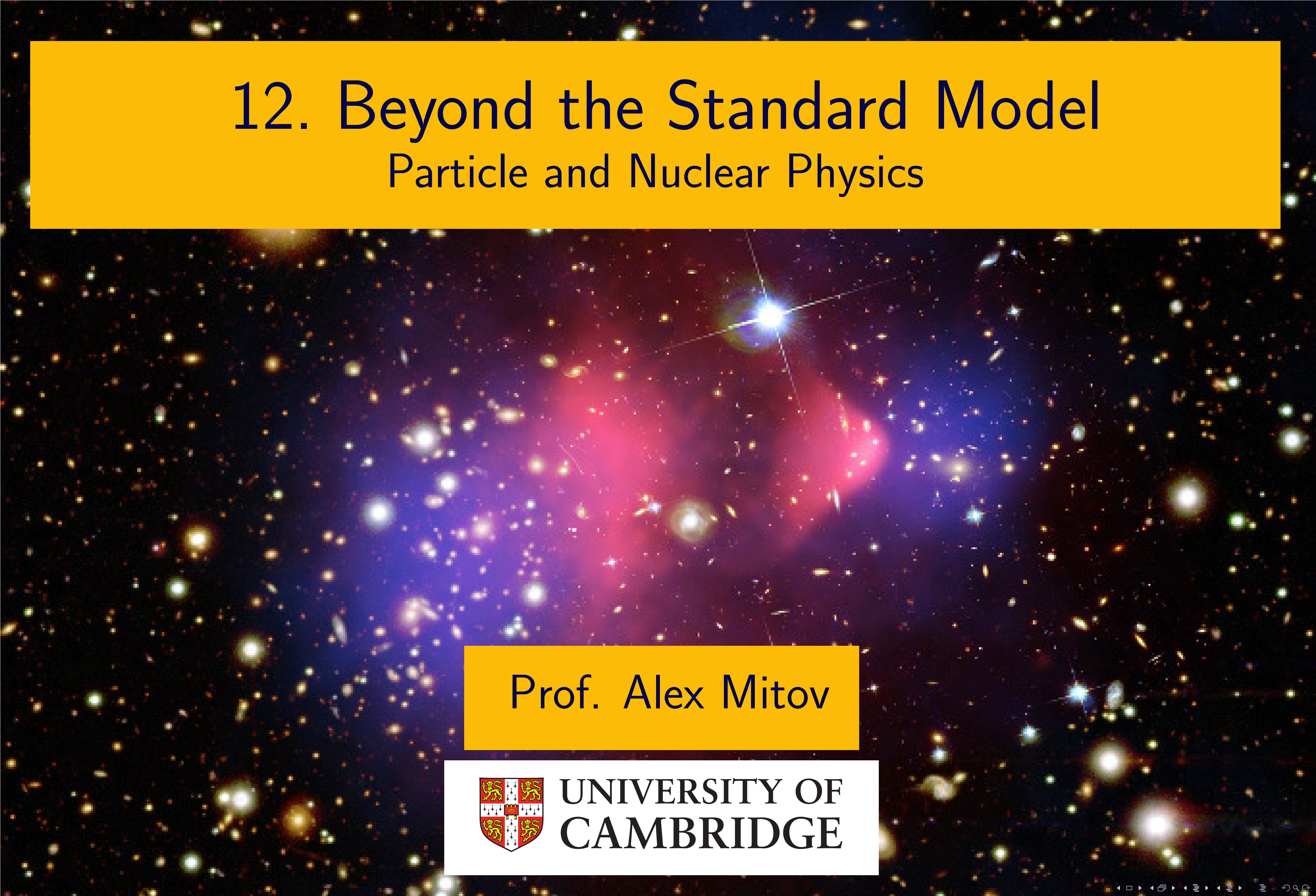

12. Beyond the Standard Model

Particle and Nuclear Physics

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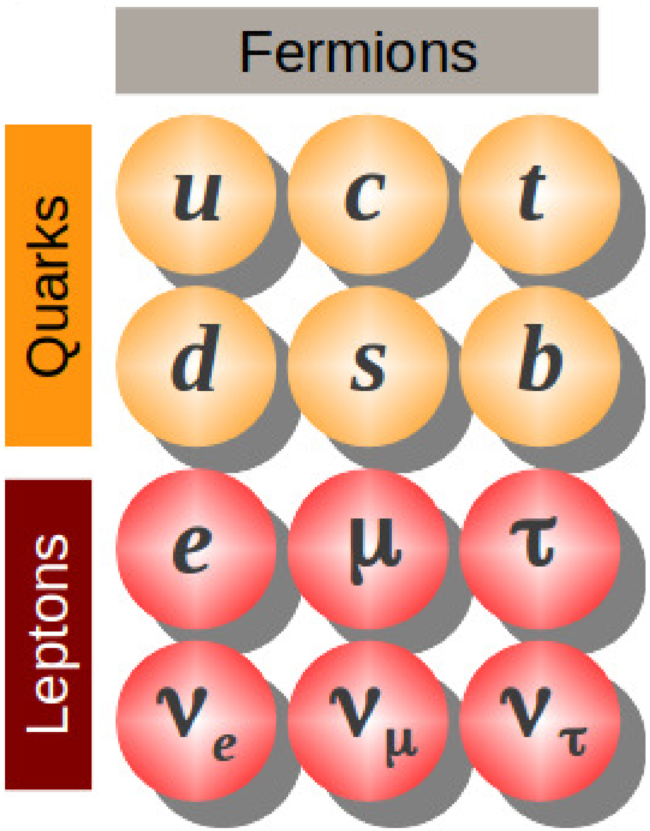
In this section...

- Summary of the Standard Model
- Problems with the Standard Model
- Neutrino oscillations
- Supersymmetry

The Standard Model (2012)

Matter: point-like spin $\frac{1}{2}$ Dirac fermions

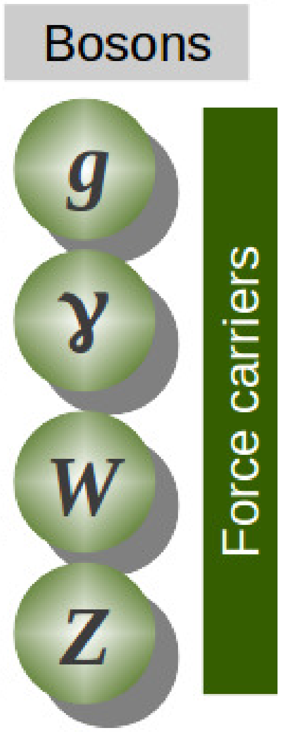


+ antiparticles

	Fermion		Charge $[e]$	Mass
1^{st} gen.	Electron	e^-	-1	$0.511\ \text{MeV}$
	Electron neutrino	ν_e	0	~ 0
	Down quark	d	$-1/3$	$4.8\ \text{MeV}$
	Up quark	u	$+2/3$	$2.3\ \text{MeV}$
2^{nd} gen.	Muon	μ^-	-1	$106\ \text{MeV}$
	Muon neutrino	ν_μ	0	~ 0
	Strange quark	s	$-1/3$	$95\ \text{MeV}$
	Charm quark	c	$+2/3$	$1.3\ \text{GeV}$
3^{rd} gen.	Tau	τ^-	-1	$1.78\ \text{GeV}$
	Tau neutrino	ν_τ	0	~ 0
	Bottom quark	b	$-1/3$	$4.7\ \text{GeV}$
	Top quark	t	$+2/3$	$173\ \text{GeV}$

The Standard Model (2012)

Forces: mediated by spin 1 bosons



Force	**Particle**	**Mass**
Electromagnetic	Photon γ	0
Strong	8 gluons g	0
Weak (CC)	$W^{\pm}$	80.4 GeV
Weak (NC)	Z	91.2 GeV

- The Standard Model also predicts the existence of a spin-0 Higgs boson which gives all particles their masses via its interactions. Evidence from LHC confirms this, with $m_H \sim 125\ \mathrm{GeV}$.

- The Standard Model successfully describes all existing particle physics data, with the exception of one

  $\Rightarrow$ Neutrino Oscillations $\Rightarrow$ Neutrinos have mass

  In the SM, neutrinos are treated as massless; right-handed states do not exist $\Rightarrow$ indication of physics Beyond the Standard Model

Problems with the Standard Model

The Standard Model successfully describes all existing particle physics data (though question marks over the neutrino sector).

But: many (too many?) input parameters:

- Quark and lepton masses
- Quark charge
- Couplings $\alpha_{\rm EM}$, $\sin^2\theta_W$, α_s
- Quark (+ neutrino) generation mixing – $V_{\rm CKM}$

23 free parameters in SM
- 9 fermion masses (e, μ, τ, u, d, s, c, b, t)
- 4 CKM: 3 mixing angles + CPV phase
- 4 PMNS: 3 mixing angles + CPV phase
- 3 gauge couplings: U(1), SU(2), SU(3)
- 3 other: QCD vacuum angle (strong CPV), Higgs VEV, Higgs mass

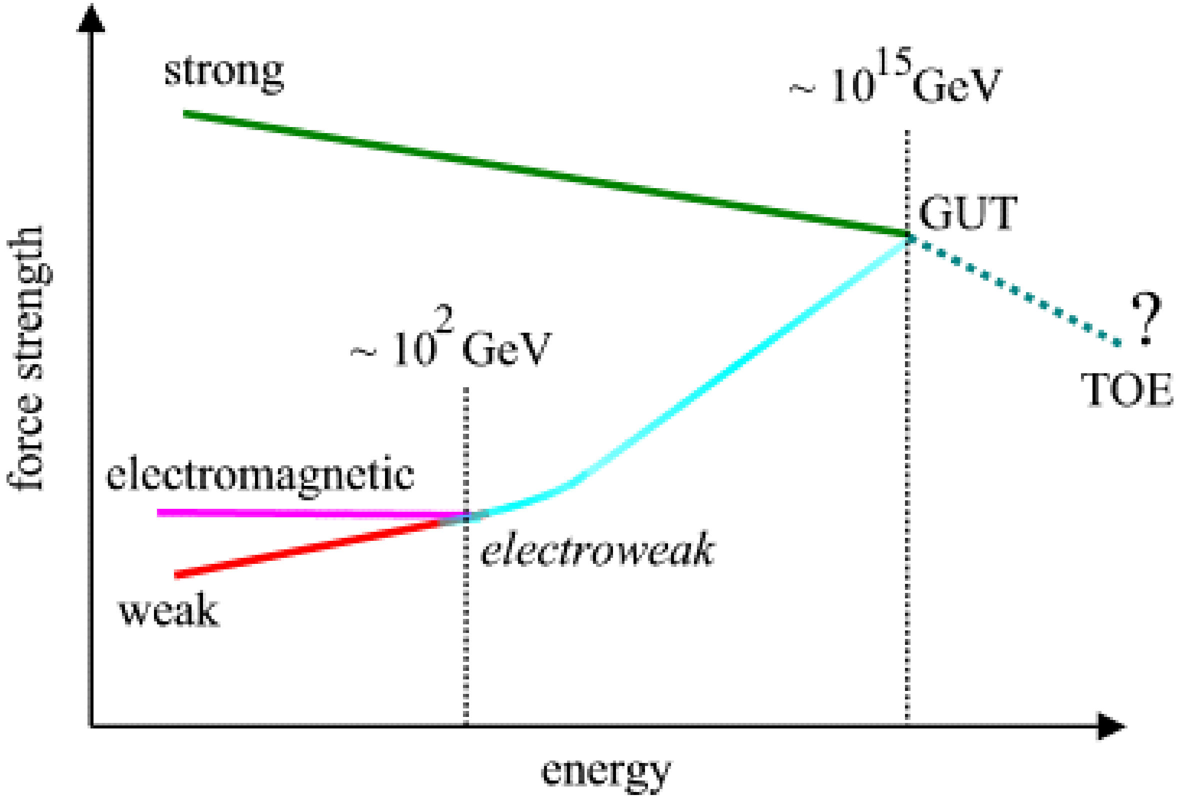
and: many unanswered questions:

- Why so many free parameters?
- Why only three generations of quarks and leptons?
- Where does mass come from? (Higgs boson probably OK)
- Why is the neutrino mass so small and the top quark mass so large?
- Why are the charges of the p and e identical?
- What is responsible for the observed matter-antimatter asymmetry?
- How can we include gravity?

etc

Beyond the Standard Model – further unification??

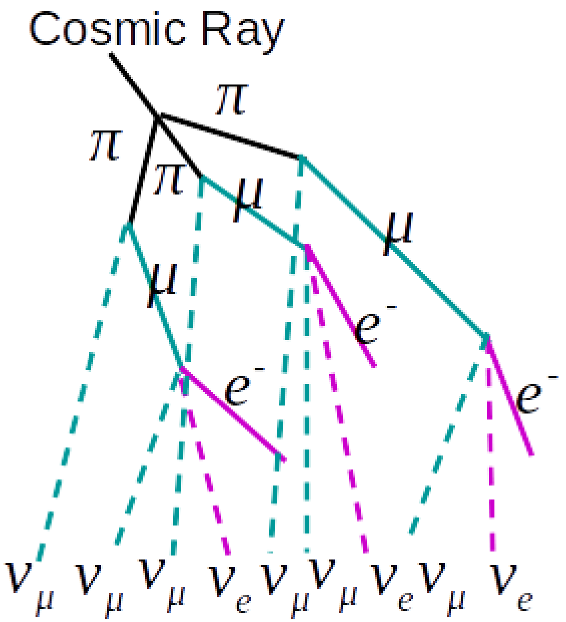
Grand Unification Theories (GUTs) aim to unite the strong interaction with the electroweak interaction. Underpins many ideas about physics beyond the Standard Model.
The strength of the interactions depends on energy:



- Suggests unification of all forces at $\sim 10^{15}$ GeV?
- Strength of Gravity only significant at the Planck Mass $\sim 10^{19}$ GeV

Neutrino Oscillations

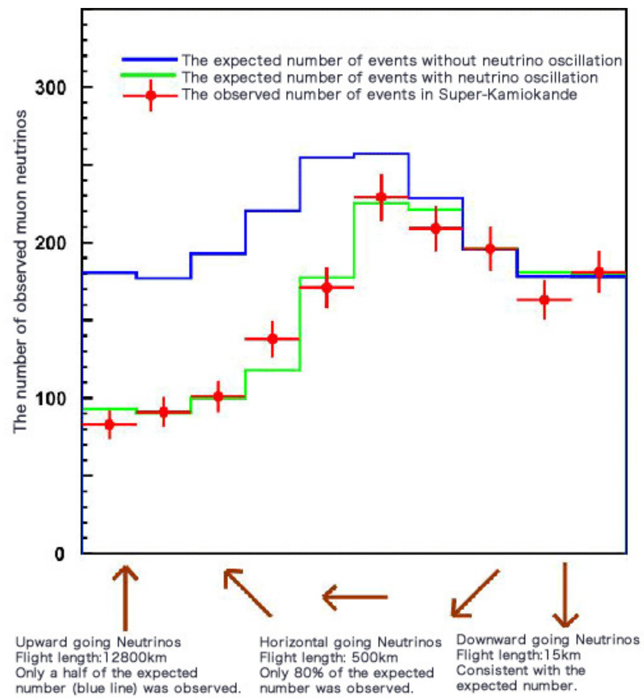
In 1998 the Super-Kamiokande experiment announced convincing evidence for neutrino oscillations implying that neutrinos have mass.



$$\pi \to \mu\nu_\mu$$
$$\hookrightarrow e\nu_\mu\bar{\nu}_e$$

Expect

$$\frac{N(\nu_\mu)}{N(\nu_e)} \sim 2$$



Super-Kamiokande results indicate a deficit of ν_μ from the upwards direction. Upward neutrinos created further away from the detector.

- Interpreted as $\nu_\mu \to \nu_\tau$ oscillations
- Implies neutrino mixing and neutrinos have mass

Detecting Neutrinos

Neutrinos are detected by observing the lepton produced in charged current interactions with nuclei. e.g. $\nu_e + N \to e^- + X$ $\bar{\nu}_\mu + N \to \mu^+ + X$

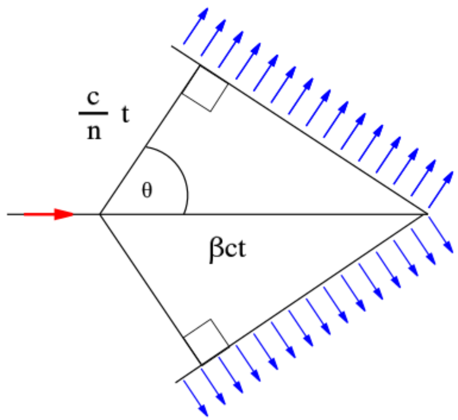
Size Matters:

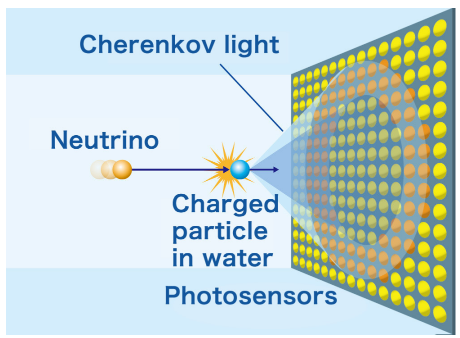
- Neutrino cross-sections on nucleons are tiny; $\sim 10^{-42}(E_\nu/\ \mathrm{GeV})\mathrm{m}^2$
- Neutrino mean free path in water $\sim$ light-years.
- Require very large mass, cheap and simple detectors.
- Water Čerenkov detection

Čerenkov radiation

- Light is emitted when a charged particle traverses a dielectric medium
- A coherent wavefront forms when the velocity of a charged particle exceeds c/n ($n =$ refractive index)
- Čerenkov radiation is emitted in a cone i.e. at fixed angle with respect to the particle.

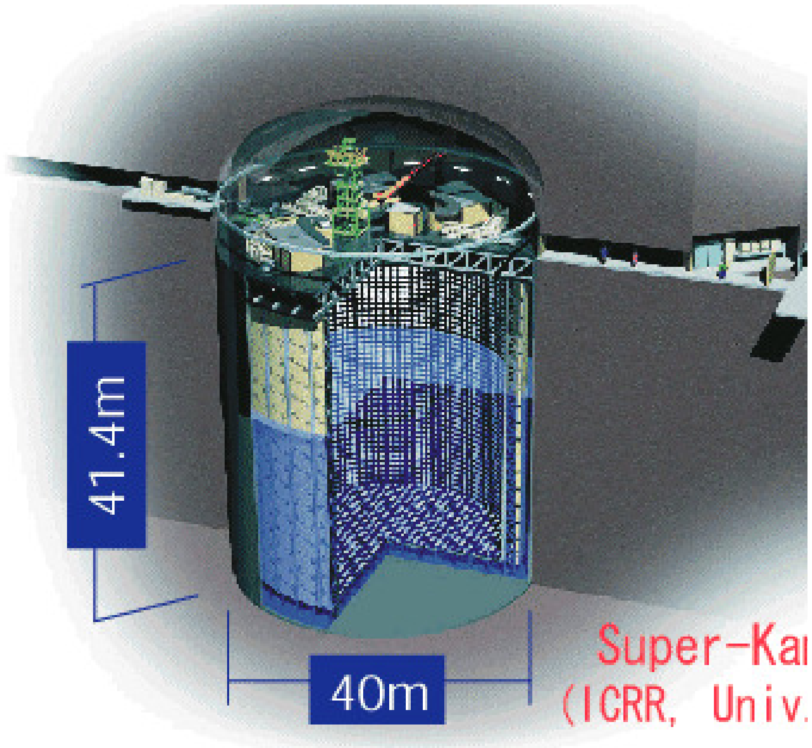
$$\cos\theta_C = \frac{c}{nv} = \frac{1}{n\beta}$$

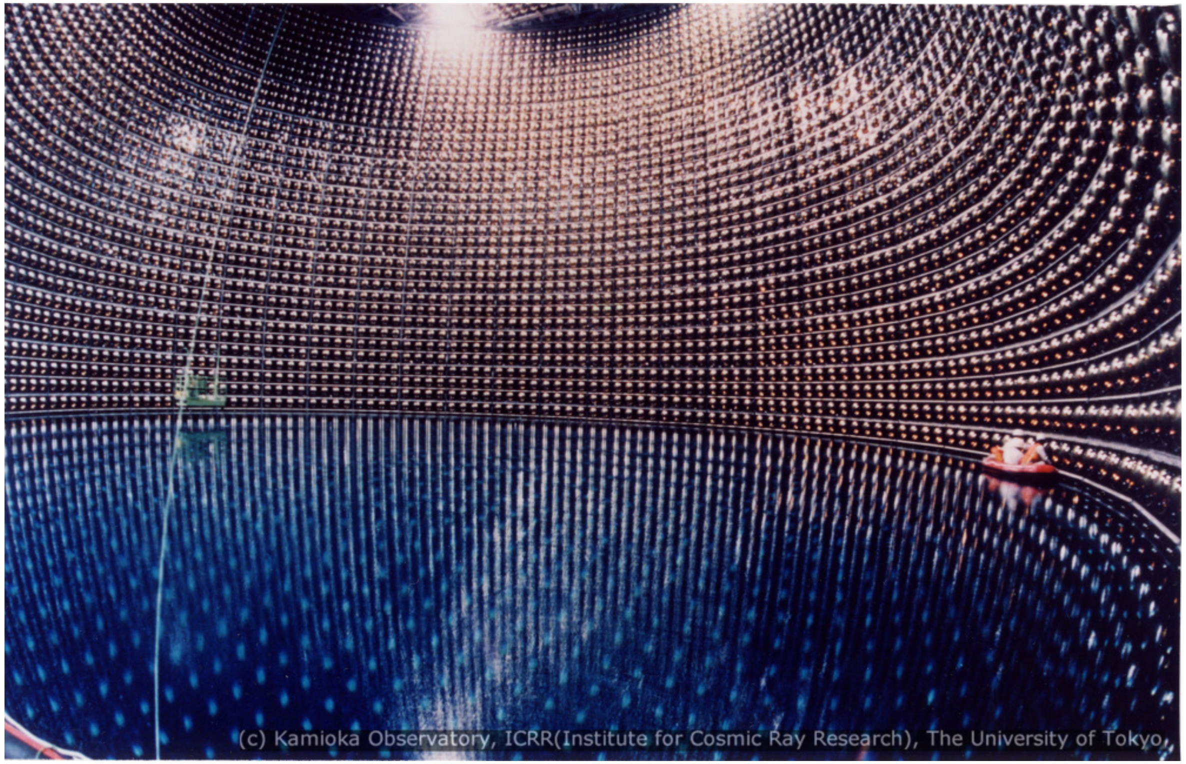




Super-Kamiokande

Super-Kamiokande is a Water Čerenkov detector sited in Kamioka, Japan



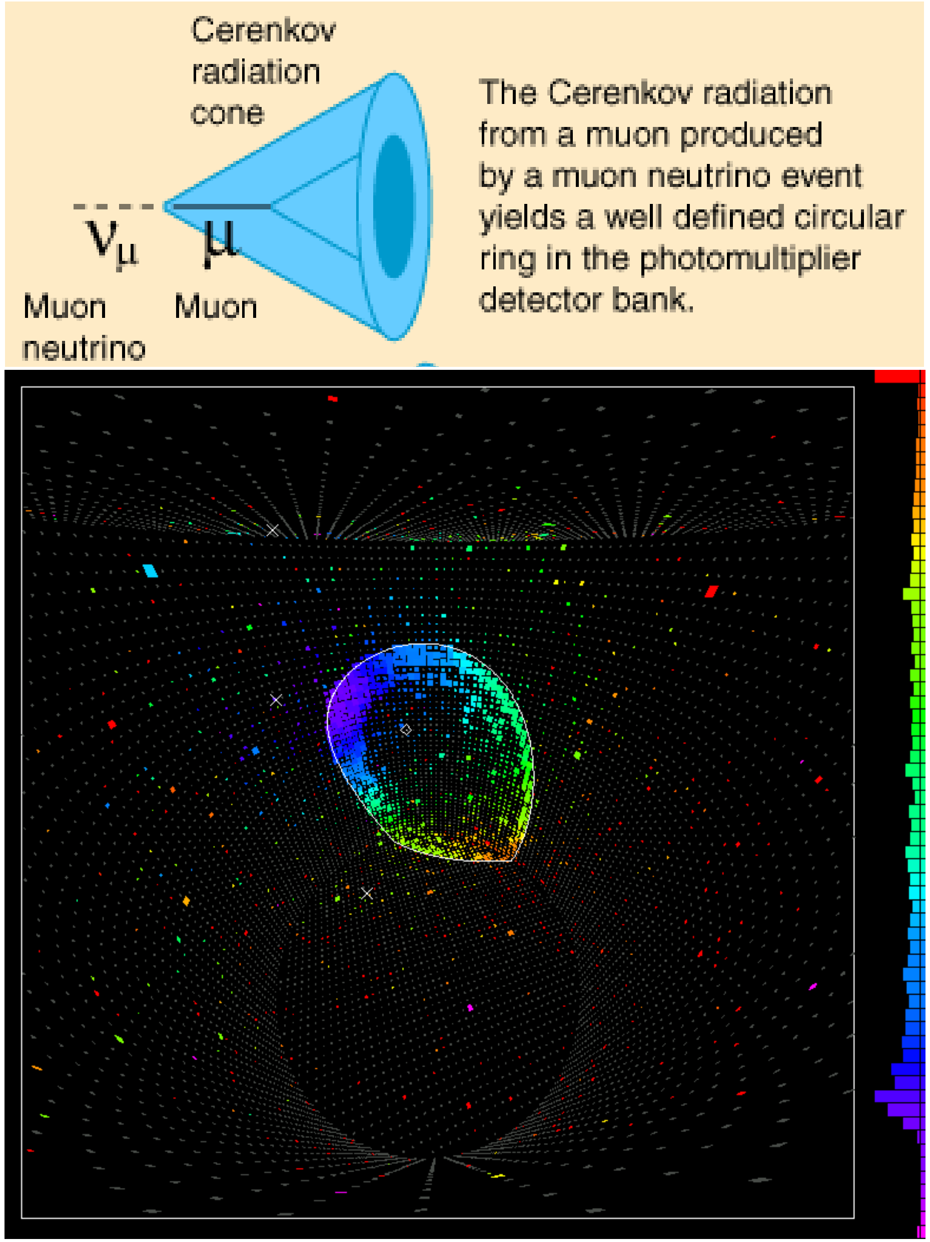


$50,000$ tons of water

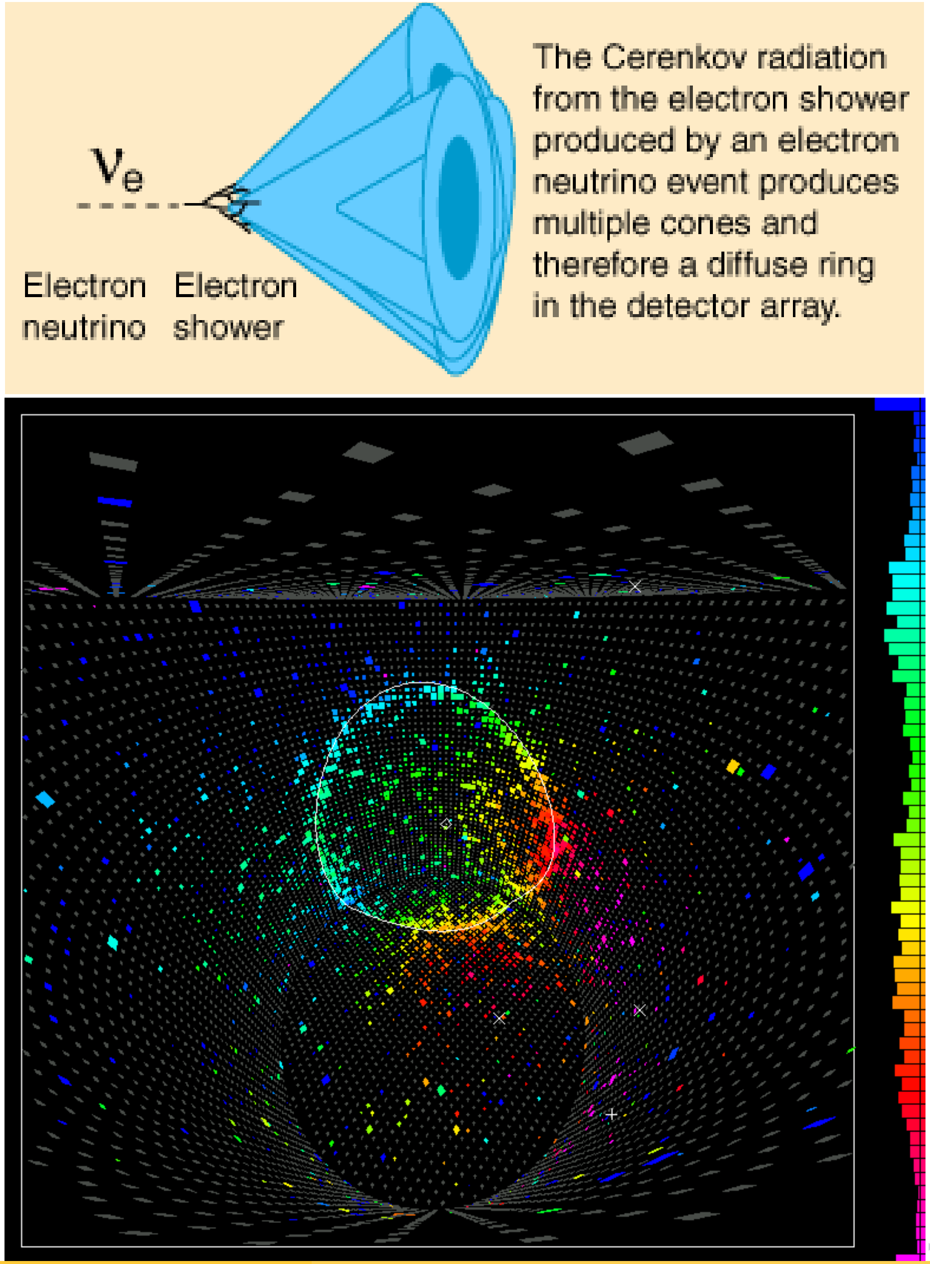
Surrounded by $11,146 \times 50\,\mathrm{cm}$ diameter, photo-multiplier tubes

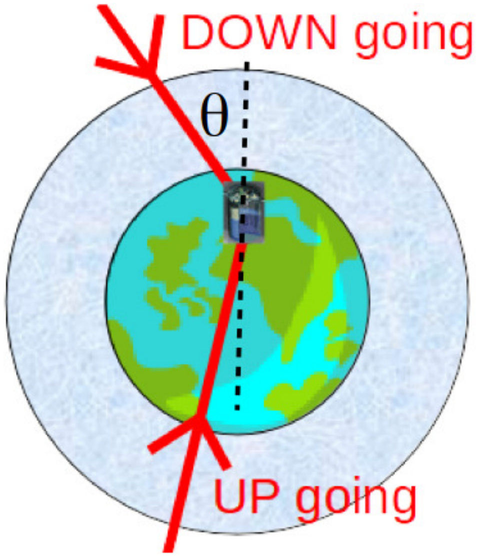
Super-Kamiokande *Examples of events*

$$\nu_\mu + N \to \mu^- + X$$



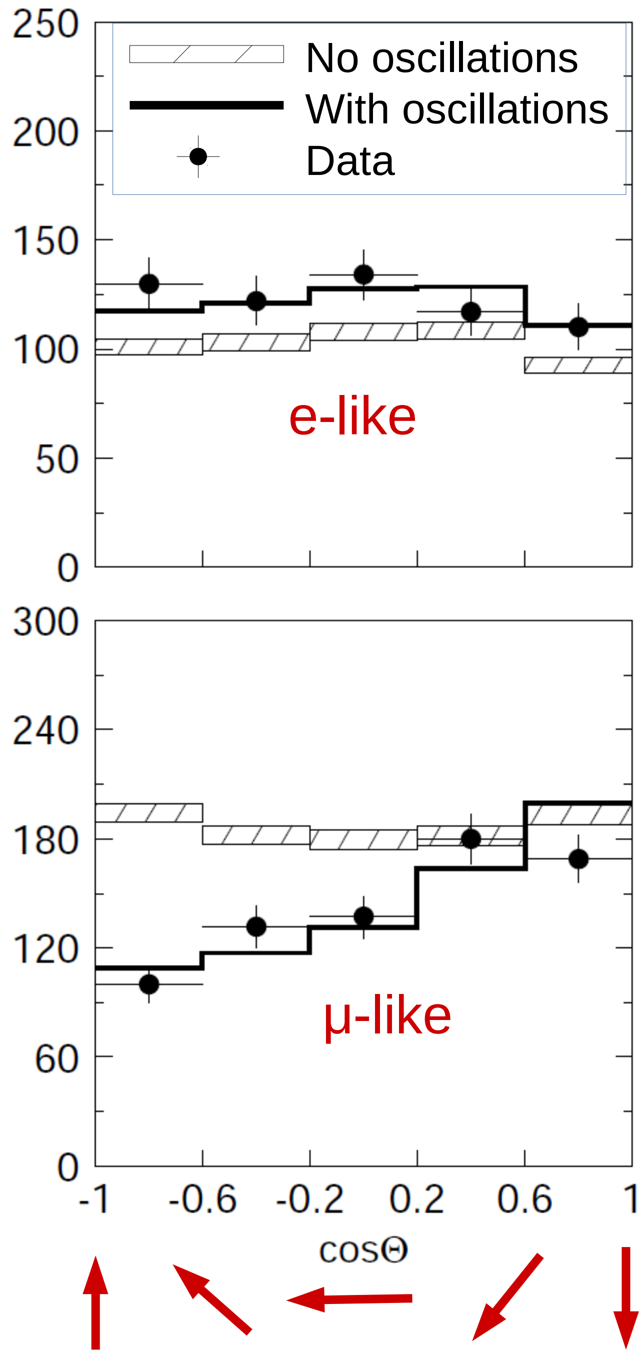
$$\nu_e + N \to e^- + X$$





Expect

- Isotropic (flat) distributions in $\cos\theta$
- $N(\nu_\mu) \sim 2N(\nu_e)$



Observe

- Deficit of ν_μ from **below**
- Whereas ν_e look as expected

Interpretation

- $\nu_\mu \to \nu_\tau$ oscillations
- $\Rightarrow$ neutrinos have mass

Neutrino Mixing

The quark states which take part in the weak interaction (d', s') are related to the flavour (mass) states (d, s)

Weak Eigenstates
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$
Mass Eigenstates

Cabibbo angle $\theta_C \sim 13^\circ$

Suppose the same thing happens for neutrinos. Consider only the first two generations for simplicity.

Weak Eigenstates

= flavour eigenstates
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$
Mass Eigenstates

Mixing angle θ

e.g. in π^+ decay produce μ^+ and ν_μ i.e. the neutrino state that couples to the weak interaction.

The ν_μ corresponds to a linear combination of the states with definite mass, ν_1 and ν_2

$$\nu_e = +\nu_1 \cos\theta + \nu_2 \sin\theta$$
$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta$$

or expressing the mass eigenstates in terms of the weak eigenstates

$$\nu_1 = +\nu_e \cos\theta - \nu_\mu \sin\theta$$
$$\nu_2 = +\nu_e \sin\theta + \nu_\mu \cos\theta$$

Neutrino Mixing

Neutrino Mixing

Neutrino Mixing

Neutrino Mixing

Suppose a muon neutrino with momentum $\vec{p}$ is produced in a weak decay, e.g. $\pi^+ \to \mu^+ \nu_\mu$

At $t = 0$, the wavefunction

$$\psi(\vec{p}, t = 0) = \nu_\mu(\vec{p}) = \nu_2(\vec{p}) \cos\theta - \nu_1(\vec{p}) \sin\theta$$

The time evolution of ν_1 and ν_2 will be different if they have different masses

$$\nu_1(\vec{p}, t) = \nu_1(\vec{p}) \mathrm{e}^{-iE_1 t} \; ; \qquad \nu_2(\vec{p}, t) = \nu_2(\vec{p}) \mathrm{e}^{-iE_2 t}$$

After time t, state will in general be a mixture of ν_e and ν_μ

$$\begin{aligned}
\psi(\vec{p}, t) &= \nu_2(\vec{p}) \mathrm{e}^{-iE_2 t} \cos\theta - \nu_1(\vec{p}) \mathrm{e}^{-iE_1 t} \sin\theta \\
&= [\nu_e(\vec{p}) \sin\theta + \nu_\mu(\vec{p}) \cos\theta]\, \mathrm{e}^{-iE_2 t} \cos\theta - [\nu_e(\vec{p}) \cos\theta - \nu_\mu(\vec{p}) \sin\theta]\, \mathrm{e}^{-iE_1 t} \sin\theta \\
&= \nu_\mu(\vec{p}) \left[\cos^2\theta \mathrm{e}^{-iE_2 t} + \sin^2\theta \mathrm{e}^{-iE_1 t}\right] + \nu_e(\vec{p}) \left[\sin\theta \cos\theta \left(\mathrm{e}^{-iE_2 t} - \mathrm{e}^{-iE_1 t}\right)\right] \\
&= c_\mu \nu_\mu(\vec{p}) + c_e \nu_e(\vec{p})
\end{aligned}$$

Neutrino Mixing

Probability of oscillating into ν_e

$$P(\nu_e) = |c_e|^2 = \left|\sin\theta\cos\theta\left(\mathrm{e}^{-iE_2 t} - \mathrm{e}^{-iE_1 t}\right)\right|^2$$
$$= \frac{1}{4}\sin^2 2\theta\left(\mathrm{e}^{-iE_2 t} - \mathrm{e}^{-iE_1 t}\right)\left(\mathrm{e}^{iE_2 t} - \mathrm{e}^{iE_1 t}\right)$$
$$= \frac{1}{4}\sin^2 2\theta\left(2 - \mathrm{e}^{i(E_2 - E_1)t} - \mathrm{e}^{-i(E_2 - E_1)t}\right)$$
$$= \sin^2 2\theta \sin^2\left[\frac{(E_2 - E_1)t}{2}\right]$$

But $$E = \sqrt{\vec{p}^2 + m^2} = \vec{p}\sqrt{1 + \frac{m^2}{\vec{p}^2}} \sim \vec{p} + \frac{m^2}{2\vec{p}}$$ for $m \ll E$

$1 + x \sim (1 + x/2)^2$

when x is small, can ignore x^2 term

$$\Rightarrow E_2(\vec{p}) - E_1(\vec{p}) \sim \frac{m_2^2 - m_1^2}{2\vec{p}} \sim \frac{m_2^2 - m_1^2}{2E}$$

$$\Rightarrow P(\nu_\mu \to \nu_e) = \sin^2 2\theta \sin^2\left[\frac{(m_2^2 - m_1^2)t}{4E}\right]$$

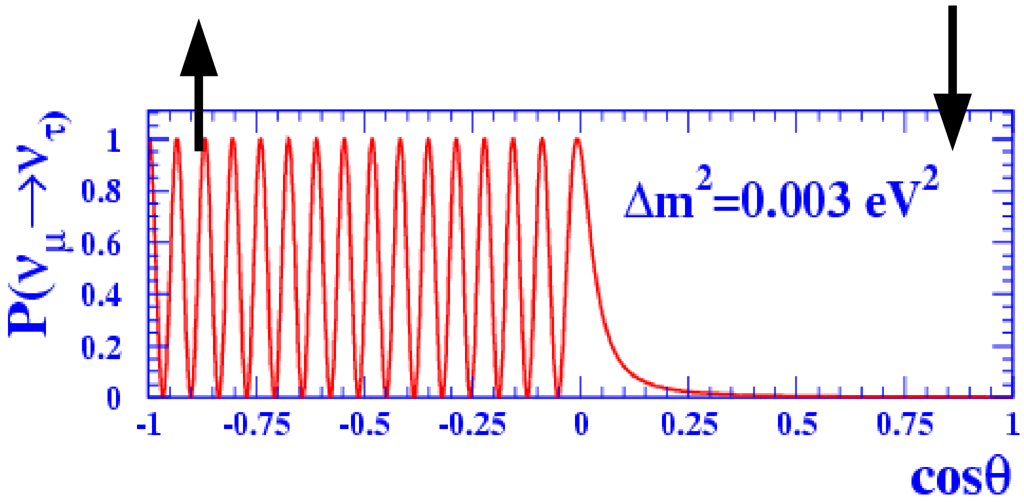
Neutrino Mixing

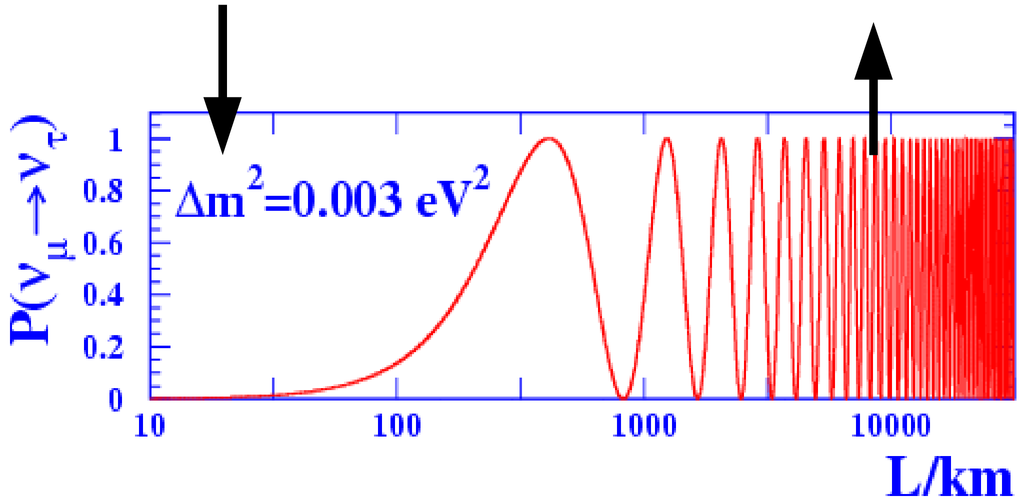
For $\nu_\mu \to \nu_\tau$ $\quad P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\left[\frac{(m_3^2 - m_2^2)t}{4E}\right] = \sin^2 2\theta \sin^2\left[\frac{1.27\Delta m^2 L}{E_\nu}\right]$

where L is the distance travelled in km,
$\Delta m^2 = m_3^2 - m_2^2$ is the mass difference in $(\mathrm{eV})^2$
and E_ν is the neutrino energy in GeV.

Interpretation of Super-Kamiokande Results

For $E(\nu_\mu) = 1\ \mathrm{GeV}$ (typical of atmospheric neutrinos)





Results are consistent with $\nu_\mu \to \nu_\tau$ oscillations:

$$|m_3^2 - m_2^2| \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2; \qquad \sin^2 2\theta \sim 1$$

Neutrino Mixing – Comments

- Neutrinos almost certainly have mass
- Neutrino oscillation only sensitive to mass differences
- More evidence for neutrino oscillations

  Solar neutrinos (SNO experiment)

  Reactor neutrinos (KamLand)

  suggest $|m_2^2 - m_1^2| \sim 8 \times 10^{-5}\ \mathrm{eV}^2$.
- More recent experiments use neutrino beams from accelerators or reactors; observe energy spectrum of neutrinos at a distant detector.
- At fixed L, observation of the values of E_ν at which minima/maxima are seen determines Δm^2, while depth of minima determine $\sin^2 2\theta$.
- Note all these experiments only tell us about mass **differences**.
- Best constraint on absolute mass comes from the end point in Tritium β-decay, $m(\nu_e) < 2\ \mathrm{eV}$.

Three-flavour oscillations

This whole framework can be generalised...

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where

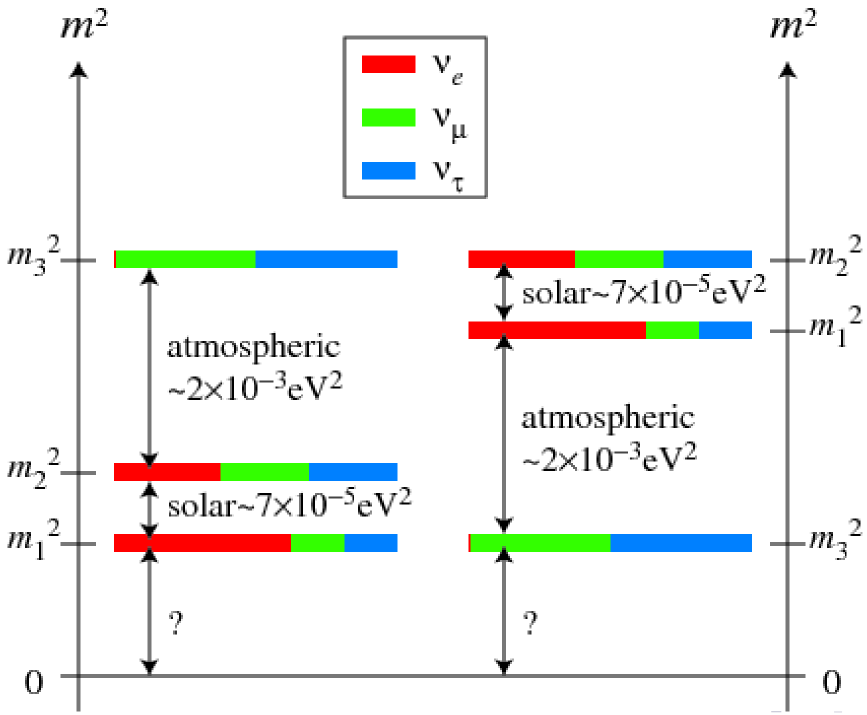
$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12} & 0 & s_{13}\mathrm{e}^{-i\delta} \\ 0 & 1 & 0 \\ -s_{23}\mathrm{e}^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

defining $\cos\theta_{12} = c_{12}$ etc.

This is an active field!

Current status...

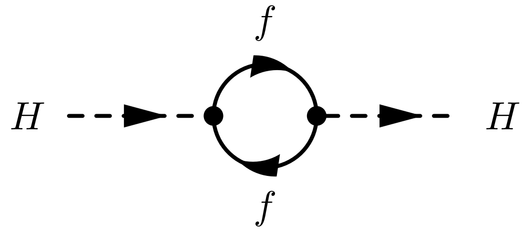
- $\sin^2\theta_{12} = 0.304 \pm 0.014$
- $\sin^2\theta_{23} = 0.51 \pm 0.06$
- $\sin^2\theta_{13} = 0.0219 \pm 0.0012$



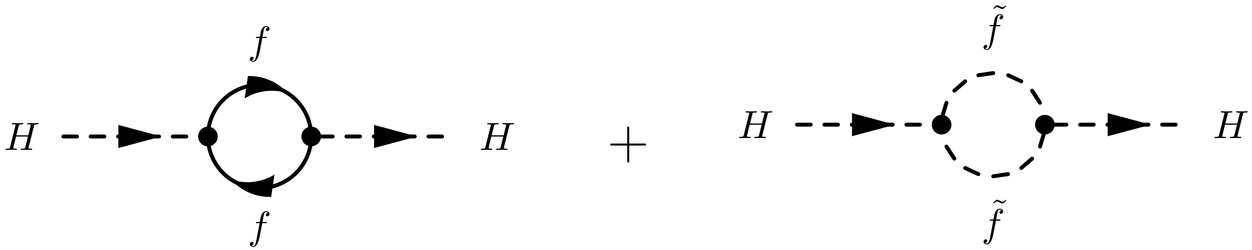
Supersymmetry (SUSY)

A significant problem is to explain why the Higgs boson is so light.

- The effect of loop corrections on the Higgs mass should be to drag it up to the highest energy scale in the problem (i.e. unification, or Planck mass).

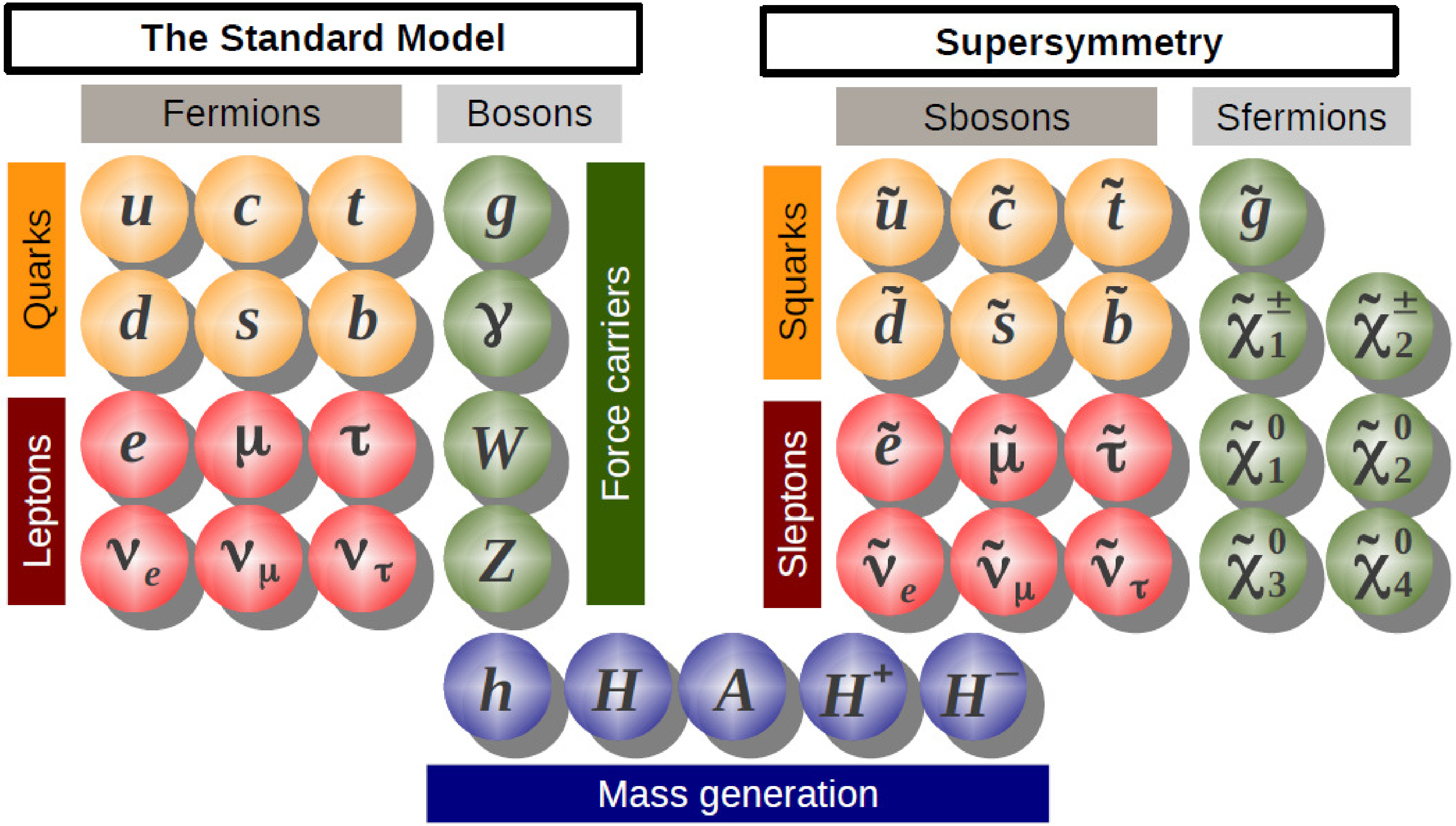


- One attractive solution is to introduce a new space-time symmetry, "supersymmetry" which links fermions and bosons (the only way to extend the Poincaré symmetry of special relativity and respect quantum field theory.)
- Each fermion has a boson partner, and vice versa, with the same couplings. Boson and fermion loops contribute with opposite sign, giving a natural cancellation in their effect on the Higgs mass.



- Must be a broken symmetry, because we clearly don't see bosons and fermions of the same mass.
- However, this doubles the particle content of the model, without any direct evidence (yet), and introduces lots of new unknown parameters.

The Supersymmetric Standard Model

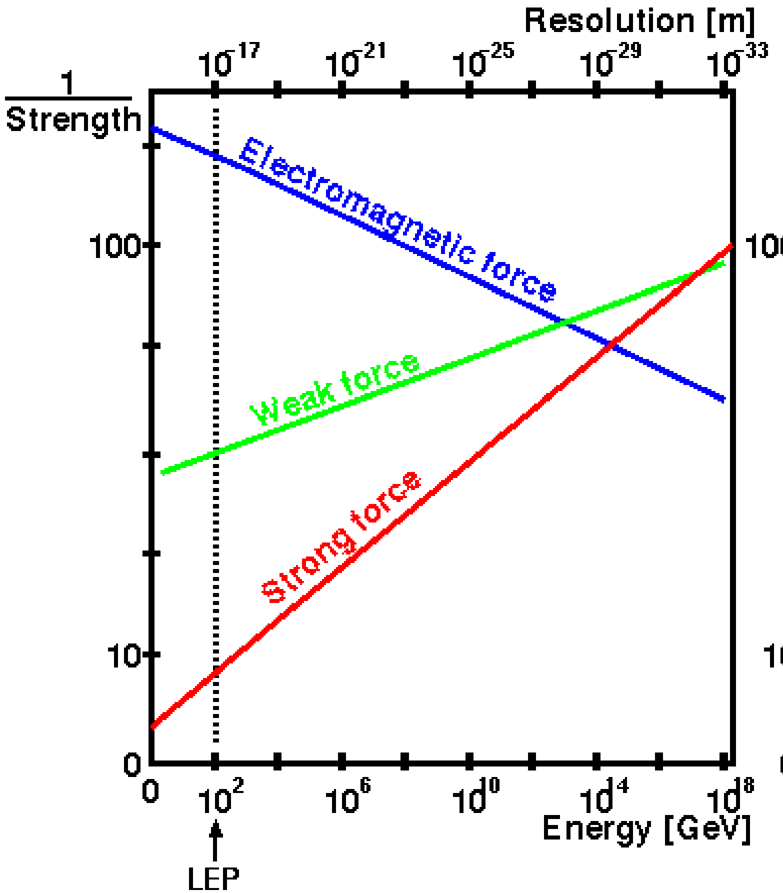


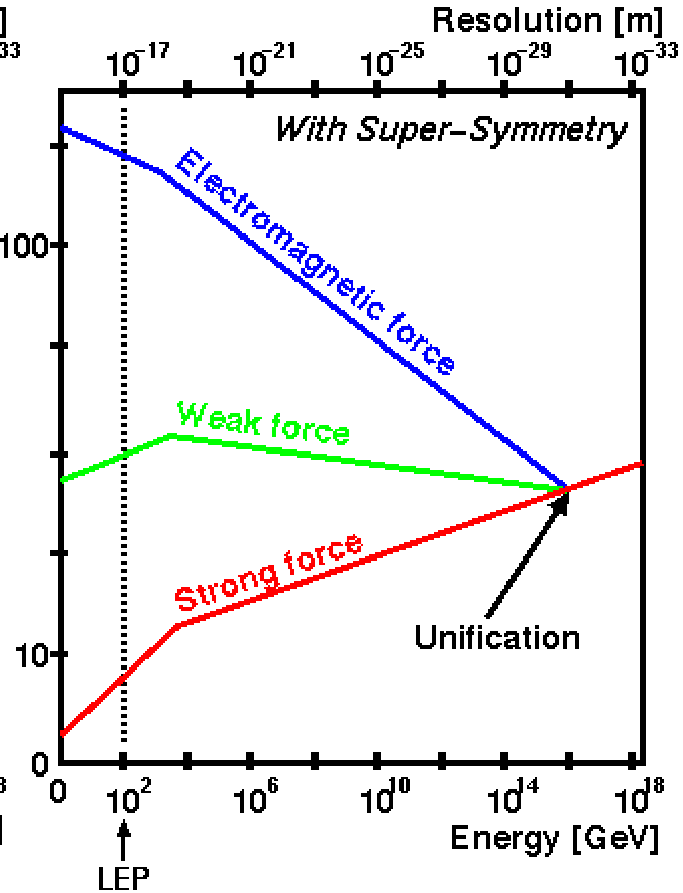
$$\mathrm{SM}: \quad W^{\pm},\ W^0,\ B \xrightarrow{\text{mixing}} W^{\pm},\ Z,\ \gamma \qquad \mathrm{SUSY}: \quad \tilde{H}_u^0,\ \tilde{H}_d^0,\ \tilde{W}^0,\ \tilde{B}^0 \xrightarrow{\text{mixing}} \tilde{\chi}_1^0,\ \tilde{\chi}_2^0,\ \tilde{\chi}_3^0,\ \tilde{\chi}_4^0$$

$$\tilde{H}_u^+,\ \tilde{H}_d^-,\ \tilde{W}^+,\ \tilde{W}^- \xrightarrow{\text{mixing}} \tilde{\chi}_1^{\pm},\ \tilde{\chi}_2^{\pm}$$

SUSY and Unification

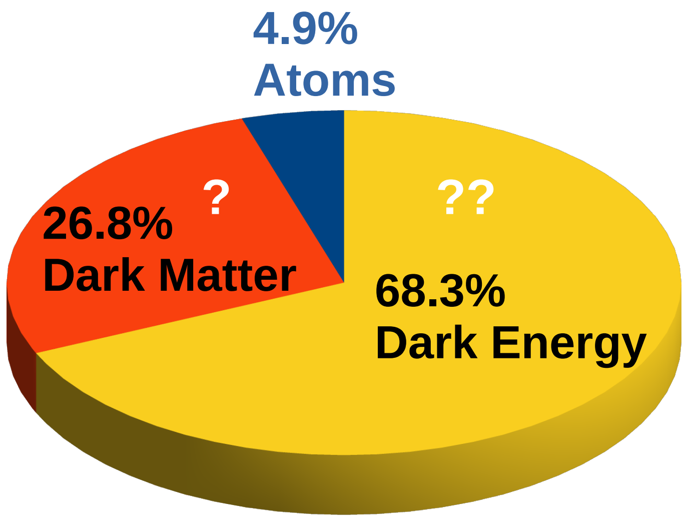
- In the Standard Model, the interaction strengths are not quite unified at very high energy.
- Add SUSY, the running of the couplings is modified, because sparticle loops contribute as well as particle loops.
- Details depend on the version of SUSY, but in general unification much improved.





SUSY and cosmology

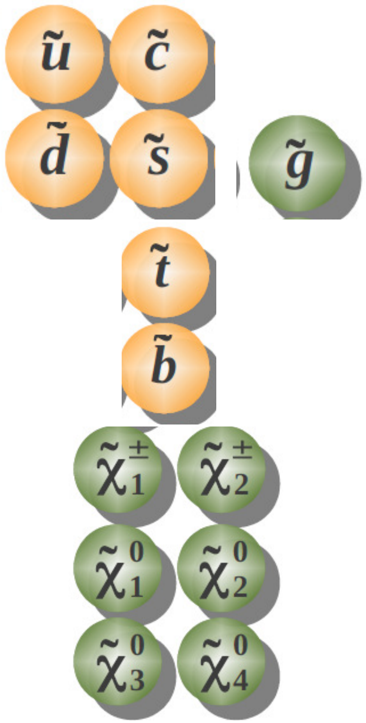
- SUSY, or any unified theory, tends to have potential problems with explaining the non-observation of proton decay.
- For this reason, many versions of SUSY introduce a conserved quantity "R-parity", which means that sparticles have to be produced in pairs.
- A consequence is that the lightest sparticle would have to be stable. In many scenarios this would be a "neutralino" $\tilde{\chi}_1^0$ (a mixture of neutral "gauginos" and "Higgsinos").
- Cosmologists tell us that $\sim 25\%$ of the mass in the universe is in the form of "dark matter", which interacts gravitationally, but otherwise only weakly.
- The lightest sparticle could be a candidate for the "WIMPs" (Weakly Interacting Massive Particles) which could comprise dark matter.

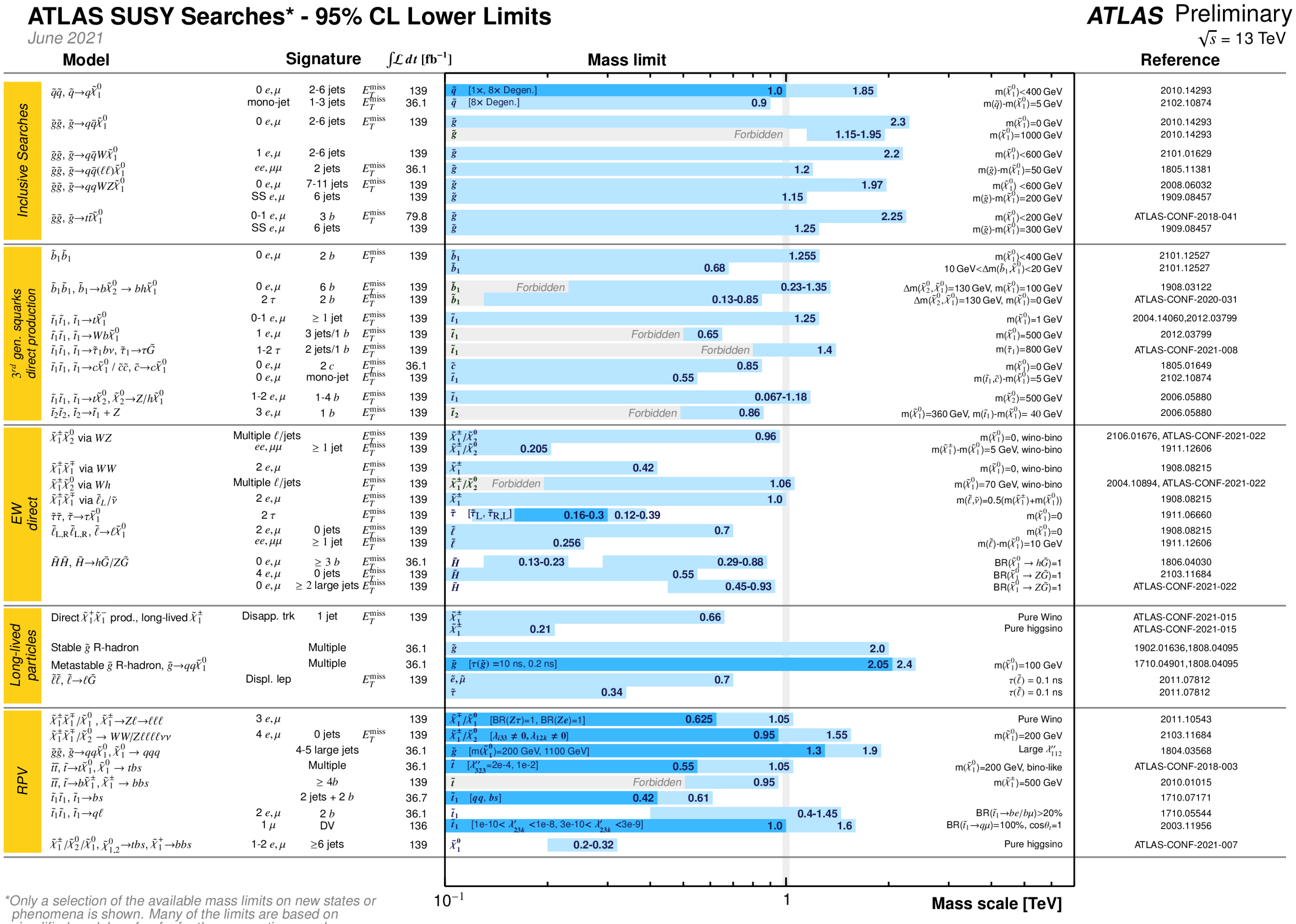


- So there are several different reasons why SUSY is attractive.

However, no sign of supersymmetry yet...

On general grounds, some sparticles ought to be seen at energies around 1 TeV or lower. So LHC ought to be able to see them, especially squarks+gluinos (high σ @LHC).

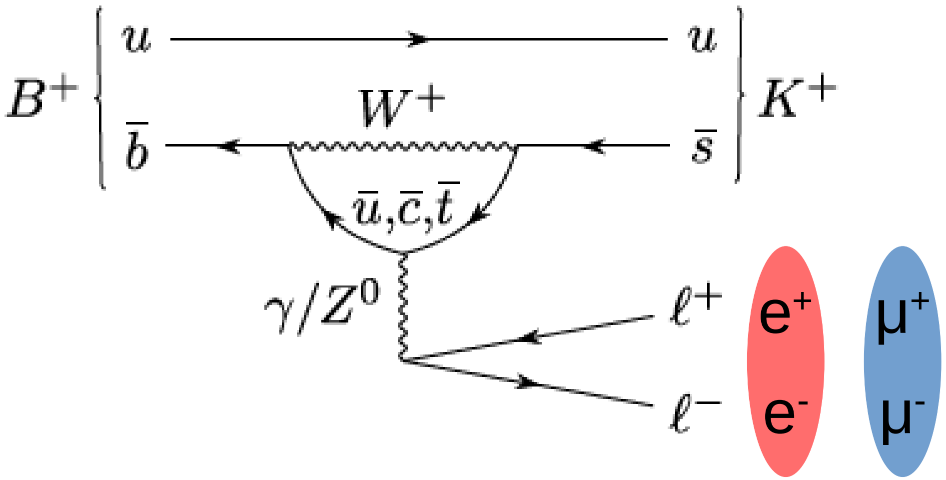


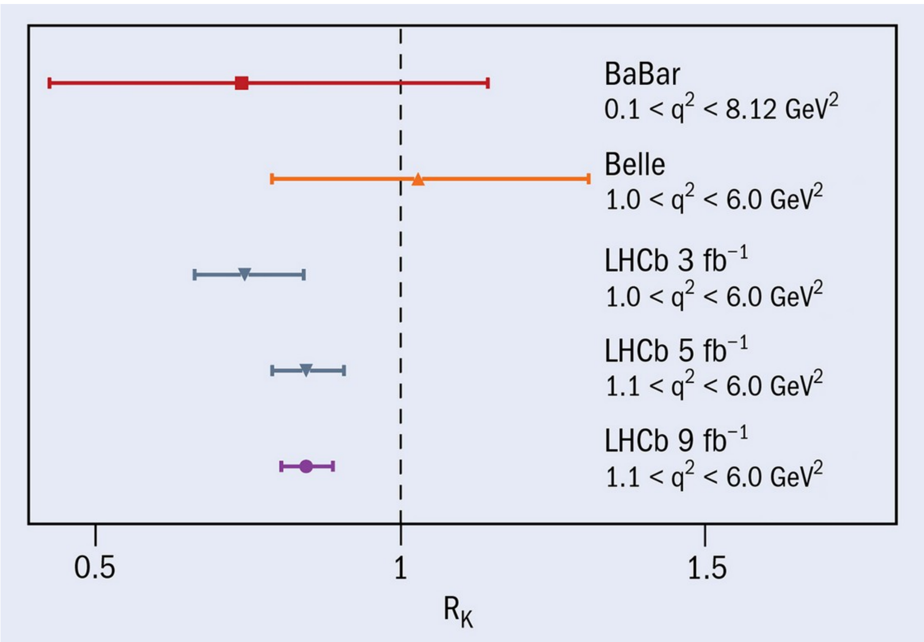


Signs of anything else?

(non-examinable)

LHCb Flavour Anomalies





Lepton universality in SM predicts $R = \frac{\mu\mu}{ee} = 1$

Test using rare decays of B mesons

- easy to see deviations from small values
- precise theory predictions

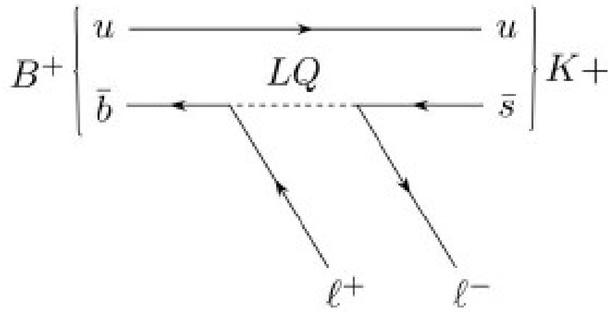
$$R_K = 0.85 \pm 0.04(stat.) \pm 0.01(syst.)$$

3 standard deviations from prediction.
Evidence of something new!

5 std.dev is gold standard for discovery.

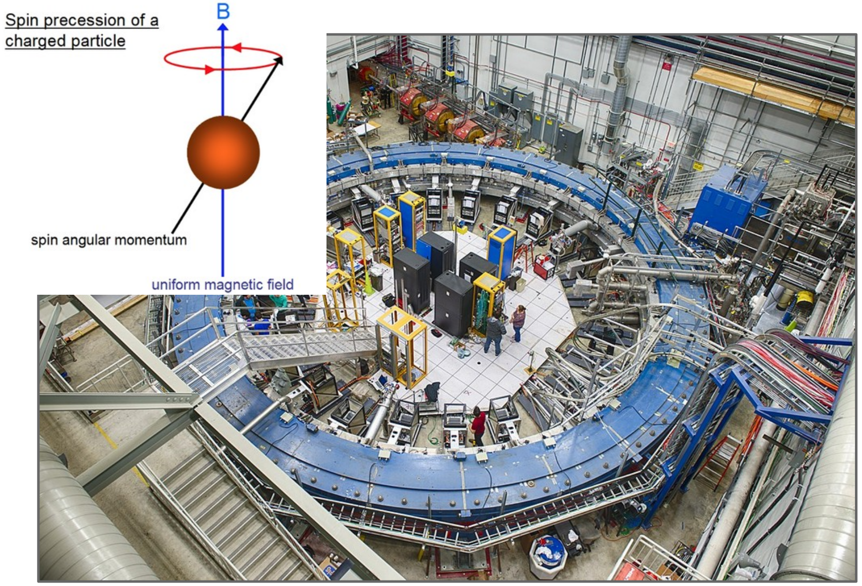
Similar effects seen in several rare decay modes.

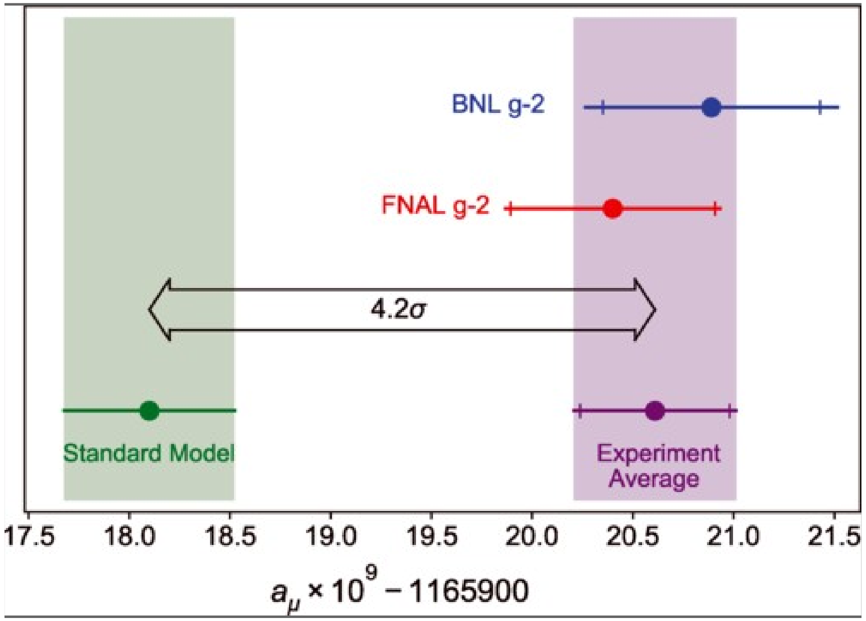
This might be the first glimpse of new particles affecting decay rates, e.g. Leptoquarks



Signs of anything else? *(non-examinable)*

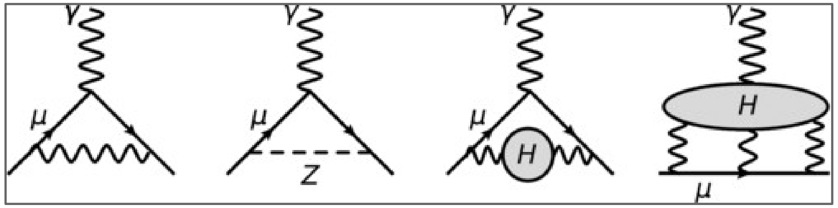
Muon g-2 Anomaly

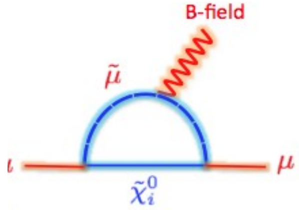




Measure muon spin precession in magnetic field.
Precision test of QED – precession frequency depends on how much it interacts with the magnetic field.

All known particles contribute to the muon's magnetic moment. Measure this very precisely and look for deviations.





20 year anomaly has been confirmed with a new measurement at Fermilab – measured muon magnetic moment to 0.46 ppm.
4.2 standard deviations from prediction.
Evidence of something new! Perhaps smuons?

Follow the results from LHC yourself!

To date (2024) LHC has taken only ∼5% of its planned total dataset.
Stay tuned!!

http://atlas.ch
http://cms.web.cern.ch
http://lhcb-public.web.cern.ch/lhcb-public/

Summary

- Over the past 50 years our understanding of the fundamental particles and forces of nature has changed beyond recognition.
- The Standard Model of particle physics is an enormous success. It has been tested to very high precision and can model almost all experimental observations so far.
- The Higgs "hole" is now becoming closed, though some other aspects of the SM are not quite yet under as much experimental "control" as one might wish for (the neutrino sector, the CKM matrix, etc).
- Good reasons to expect that the next few years will bring many more (un)expected surprises (more Higgs or gauge bosons, SUSY?).

Problem Sheet: q.29-30

Up next...
Section 13: Nuclear Physics, Basic Nuclear Properties