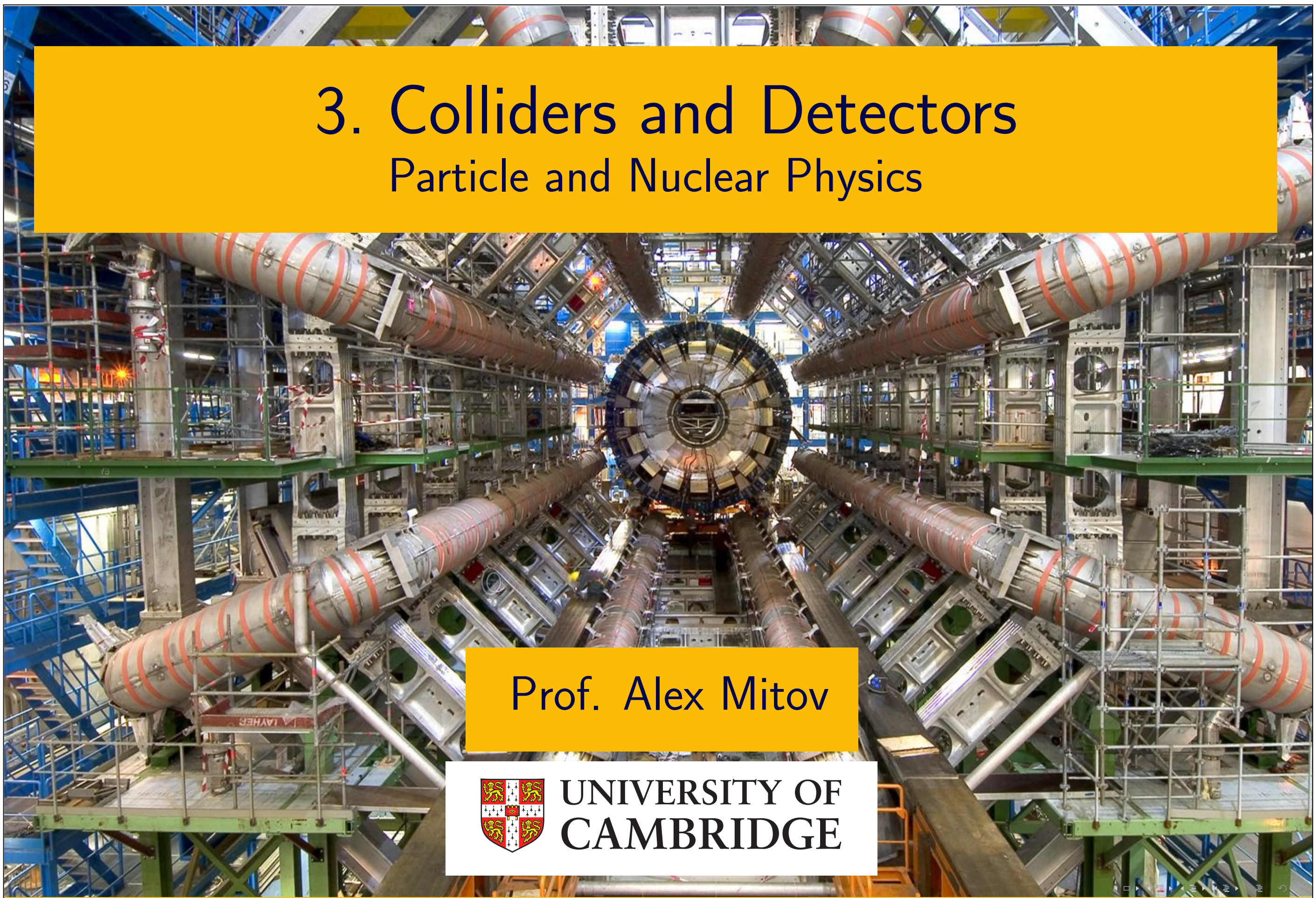
Particle and Nuclear Physics

Handout #2

Particle Physics

Lent/Easter Terms 2025

Prof. Alex Mitov



3. Colliders and Detectors
Particle and Nuclear Physics

Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- Physics of colliders
- Different types of detectors
- How to detect and identify particles

Colliders and $\sqrt{s}$

Consider the collision of two particles:

$$\longrightarrow \quad \cdot \quad \longleftarrow$$

$$p_1 = (E_1, \vec{p}_1) \quad p_2 = (E_2, \vec{p}_2)$$

The invariant quantity $s = E_{CM}^2 = (p_1 + p_2)^2$

$$= (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

$$= E_1^2 - |\vec{p}_1|^2 + E_2^2 - |\vec{p}_2|^2 + 2E_1E_2 - 2\vec{p}_1.\vec{p}_2$$

$$= m_1^2 + m_2^2 + 2(E_1E_2 - |\vec{p}_1||\vec{p}_2|\cos\theta)$$

θ is the angle between the momentum three-vectors
$\sqrt{s}$ is the energy in the centre-of-mass frame; it is the amount of energy available to the interaction e.g. in particle-antiparticle annihilation it is the maximum energy/mass of particle(s) that can be produced.

Colliders and $\sqrt{s}$

Fixed Target Collision

$$\longrightarrow \quad \cdot$$

$$p_1 = (E_1, \vec{p}_1) \quad p_2 = (m_2, 0)$$

$$s = m_1^2 + m_2^2 + 2E_1m_2$$

$$\text{For } E_1 \gg m_1, m_2$$

$$s \sim 2E_1m_2 \quad \Rightarrow \quad \sqrt{s} \sim \sqrt{2E_1m_2}$$

e.g. 450 GeV proton hitting a proton at rest:

$$\sqrt{s} \sim \sqrt{2 \times 450 \times 1} \sim 30 \text{ GeV}$$

Collider Experiment

$$\longrightarrow \quad \cdot \quad \longleftarrow$$

$$p_1 = (E_1, \vec{p}_1) \quad p_2 = (E_2, \vec{p}_2)$$

$$s = m_1^2 + m_2^2 + 2(E_1E_2 - |\vec{p}_1||\vec{p}_2|\cos\theta)$$

$$\text{For } E_1 \gg m_1, m_2 \quad |\vec{p}| = E, \quad \theta = \pi$$

$$s = 2(E^2 - E^2\cos\theta) = 4E^2 \quad \Rightarrow \quad \sqrt{s} = 2E$$

e.g. 450 GeV proton colliding with a 450 GeV proton:

$$\sqrt{s} \sim 2 \times 450 = 900 \text{ GeV}$$

In a fixed target experiment most of the proton's energy is wasted providing forward momentum to the final state particles rather than being available for conversion into interesting particles.

Colliders

To produce and discover heavy new particles, we need high E_{CM}.
Need to collide massive particles at high energies!

Accelerate charged particles using RF high-voltage

Energy gained with each electric field $\Delta E = qV$
Limited by space! SLAC 3.2 km long, reached $E_e = 50\ \mathrm{GeV}$

Colliders

To produce and discover heavy new particles, we need high E_{CM}.
Need to collide massive particles at high energies!

Accelerate charged particles using RF high-voltage, bend using magnets.

High power magnets needed

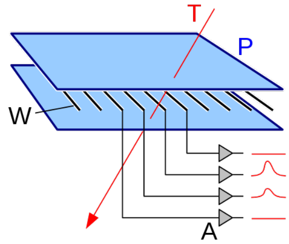
$$B = \frac{p[\ \mathrm{GeV}]}{0.3r[\mathrm{m}]}$$

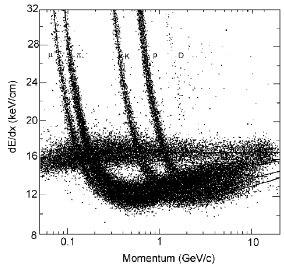
Limited by synchrotron radiation

$$\text{radiated energy per orbit} = \frac{E^4}{m^4 r}$$

Detecting Particles *Trackers*

Trackers detect ionisation loss
$\Rightarrow$ only detect charged particles
e.g. multiwire proportional chambers, cloud chambers

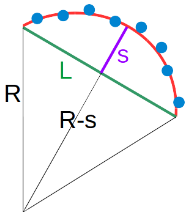




Ionisation loss given by Bethe-Block formula depends on particle charge q and speed β, γ (not mass)

$$-\frac{dE}{dx} = \frac{4\pi N_0 q^2 \alpha^2 (\hbar c)^2}{m_e \beta^2} \frac{Z}{A} \left[\log\left(\frac{2m_e\gamma^2\beta^2}{I}\right) - \beta^2 \right]$$

Immerse tracker in $\vec{B}$ to measure track radius, and thus particle momentum p.
Measure sagitta s from track arc $\rightarrow$ curvature R



$$R = \frac{L^2}{8s} + \frac{s}{2} \sim \frac{L^2}{8s}$$

$$p = 0.3B\left(\frac{L^2}{8s}\right)$$

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8p}{0.3BL^2}\sigma_s$$

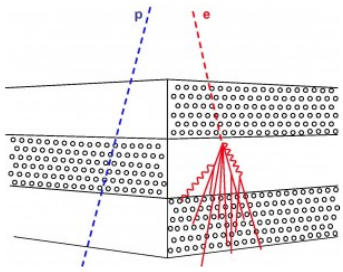
High-p particles have high radius of curvature
$\Rightarrow$ track almost straight.
Low-p particles have small radius of curvature
$\Rightarrow$ measure with high accuracy.

$$\frac{\sigma_p}{p} \propto p$$

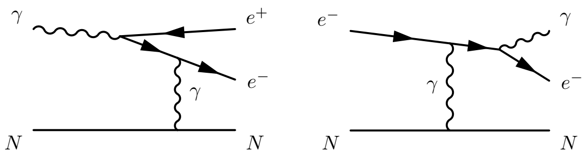
Detecting Particles *Calorimeters*

Calorimeters detect EM/hadronic showers using layers of absorber and scintillating material

High-density material interacts with the particle and initiates shower.



Electromagnetic calorimeter $(e^\pm, \gamma)$

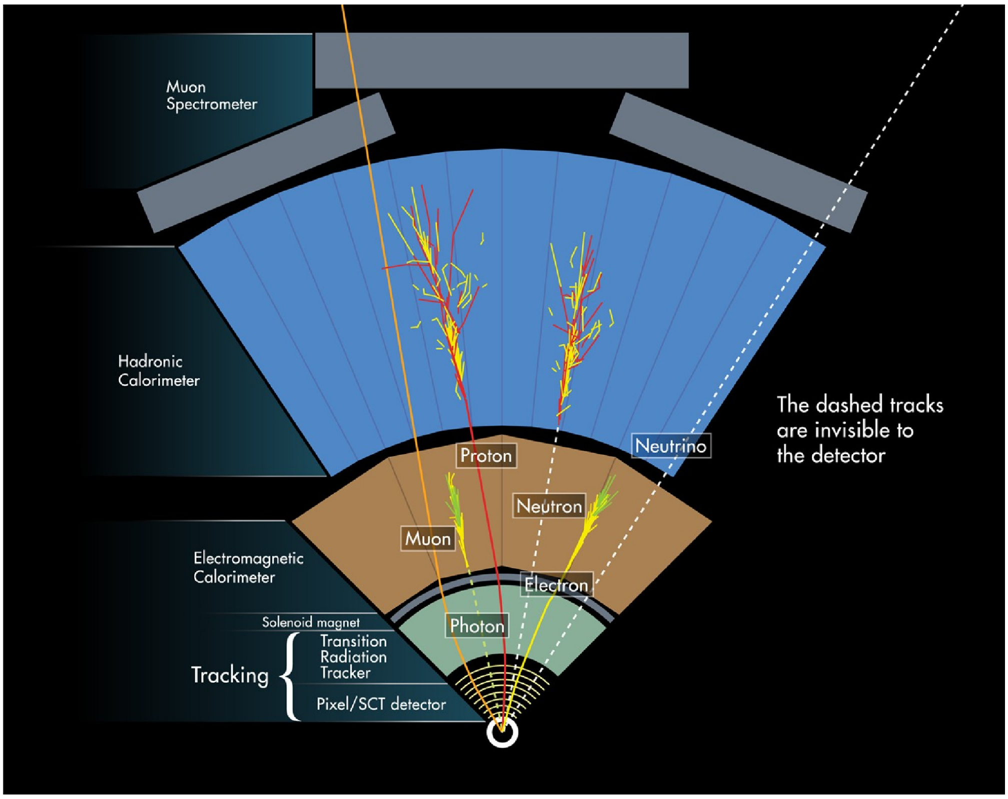


Hadronic calorimeter $(p, n, \pi, K\ldots)$
Nuclear interaction length $>$ radiation length.
Use more (denser) material.

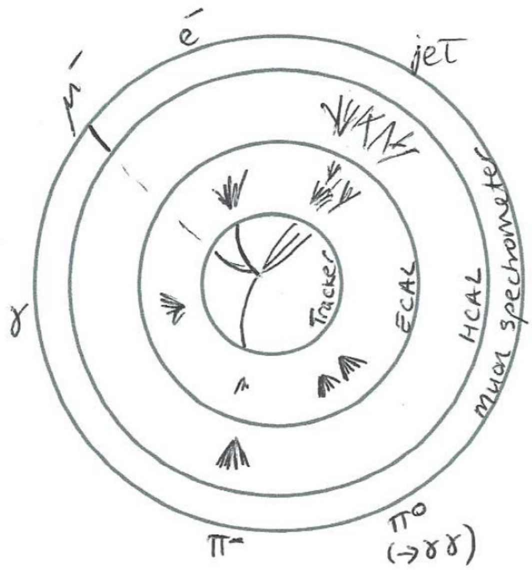
High-energy particles produce showers with many particles
$\Rightarrow$ measure with high accuracy.
Low-energy particles produce showers with few particles
$\Rightarrow$ low accuracy.

$$\frac{\sigma_E}{E} \propto \frac{\sqrt{N}}{E} = \frac{1}{\sqrt{E}}$$

Detector design

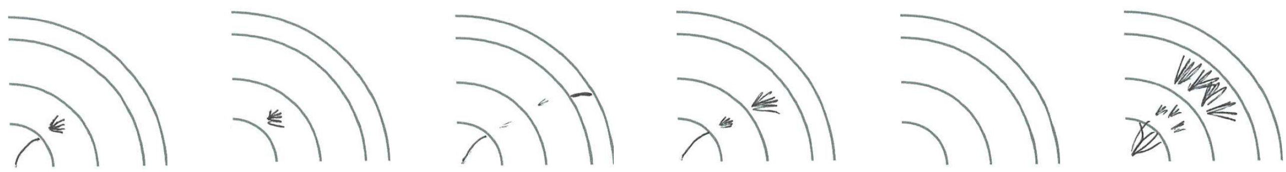


Particle Signatures



Different particles leave different signals in the various detector components allowing almost unambiguous identification.

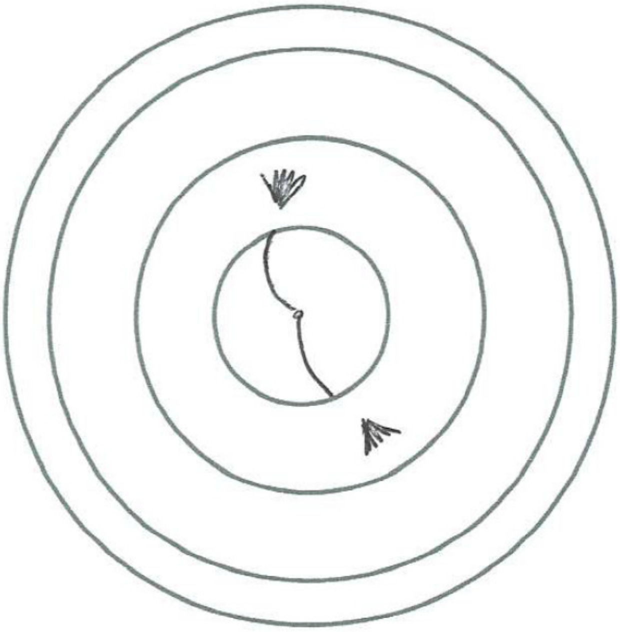
$e^{\pm}$: Track + EM energy
γ: No track + EM energy
$\mu^{\pm}$: Track, small calo energy deposits, penetrating
$\tau^{\pm}$: decay, observe decay products
ν: not detected (need specialised detectors)
hadrons: track (if charged) + calo energy deposits
quarks: seen as jets of hadrons



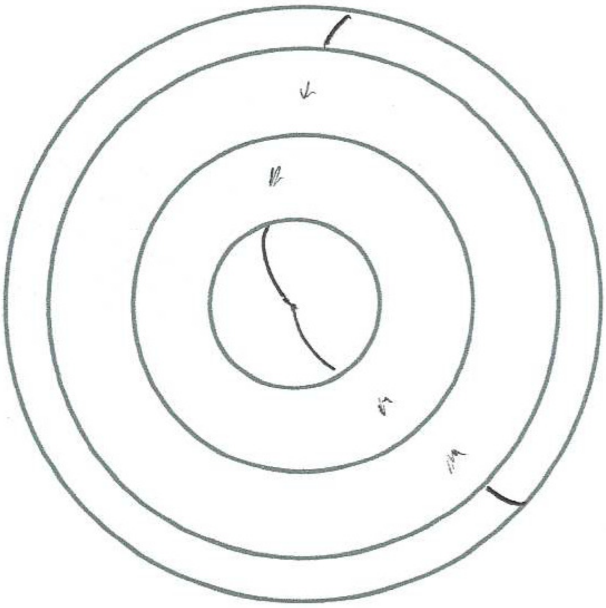
electron photon muon pion neutrino jet

Particle Signatures *Examples*

$e^+e^- \to Z \to e^+e^-$

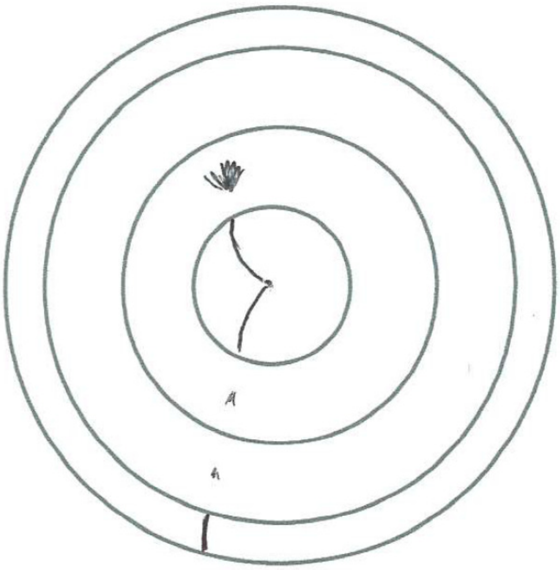


$e^+e^- \to Z \to \mu^+\mu^-$



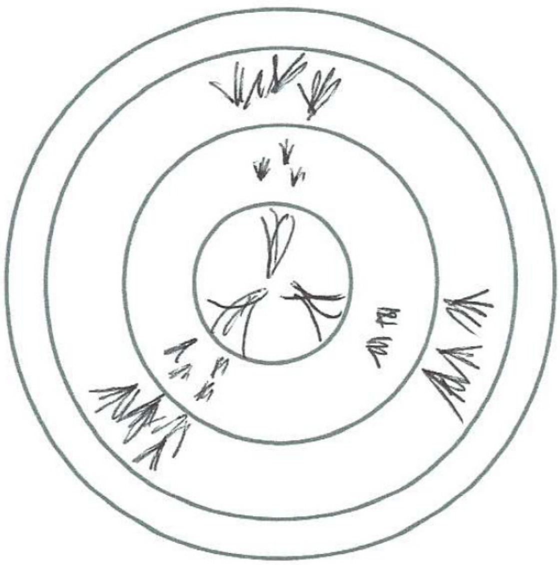
Particle Signatures *Examples*

$e^+e^- \to Z \to \tau^+\tau^-$



Taus decay within the detector (lifetime $\sim 10^{-13}$ s).
Here $\tau^- \to e^-\bar{\nu}_e\nu_\tau$, $\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau$

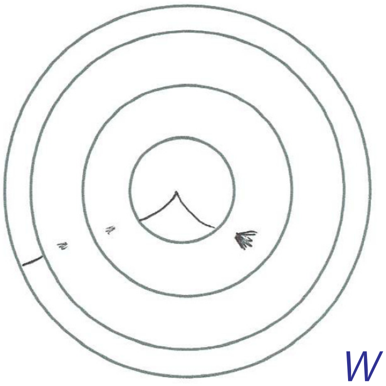
$e^+e^- \to Z \to q\bar{q}$



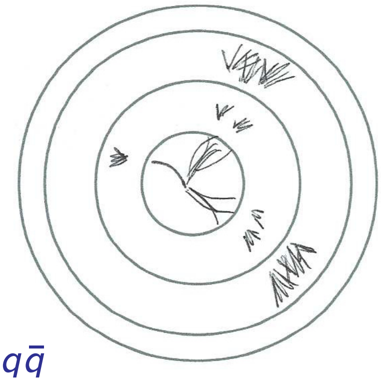
3-jet event (gluon emitted by $q/\bar{q}$)

Particle Signatures *Examples*

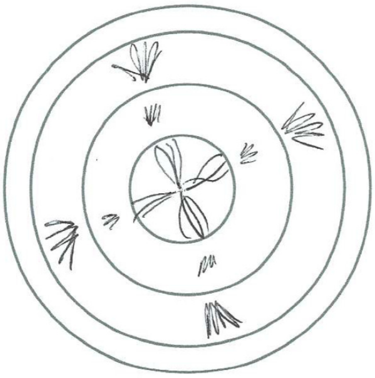
$W^+W^- \to e\nu\mu\nu$



$W^+W^- \to q\bar{q}e\nu$



$W^+W^- \to q\bar{q}q\bar{q}$



Example

e^+e^- collider with typical cylinder detector.
In one event, two electrons are detected:

1. e^+, $E_{\rm cluster} = 44.7 \pm 1.2\ \mathrm{GeV}$, $|\vec{p}_{\rm track}| = 46.0 \pm 3.2\ \mathrm{GeV}$
2. e^-, $E_{\rm cluster} = 46.0 \pm 1.2\ \mathrm{GeV}$, $|\vec{p}_{\rm track}| = 49.5 \pm 3.5\ \mathrm{GeV}$

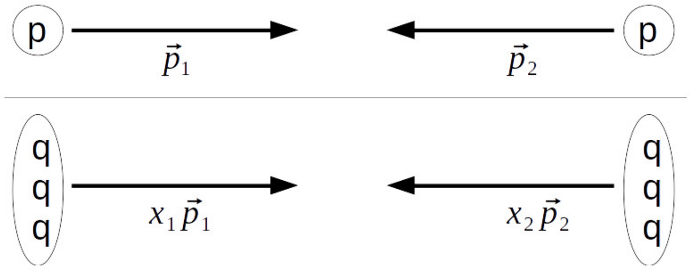
For this event we need

- Lowest order Feynman diagram
- Detector signature
- Invariant mass

Example

Consider pp collisions.

Calculate the reduced E_{CM} assuming the colliding quarks carry a fraction x_1 and x_2 of the proton energy.



Summary

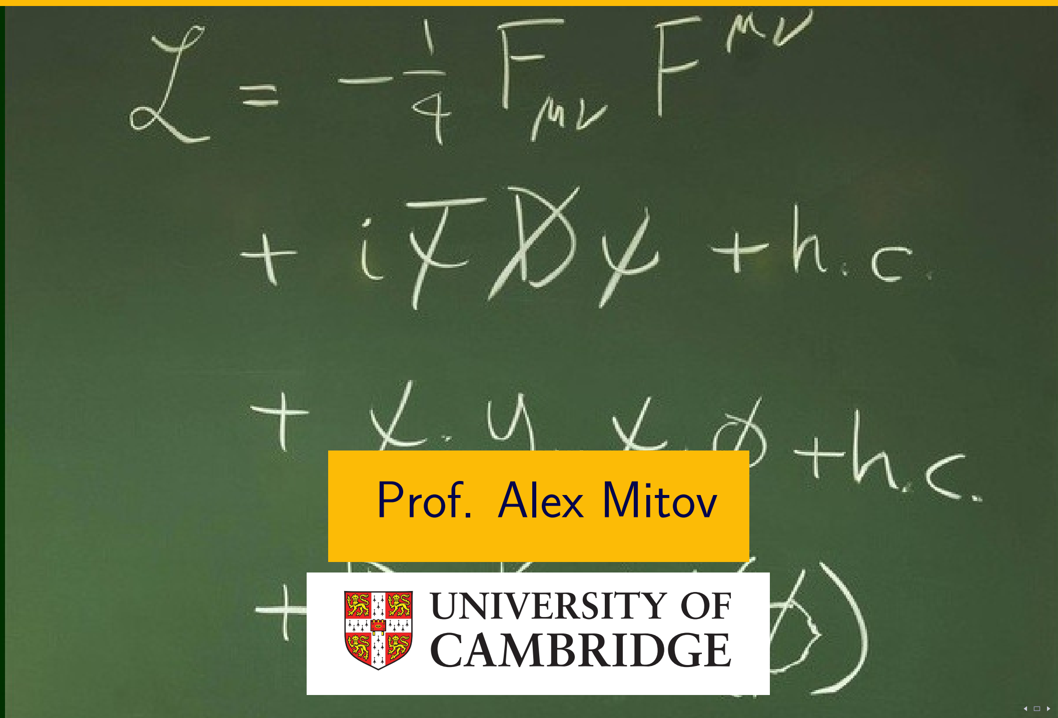
- For high $\sqrt{s}$:
  - Prefer colliders over fixed target collisions
  - Prefer circular colliders with high power magnets
  - Prefer to collide high mass particles
- Trackers to trace the path of charged particles
- Calorimeters to stop and measure the energy of particles
- Detector design and particle signatures

Problem Sheet: q.7-9

Up next...
Section 4: The Standard Model

4. The Standard Model

Particle and Nuclear Physics



Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

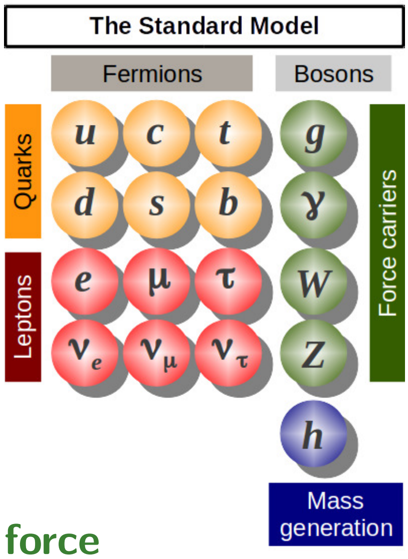
- Standard Model particle content
- Klein-Gordon equation
- Antimatter
- Interaction via particle exchange
- Virtual particles

The Standard Model

Spin-1/2 fermions — Charge (units of e)

		Charge (units of e)
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{2}{3}$ $-\frac{1}{3}$
Leptons	$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$	-1 0

Plus antileptons and antiquarks



Spin-1 bosons

		Mass (GeV/c^2)	
Gluon	g	0	**Strong force**
Photon	γ	0	**EM force**
W and Z bosons	$W^\pm, Z$	91.2, 80.3	**Weak force**

Spin-0 bosons

Higgs	h	125	**Mass generation**

Theoretical Framework

	Macroscopic	Microscopic
Slow	Classical Mechanics	Quantum Mechanics
Fast	Special Relativity	Quantum Field Theory

The Standard Model is a collection of related Gauge Theories which are Quantum Field Theories that satisfy Local Gauge Invariance.

Electromagnetism: **Quantum Electrodynamics (QED)**
1948 Feynman, Schwinger, Tomonaga (1965 Nobel Prize)

Electromagnetism + Weak: **Electroweak Unification**
1968 Glashow, Weinberg, Salam (1979 Nobel Prize)

Strong: **Quantum Chromodynamics (QCD)**
1974 Politzer, Wilczek, Gross (2004 Nobel Prize)

The Schrödinger Equation

To describe the fundamental interactions of particles we need a theory of Relativistic Quantum Mechanics

Schrödinger Equation for a free particle $\hat{E}\psi = \frac{\hat{p}^2}{2m}\psi$

with energy and momentum operators $\hat{E} = \mathrm{i}\frac{\partial}{\partial t}, \quad \hat{p} = -\mathrm{i}\boldsymbol{\nabla}$

($\hbar = 1$ natural units)

giving $\mathrm{i}\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\boldsymbol{\nabla}^2\psi$

which has plane wave solutions: $\psi(\vec{r}, t) = N\mathrm{e}^{-\mathrm{i}(Et-\vec{p}.\vec{r})}$

- 1^{st} order in time derivative
- 2^{nd} order in space derivatives

Not Lorentz Invariant!

Schrödinger equation cannot be used to describe the physics of relativistic particles.

Klein-Gordon Equation

Use the KG equation to describe the physics of relativistic particles.

From Special Relativity: $E^2 = p^2 + m^2$

use energy and momentum operators $\hat{E} = \mathrm{i}\frac{\partial}{\partial t}, \quad \hat{p} = -\mathrm{i}\boldsymbol{\nabla}$

giving $-\frac{\partial^2\psi}{\partial t^2} = -\boldsymbol{\nabla}^2\psi + m^2\psi$ $\quad\frac{\partial^2\psi}{\partial t^2} = (\boldsymbol{\nabla}^2 - m^2)\psi$ Klein-Gordon Equation

Second order in both space and time derivatives $\Rightarrow$ Lorentz invariant.

Plane wave solutions $\psi(\vec{r}, t) = N\mathrm{e}^{-\mathrm{i}(Et-\vec{p}.\vec{r})}$

but this time requiring $E^2 = \vec{p}^{\,2} + m^2$, allowing $E = \pm\sqrt{|\vec{p}|^2 + m^2}$

Negative energy solutions required to form complete set of eigenstates.

$\Rightarrow$ **Antimatter**

Antimatter *and the Dirac Equation*

In the hope of avoiding negative energy solutions, Dirac sought a linear relativistic wave equation:

$$\mathrm{i}\frac{\partial \psi}{\partial t} = (-\mathrm{i}\vec{\alpha}.\vec{\nabla} + \beta m)\psi$$

$\vec{\alpha}$ and β are appropriate 4x4 matrices.

ψ is a column vector "spinor" of four wavefunctions.

Two of the wavefunctions describe the states of a fermion, but the other two still have negative energy.

Dirac suggested the vacuum had all negative energy states filled. A hole in the negative energy "sea" could be created by exciting an electron to a positive energy state. The hole would behave like a positive energy positive charged "positron". Subsequently detected.

However, this only works for fermions...

We now interpret negative energy states differently...

Antimatter *and the Feynman-Stückelberg Interpretation*

Consider the negative energy solution in which a negative energy particle travels backwards in time.

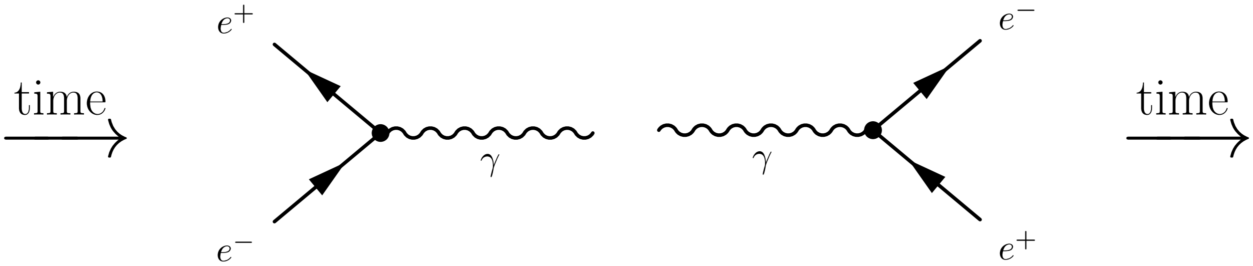
$$\mathrm{e}^{-\mathrm{i}Et} \equiv \mathrm{e}^{-\mathrm{i}(-E)(-t)}$$

Interpret as a **positive** energy **anti**particle travelling **forwards** in time.

Then all solutions can be used to describe physical states with positive energy, going forward in time.

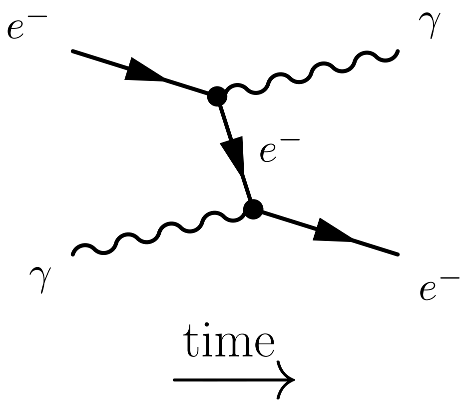
e.g.

e^+e^- annihilation pair production

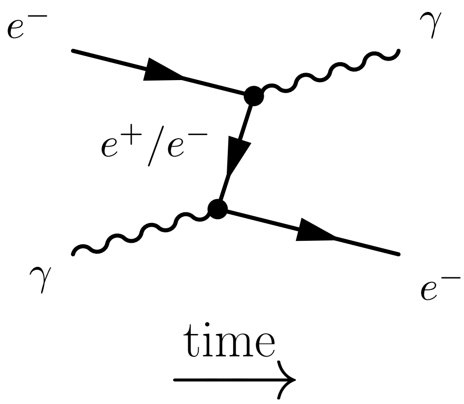


All quantum numbers carried into a vertex by the e^+ are the same as if it is regarded as an outgoing e^-, or vice versa.

Antimatter *and the Feynman-Stückelberg Interpretation*



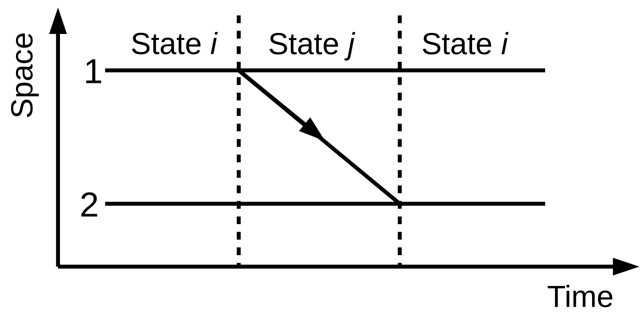
The interpretation here is easy. The first photon emitted has less energy than the electron it was emitted from. No need for “anti-particles” or negative energy states.



The emitted photon has more energy than the electron that emitted it. Either view the top vertex as “emission of a negative energy electron travelling backwards in time” or “absorption of a positive energy positron travelling forwards in time”.

Interaction via Particle Exchange

Consider two particles, fixed at $\vec{r}_1$ and $\vec{r}_2$, which exchange a particle of mass m.



$$p^\mu = (E, \vec{p})$$

$$E = E_j - E_i$$

Calculate the shift in energy of state i due to this exchange (using second order perturbation theory):

$$\Delta E_i = \sum_{j \neq i} \frac{\langle i|H|j\rangle\langle j|H|i\rangle}{E_i - E_j}$$

Sum over all possible states j with different momenta

where $\langle j|H|i\rangle$ is the transition from i to j at $\vec{r}_1$
where $\langle i|H|j\rangle$ is the transition from j to i at $\vec{r}_2$

Interaction via Particle Exchange

Consider $\langle j|H|i\rangle$ (transition from $i \to j$ by emission of m at $\vec{r}_1$)

$$\psi_i = \psi_1\psi_2 \qquad \text{Original 2 particles}$$

$$\psi_j = \psi_1\psi_2\psi_3 \qquad \psi_3 = N\,\mathrm{e}^{-\mathrm{i}(Et-\vec{p}.\vec{r})}$$

ψ_3 represents a free particle with $p^\mu = (E, \vec{p})$

normalise $\psi_1^*\psi_1 = \psi_2^*\psi_2 = \psi_3^*\psi_3 = 1$

Let g be the probability of emitting m at r_1

$g/\sqrt{2E}$ is required on dimensional grounds, c.f. AQP vector potential of a photon.

$$\langle j|H|i\rangle = \int \mathrm{d}^3\vec{r}\,\psi_1^*\psi_2^*\psi_3^*\,\frac{g}{\sqrt{2E}}\,\psi_1\psi_2\,\delta^3(\vec{r}-\vec{r}_1)$$
$$= \frac{g}{\sqrt{2E}}N\mathrm{e}^{\mathrm{i}(Et-\vec{p}.\vec{r}_1)}$$

Dirac δ function

$$\int \mathrm{d}^3\vec{r}\delta^3\left(\vec{r}-\vec{r}_1\right)$$
$$= 1 \quad \text{for } \vec{r} = \vec{r}_1$$
$$= 0 \quad \text{for } \vec{r} \neq \vec{r}_1$$

Similarly $\langle i|H|j\rangle$ is the transition from j to i at $\vec{r}_2$

$$\langle i|H|j\rangle = \frac{g}{\sqrt{2E}}N\mathrm{e}^{-\mathrm{i}(Et-\vec{p}.\vec{r}_2)}$$

Shift in energy state $\Delta E_i^{1\to2} = \sum_{j\neq i}\frac{g^2}{2E}\frac{N^2\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{E_i - E_j} = \sum_{j\neq i}\frac{g^2N^2\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{-2E^2}$ $\quad (E = E_j - E_i)$

Interaction via Particle Exchange

Putting the pieces together

Different states j have different momenta $\vec{p}$ for the exchanged particle.
Therefore sum is actually an integral over all momenta:

$$\Delta E_i^{1\to2} = \int \frac{g^2N^2\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{-2E^2}\rho(p)\,\mathrm{d}p \;= \int \frac{g^2\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{-2E^2}\frac{1}{L^3}\left(\frac{L}{2\pi}\right)^3 p^2\,\mathrm{d}p\,\mathrm{d}\Omega$$
$$= -g^2\left(\frac{1}{2\pi}\right)^3\int\frac{\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{2E^2}p^2\,\mathrm{d}p\,\mathrm{d}\Omega$$

$N = \sqrt{\frac{1}{L^3}}, \quad \rho(p) = \left(\frac{L}{2\pi}\right)^3 p^2\,\mathrm{d}\Omega$

$E^2 = p^2 + m^2$

The integral can be done by taking the z-axis along $\vec{r} = \vec{r}_2 - \vec{r}_1$

Then $\vec{p}.\vec{r} = pr\cos\theta$ and $\mathrm{d}\Omega = 2\pi\,\mathrm{d}(\cos\theta)$

$$\Delta E_i^{1\to2} = -\frac{g^2}{2(2\pi)^2}\int_0^\infty\frac{p^2}{p^2+m^2}\frac{\mathrm{e}^{\mathrm{i}\vec{p}.\vec{r}} - \mathrm{e}^{-\mathrm{i}\vec{p}.\vec{r}}}{\mathrm{i}pr}\mathrm{d}p \qquad \text{(see Appendix D)}$$

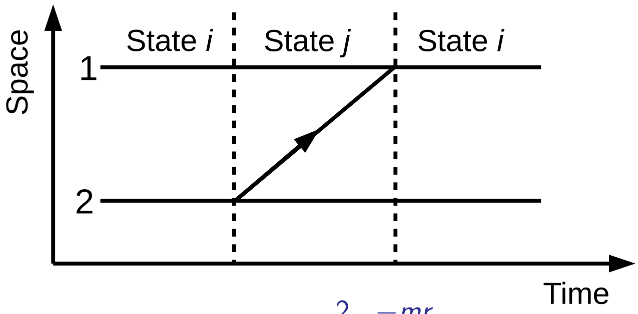
Write this integral as one half of the integral from $-\infty$ to $+\infty$, which can be done by residues giving

$$\Delta E_i^{1\to2} = -\frac{g^2}{8\pi}\frac{\mathrm{e}^{-mr}}{r}$$

Interaction via Particle Exchange

Final stage

Can also exchange particle from 2 to 1:



Get the same result: $\Delta E_i^{2\to 1} = -\frac{g^2}{8\pi}\frac{\mathrm{e}^{-mr}}{r}$

Total shift in energy due to particle exchange is

$$\Delta E_i = -\frac{g^2}{4\pi}\frac{\mathrm{e}^{-mr}}{r} \qquad \text{Yukawa Potential}$$

Attractive force between two particles, decreasing exponentially with range r.

Yukawa Potential

Hideki Yukawa
1949 Nobel Prize

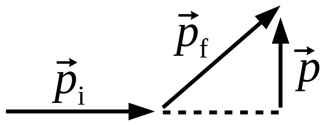
$$V(r) = -\frac{g^2}{4\pi}\frac{\mathrm{e}^{-mr}}{r} \qquad \text{Yukawa Potential}$$

- Characteristic range $= 1/m$
  (Compton wavelength of exchanged particle)
- For $m \to 0$, $V(r) = -\frac{g^2}{4\pi r}$ infinite range (Coulomb-like)

Yukawa potential with $m = 139\ \mathrm{MeV}/c^2$ gives a good description of long range part of the interaction between two nucleons and was the basis for the prediction of the existence of the pion.

Scattering from the Yukawa Potential

Consider elastic scattering (no energy transfer)



Born Approximation $$M_{\mathrm{fi}} = \int \mathrm{e}^{\mathrm{i}\vec{p}.\vec{r}} V(r)\, \mathrm{d}^3\vec{r}$$

Yukawa Potential $$V(r) = -\frac{g^2}{4\pi}\frac{\mathrm{e}^{-mr}}{r}$$

$$M_{\mathrm{fi}} = -\frac{g^2}{4\pi}\int \frac{\mathrm{e}^{-mr}}{r}\mathrm{e}^{\mathrm{i}\vec{p}.\vec{r}}\,\mathrm{d}^3\vec{r} = -\frac{g^2}{|\vec{p}|^2 + m^2}$$

$q^{\mu} = (E, \vec{p})$

$q^2 = E^2 - |\vec{p}|^2$

q^2 is invariant

"Virtual Mass"

The integral can be done by choosing the z-axis along $\vec{r}$, then $\vec{p}.\vec{r} = pr\cos\theta$ and $\mathrm{d}^3\vec{r} = 2\pi r^2\,\mathrm{d}r\,\mathrm{d}(\cos\theta)$

For elastic scattering, $q^{\mu} = (0, \vec{p})$, $q^2 = -|p|^2$ and exchanged massive particle is highly "virtual"

$$M_{\mathrm{fi}} = \frac{g^2}{q^2 - m^2}$$

Virtual Particles

Forces arise due to the exchange of unobservable virtual particles.

- The effective mass of the virtual particle, q^2, is given by
$$q^2 = E^2 - |\vec{p}|^2$$
and is not equal to the physical mass m, i.e. it is off-shell mass.
- The mass of a virtual particle can be +ve, -ve or imaginary.
- A virtual particle which is off-mass shell by amount Δm can only exist for time and range
$$t \sim \frac{\hbar}{\Delta m c^2} = \frac{1}{\Delta m}, \qquad \text{range} = \frac{\hbar}{\Delta m c} = \frac{1}{\Delta m} \qquad \hbar = c = 1$$
- If $q^2 = m^2$, the the particle is real and can be observed.

Virtual Particles

For virtual particle exchange, expect a contribution to the matrix element of

$$M_{\rm fi} = \frac{g^2}{q^2 - m^2}$$

where	g	Coupling constant
	g^2	Strength of interaction
	m^2	Physical (on-shell) mass
	q^2	Virtual (off-shell) mass
	$\frac{1}{q^2 - m^2}$	Propagator

Qualitatively: the propagator is inversely proportional to how far the particle is off-shell. The further off-shell, the smaller the probability of producing such a virtual state.

- For $m \to 0$; e.g. single γ exchange, $M_{\rm fi} = g^2/q^2$
- For $q^2 \to 0$, very low momentum transfer EM scattering (small angle)

Virtual Particles *Example*

Summary

- SM particles: 12 fermions, 5 spin-1 bosons, 1 spin-0 boson.
- Need relativistic wave equations to describe particle interactions. Klein-Gordon equation (bosons), Dirac equation (fermions).
- Negative energy solutions describe antiparticles.
- The exchange of a massive particle generates an attractive force between two particles.
- Yukawa potential $V(r) = -\frac{g^2}{4\pi}\frac{\mathrm{e}^{-mr}}{r}$
- Exchanged particles may be virtual.

Problem Sheet: q.10

Up next...
Section 5: Feynman Diagrams

5. Feynman Diagrams
Particle and Nuclear Physics

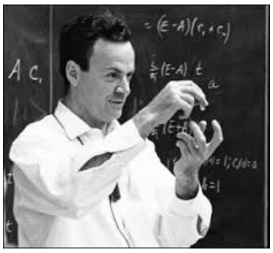
Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- Introduction to Feynman diagrams.
- Anatomy of Feynman diagrams.
- Allowed vertices.
- General rules

Feynman Diagrams

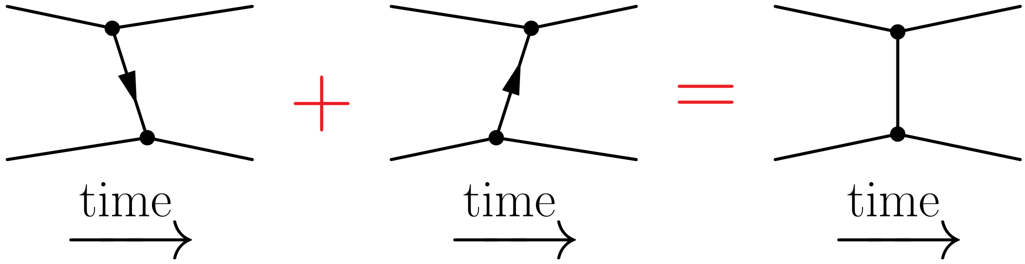


Richard Feynman
1965 Nobel Prize

The results of calculations based on a single process in Time-Ordered Perturbation Theory (sometimes called old-fashioned, OFPT) depend on the reference frame.

The sum of all time orderings is frame independent and provides the basis for our relativistic theory of Quantum Mechanics.

A Feynman diagram represents the sum of **all** time orderings



Feynman Diagrams

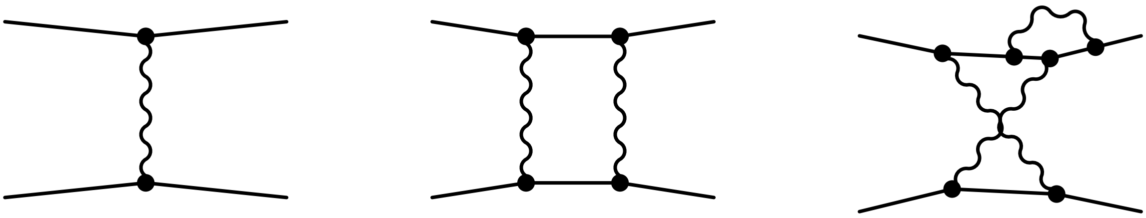
Each Feynman diagram represents a term in the perturbation theory expansion of the matrix element for an interaction.

Normally, a full matrix element contains an infinite number of Feynman diagrams.

Total amplitude $\quad M_{fi} = M_1 + M_2 + M_3 + \ldots$

Total rate $\quad \Gamma_{fi} = 2\pi |M_1 + M_2 + M_3 + \ldots|^2 \rho(E) \quad$ Fermi's Golden Rule

But each vertex gives a factor of g, so if g is small (i.e. the perturbation is small) only need the first few. *(Lowest order = fewest vertices possible)*



g^2 $\qquad$ g^4 $\qquad$ g^6

Example: QED $\quad g = e = \sqrt{4\pi\alpha} \sim 0.30, \quad \alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$

Feynman Diagrams

Perturbation Theory

Calculating Matrix Elements from Perturbation Theory from first principles is cumbersome – so we don't usually use it.

- Need to do time-ordered sums of (on mass shell) particles whose production and decay does not conserve energy and momentum.

Feynman Diagrams

Represent the maths of Perturbation Theory with Feynman Diagrams in a very simple way (to arbitrary order, if couplings are small enough). Use them to calculate matrix elements.

- Approx size of matrix element may be estimated from the simplest valid Feynman Diagram for given process.
- Full matrix element requires infinite number of diagrams.
- Now only need one exchanged particle, but it is now off mass shell, however production/decay now conserves energy and momentum.

Anatomy of Feynman Diagrams

Feynman devised a pictorial method for evaluating matrix elements for the interactions between fundamental particles in a few simple rules. We shall use Feynman diagrams extensively throughout this course.

Topological features of Feynman diagrams are straightforwardly associated with terms in the Matrix element

Represent particles (and antiparticles):

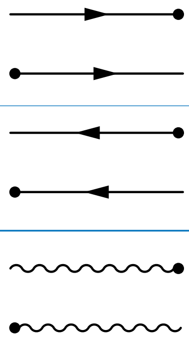
Spin 1/2	Quarks and Leptons	
Spin 1	γ, $W^{\pm}$, Z	
	g	

And each interaction point (vertex) with a •
Each vertex contributes a factor of the coupling constant, g.

Anatomy of Feynman Diagrams

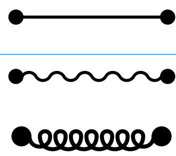
External lines (visible real particles)

Spin 1/2	Particle	Incoming
		Outgoing
	Antiparticle	Incoming
		Outgoing
Spin 1	Particle	Incoming
		Outgoing



Internal lines (propagators; virtual particles)

Spin 1/2	Particle/antiparticle	Each propagator gives a factor of $\dfrac{1}{q^2 - m^2}$
Spin 1	γ, $W^{\pm}$, Z	
	g	



Vertices

A vertex represents a point of interaction: either EM, weak or strong.

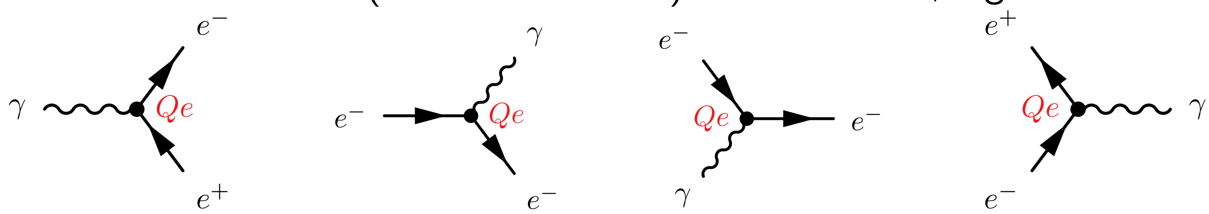
The strength of the interaction is denoted by g

EM interaction: $g = Qe$ (sometimes denoted as $Q\sqrt{\alpha}$, where $\alpha = e^2/4\pi$)

Weak interaction: $g = g_W$

Strong interaction: $g = \sqrt{\alpha_s}$

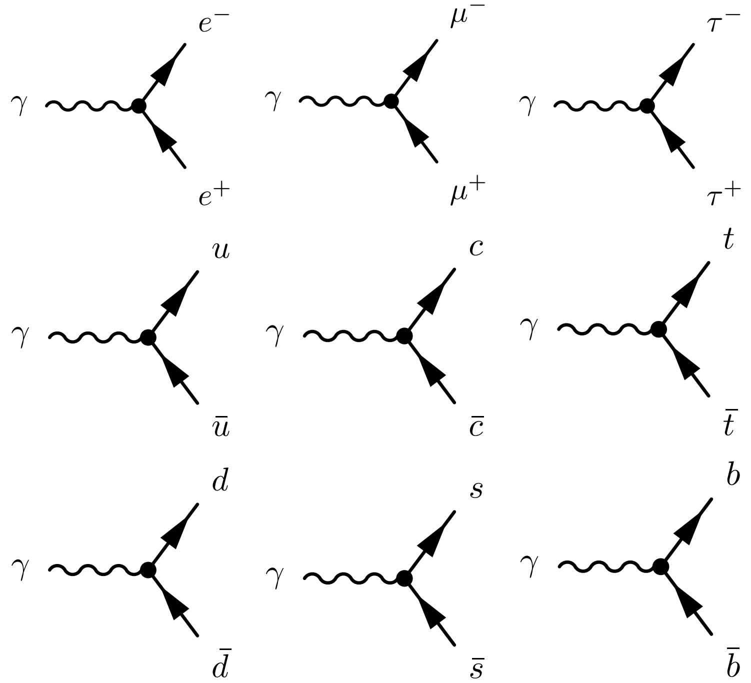
A vertex will have three (in rare cases four) lines attached, e.g.



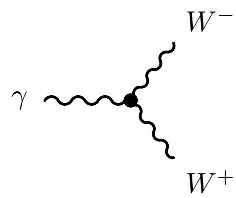
At each vertex, conserve energy, momentum, angular momentum, charge, lepton number ($L_e = +1$ for e^-, ν_e, $= -1$ for $e^+, \bar{\nu}_e$, similar for L_μ, L_τ), baryon number ($B = \frac{1}{3}(n_q - n_{\bar{q}})$), strangeness ($S = -(n_s - n_{\bar{s}})$) & parity – except in weak interactions.

Allowed Vertices *EM*

- must involve a photon γ, and charged particles
- coupling strength Qe Q=charge



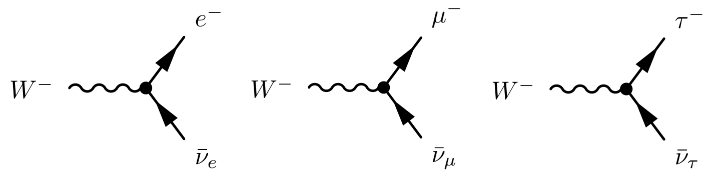
Triple Gauge Vertex

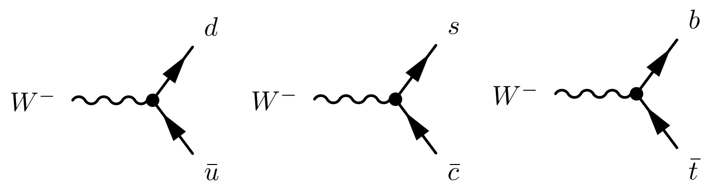


Allowed Vertices *Weak*

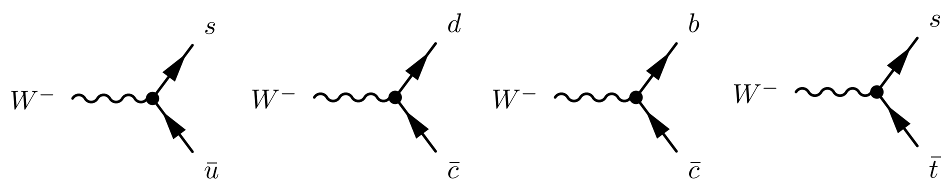
- must involve a gauge vector boson Z or $W^\pm$
- coupling strength g_W
- tip: if you see a ν or $\bar{\nu}$, it must be a weak interaction

with $W^\pm$





$\Rightarrow$ Same family quarks are **Cabibbo favoured**

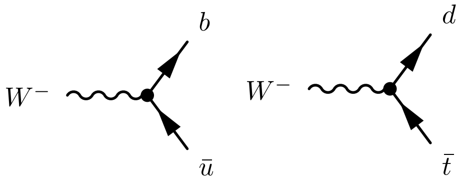


$\Rightarrow$ Cross one family **Cabibbo suppressed**

Allowed Vertices *Weak*

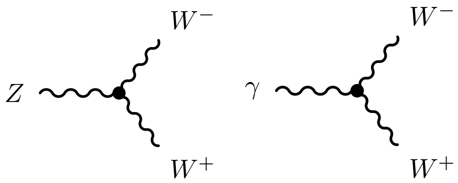
- must involve a gauge vector boson Z or $W^{\pm}$
- coupling strength g_W
- tip: if you see a ν or $\bar{\nu}$, it must be a weak interaction

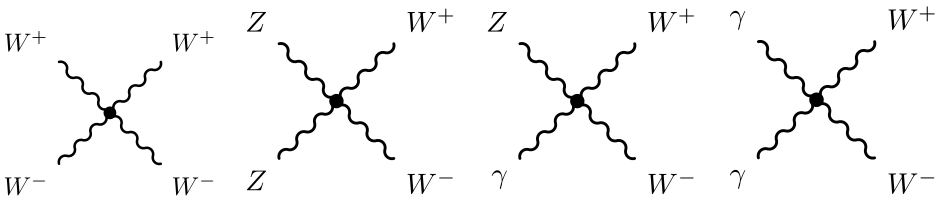
with $W^{\pm}$



$\Rightarrow$ Cross two families
Doubly Cabibbo suppressed

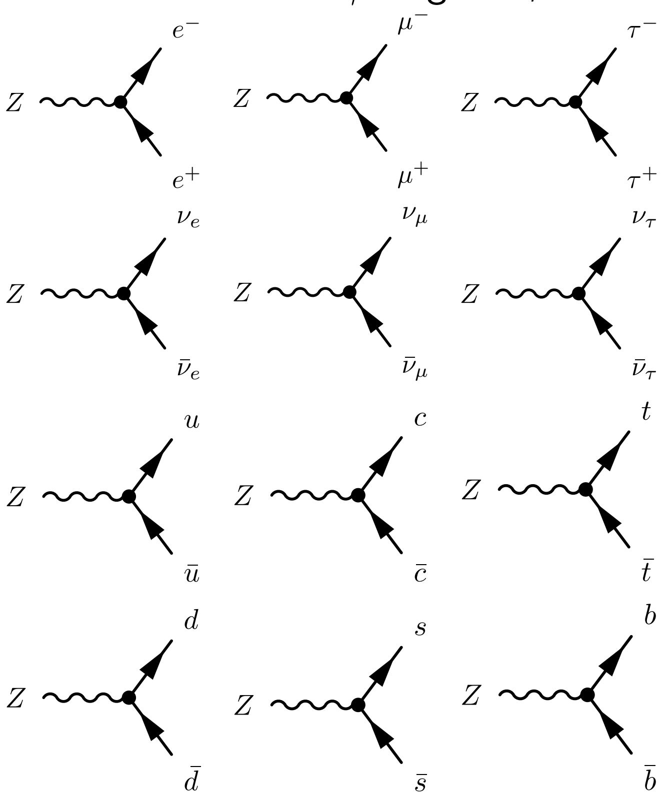
Also, Triple/Four Gauge Vertex



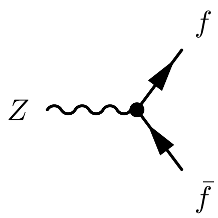


Allowed Vertices *Weak*

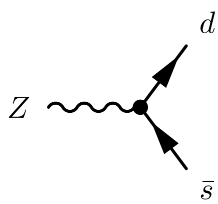
with Z Same as γ diagrams, but also vertices with ν



i.e.

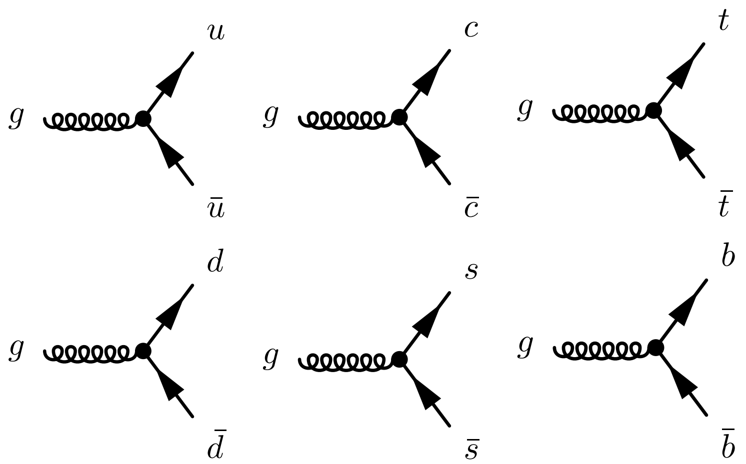


Not Allowed:
Flavour Changing
Neutral Currents (FCNC)

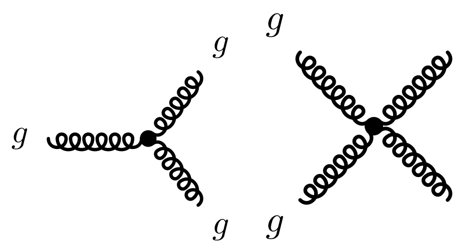


Allowed Vertices *Strong*

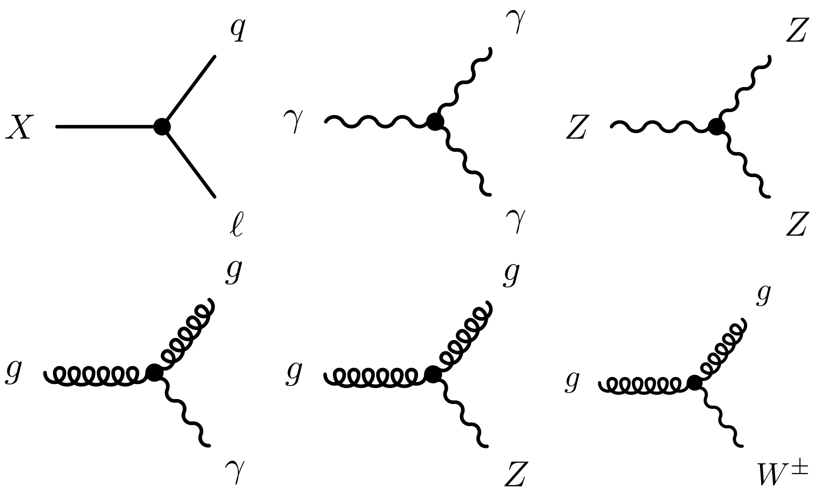
- must involve a gluon g and/or quark q
- coupling strength $\sqrt{\alpha_s}$
- conserve strangeness, charm etc



Also, Triple Gauge Vertex

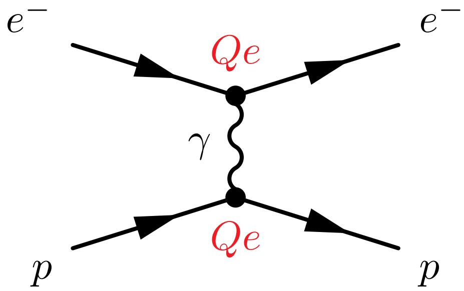


Forbidden Vertices



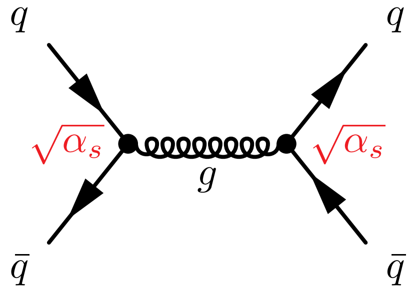
Examples

Electromagnetic



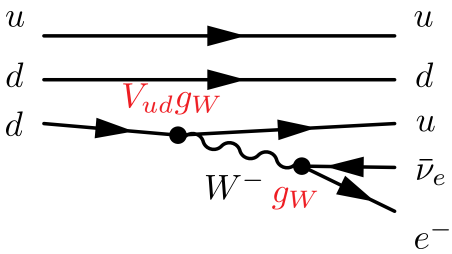
$$M \sim \frac{(e)^2}{q^2}$$

Strong



$$M \sim \frac{(\sqrt{\alpha_s})^2}{q^2}$$

Weak



$$M \sim \frac{V_{ud}g_W^2}{q^2 - m_W^2}$$

Drawing Feynman Diagrams

A Feynman diagram is a pictorial representation of the matrix element describing particle decay or interaction

$$a \to b + c + \ldots \qquad a + b \to c + d$$

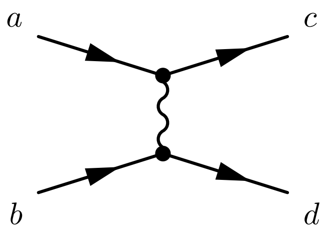
To draw a Feynman diagram and determine whether a process is allowed, follow the five basic steps below:

1. Write down the initial and final state particles and antiparticles and note the quark content of all hadrons.
2. Draw the simplest Feynman diagram using the Standard Model vertices. Bearing in mind:
   - Similar diagrams for particles/antiparticles
   - Never have a vertex connecting a lepton to a quark
   - Only the weak charged current ($W^{\pm}$) vertex changes flavour
     within generations for leptons
     within/between generations for quarks

Drawing Feynman Diagrams *Particle scattering*

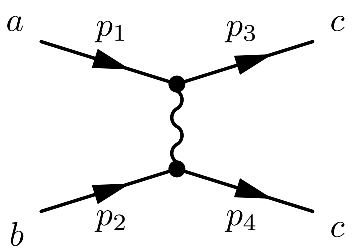
- If all are particles (or all are antiparticles), only scattering diagrams involved e.g. $a + b \to c + d$

Initial State



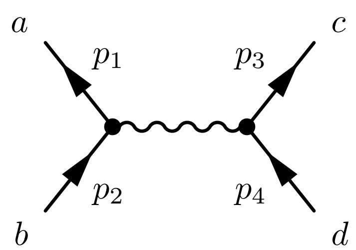
Final State

- If particles and antiparticles, may be able to have scattering and/or annihilation diagrams e.g. $a + b \to c + d$ (Mandelstam variables s, t, u)



"t-channel",

$q^2 = t = (p_1 - p_3)^2 = (p_2 - p_4)^2$



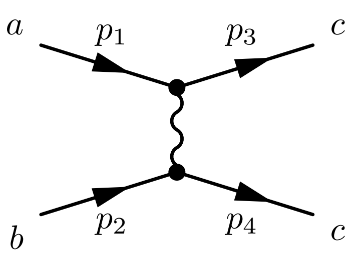
"s-channel",

$q^2 = s = (p_1 + p_2)^2 = (p_3 + p_4)^2$

Drawing Feynman Diagrams *Identical Particles*

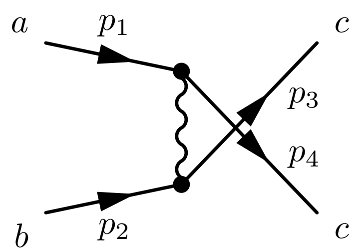
If we have identical particles in final state, e.g. $a + b \to c + c$
may not know which particle comes from which vertex.

Two possibilities are separate final Feynman diagrams:



"t-channel",

$q^2 = t = (p_1 - p_3)^2 = (p_2 - p_4)^2$



"u-channel",

$q^2 = u = (p_1 - p_4)^2 = (p_2 - p_3)^2$

Crossing not a vertex

Drawing Feynman Diagrams

Being able to draw a Feynman diagram is a necessary, but not a sufficient condition for the process to occur. Also need to check:

3. Check that the whole system conserves
   - Energy, momentum (trivially satisfied for interactions, so long as sufficient KE in initial state. May forbid decays)
   - Charge
   - Angular momentum
4. Parity
   - Conserved in EM/Strong interaction
   - Can be violated in the Weak interaction
5. Check symmetry for identical particles in the final state
   - Bosons $\psi(1,2) = +\psi(2,1)$
   - Fermions $\psi(1,2) = -\psi(2,1)$

Finally, a process will occur via the Strong, EM and Weak interaction (in that order of preference) if steps 1 – 5 are satisfied.

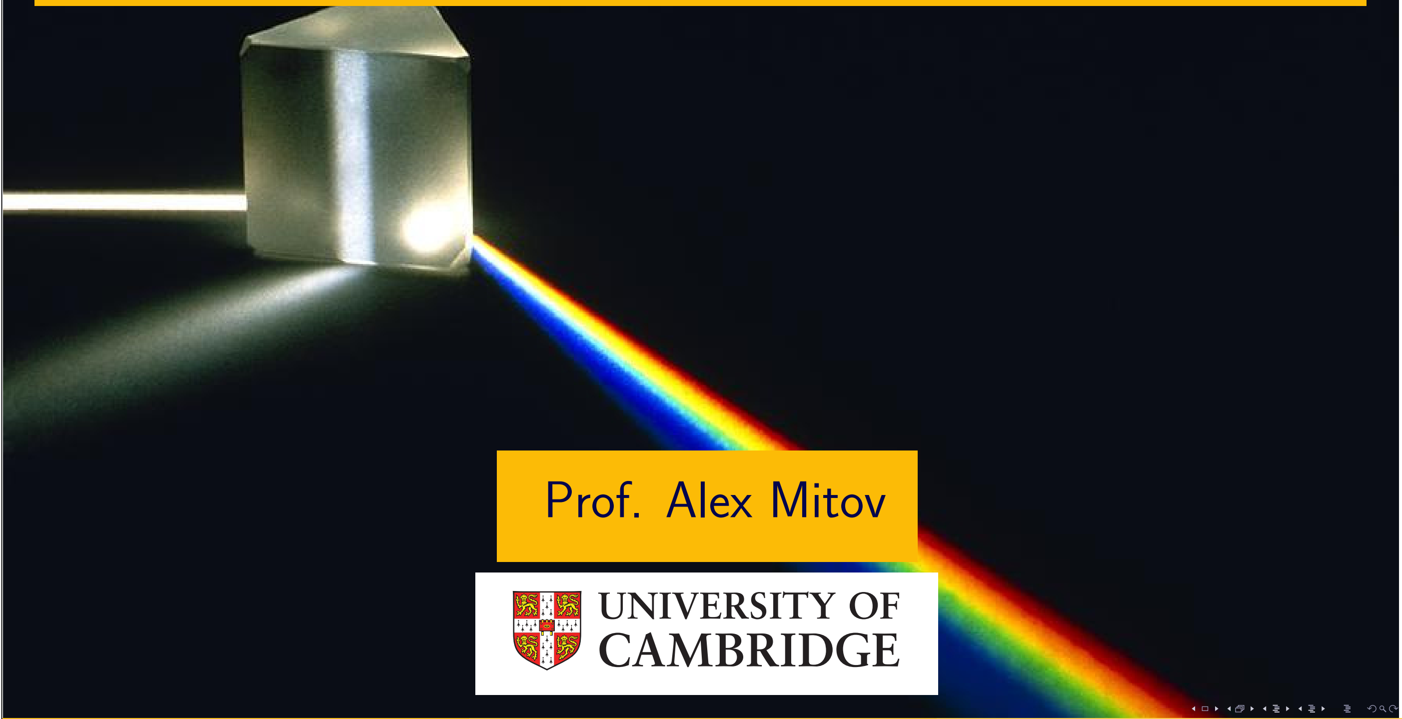
Summary

- Feynman diagrams are a core part of the course.
  Make sure you can draw them!
- Feynman diagrams are a sum over time orderings.
- Associate topological features of the diagrams with terms in matrix elements.
- Vertices $\leftrightarrow$ coupling strength between particles and field quanta
- Propagator for each internal line (off-mass shell, virtual particles)
- Conservation of quantum numbers at each vertex

Problem Sheet: q.11

Up next...
Section 6: QED

6. QED
Particle and Nuclear Physics



Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- Gauge invariance
- Allowed vertices + examples
- Scattering
- Experimental tests
- Running of alpha

QED

Quantum Electrodynamics is the gauge theory of electromagnetic interactions.

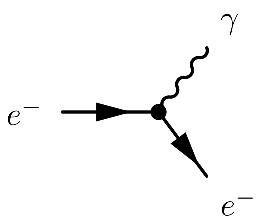
Consider a non-relativistic charged particle in an EM field:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$\vec{E}, \vec{B}$ given in term of vector and scalar potentials $\vec{A}, \varphi$

$$\vec{B} = \vec{\nabla} \times \vec{A}; \quad \vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} \qquad \text{Maxwell's Equations}$$

$$\hat{H} = \frac{1}{2m}(\hat{\vec{p}} - q\vec{A})^2 + q\varphi \qquad \text{Classical Hamiltonian}$$



Change in state of e^- requires change in field

$\Rightarrow$ Interaction via virtual γ emission

QED

Schrödinger equation $$\left[\frac{1}{2m}(\hat{\vec{p}} - q\vec{A})^2 + q\varphi\right]\psi(\vec{r}, t) = i\frac{\partial \psi(\vec{r}, t)}{\partial t}$$

is invariant under the local gauge transformation $\psi \to \psi' = \mathrm{e}^{\mathrm{i}q\alpha(\vec{r},t)}\psi$

so long as $$\vec{A} \to \vec{A} + \vec{\nabla}\alpha \; ; \quad \varphi \to \varphi - \frac{\partial \alpha}{\partial t} \qquad \text{(See Appendix E)}$$

Local Gauge Invariance requires the existence of a physical Gauge Field (photon) and completely specifies the form of the interaction between the particle and field.

- Photons are massless

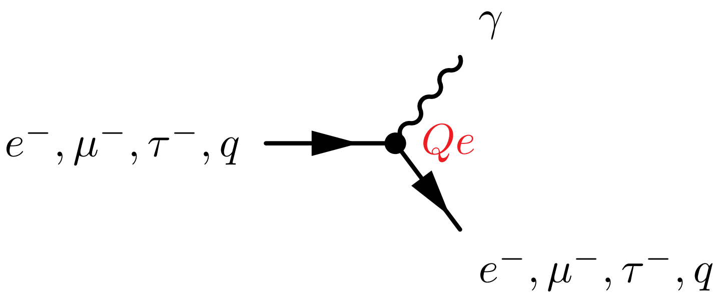
  (in order to cancel phase changes over all space-time, the range of the photon must be infinite)

- Charge is conserved – the charge q which interacts with the field must not change in space or time

QED is a gauge theory

The Electromagnetic Vertex

All electromagnetic interactions can be described by the photon propagator and the EM vertex:



The Standard Model Electromagnetic Vertex

+ antiparticles

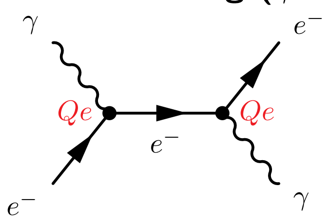
$$\alpha = \frac{e^2}{4\pi}$$

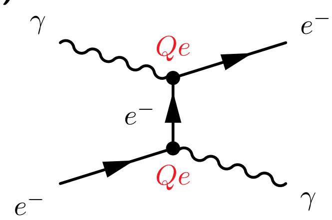
- The coupling constant is proportional to the fermion charge.
- Energy, momentum, angular momentum, parity and charge always conserved.
- QED vertex never changes particle type or flavour
  i.e. $e^- \to e^-\gamma$, but not $e^- \to q\gamma$ or $e^- \to \mu^-\gamma$

Important QED Processes

$$M \sim \frac{g^2}{q^2}, \quad \alpha = \frac{e^2}{4\pi}$$

Compton Scattering ($\gamma e^- \to \gamma e^-$)

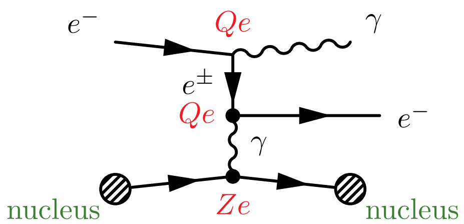




$$M \propto e^2$$

$$\sigma \propto |M|^2 \propto e^4$$
$$\propto (4\pi)^2\alpha^2$$

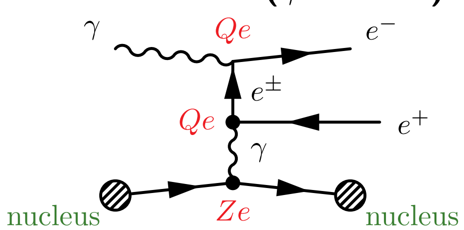
Bremsstrahlung ($e^- \to e^-\gamma$)



$$M \propto Ze^3$$

$$\sigma \propto |M|^2 \propto Z^2e^6$$
$$\propto (4\pi)^3Z^2\alpha^3$$

Pair Production ($\gamma \to e^+e^-$)



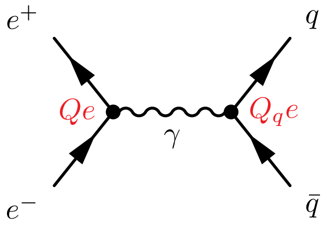
$$M \propto Ze^3$$

$$\sigma \propto |M|^2 \propto Z^2e^6$$
$$\propto (4\pi)^3Z^2\alpha^3$$

The processes $e^- \to e^-\gamma$ and $\gamma \to e^+e^-$ cannot occur for real e^-, γ due to energy & momentum conservation

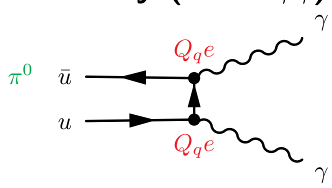
Important QED Processes

Electron-Positron Annihilation ($e^- e^+ \to q\bar{q}$)



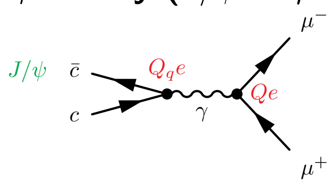
$$M \propto Q_q e^2$$
$$\sigma \propto |M|^2 \propto Q_q^2 e^4$$
$$\propto (4\pi)^2 Q_q^2 \alpha^2$$

Pion Decay ($\pi^0 \to \gamma\gamma$)



$$M \propto Q_u^2 e^2$$
$$\Gamma \propto |M|^2 \propto Q_u^4 e^4$$
$$\propto (4\pi)^2 Q_u^4 \alpha^2$$

J/ψ Decay ($J/\psi \to \mu^+\mu^-$)



$$M \propto Q_c e^2$$
$$\Gamma \propto |M|^2 \propto Q_c^2 e^4$$
$$\propto (4\pi)^2 Q_c^2 \alpha^2$$

The coupling strength determines "order of magnitude" of the matrix element.

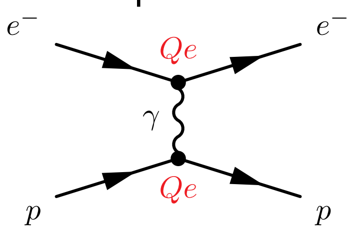
For particles interacting/decaying via EM interaction: typical values for cross-sections/ lifetimes

$$\sigma_{\rm EM} \sim 10^{-2}\ \rm mb;$$
$$\tau_{\rm EM} \sim 10^{-20}\ \rm s$$

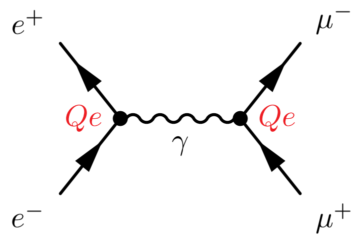
Scattering in QED *Examples*

Calculate the "spin-less" cross-sections for the two processes:

1. Electron-proton scattering



2. Electron-positron annihilation



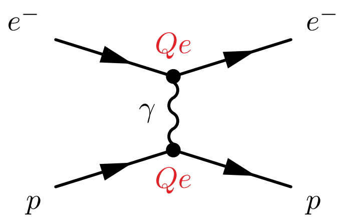
Fermi's Golden rule and Born Approximation $\quad \dfrac{d\sigma}{d\Omega} = \dfrac{E^2}{(2\pi)^2}|M|^2$

For both processes we have the same matrix element (though q^2 is different)

$$M = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

- $e^2 = 4\pi\alpha$ is the strength of the interaction.
- $1/q^2$ measures the probability that the photon carries 4-momentum $q^\mu = (E, \vec{p})$; $\quad q^2 = E^2 - |\vec{p}|^2 \quad$ i.e. smaller probability for higher mass.

Scattering in QED *1. "Spinless" $e-p$ Scattering*



$$M = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|M|^2 = \frac{E^2}{(2\pi)^2}\frac{(4\pi\alpha)^2}{q^4} = \frac{4\alpha^2E^2}{q^4}$$

q^2 is the four-momentum transfer
$$\begin{aligned} q^2 = q^\mu q_\mu &= (E_\mathrm{f} - E_\mathrm{i})^2 - (\vec{p}_\mathrm{f} - \vec{p}_\mathrm{i})^2 \\ &= E_\mathrm{f}^2 + E_\mathrm{i}^2 - 2E_\mathrm{f}E_\mathrm{i} - \vec{p}_\mathrm{f}^2 - \vec{p}_\mathrm{i}^2 + 2\vec{p}_\mathrm{f}.\vec{p}_\mathrm{i} \\ &= 2m_e^2 - 2E_\mathrm{f}E_\mathrm{i} + 2|\vec{p}_\mathrm{f}||\vec{p}_\mathrm{i}|\cos\theta \end{aligned}$$

Neglecting electron mass: i.e. $m_e = 0$ and $|\vec{p}_\mathrm{f}| = E_\mathrm{f}$

$$q^2 = -2E_\mathrm{f}E_\mathrm{i}(1-\cos\theta) = -4E_\mathrm{f}E_\mathrm{i}\sin^2\frac{\theta}{2}$$

Therefore, for elastic scattering $E_\mathrm{i} = E_\mathrm{f}$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}$$

Rutherford Scattering

same result from QED as from conventional QM

Scattering in QED *1. "Spinless" $e-p$ Scattering*

The discovery of quarks

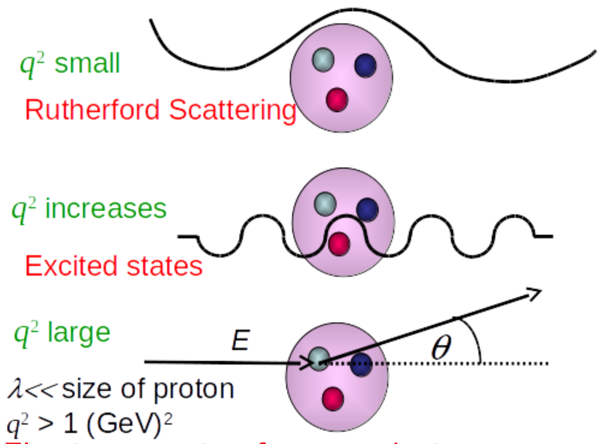
Virtual γ carries 4-momentum $q^\mu = (E, \vec{p})$

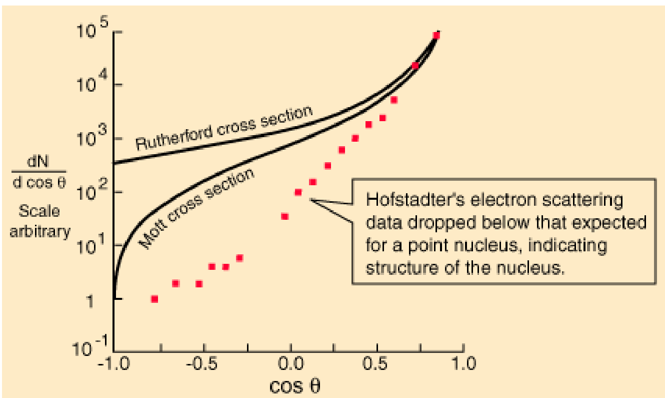
Large $q \Rightarrow$ Large $\vec{p}$, small λ $\quad |\vec{p}| = \hbar/\lambda$

Large E, large ω $\quad E = \hbar\omega$

High q wavefunction oscillates rapidly in space and time

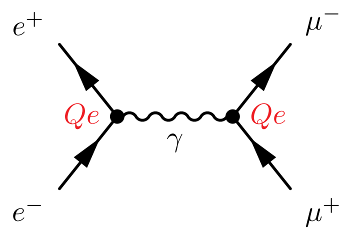
$\Rightarrow$ probes short distances and short time.





Elastic scattering from quarks in proton.

Scattering in QED 2. *"Spinless" e^+e^- Scattering*



$$M = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{E^2}{(2\pi)^2}|M|^2 = \frac{E^2}{(2\pi)^2}\frac{(4\pi\alpha)^2}{q^4} = \frac{4\alpha^2 E^2}{q^4}$$

Same formula, but different four-momentum transfer

$$q^2 = q^\mu q_\mu = (E_{e^+} + E_{e^-})^2 - (\vec{p}_{e^+} + \vec{p}_{e^-})^2$$

assuming we are in the centre-of-mass system, $E_{e^+} = E_{e^-} = E$, $\vec{p}_{e^+} = -\vec{p}_{e^-}$

$$q^2 = q^\mu q_\mu = (2E)^2 = s$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{4\alpha^2 E^2}{q^4} = \frac{4\alpha^2 E^2}{16E^4} = \frac{\alpha^2}{s}$$

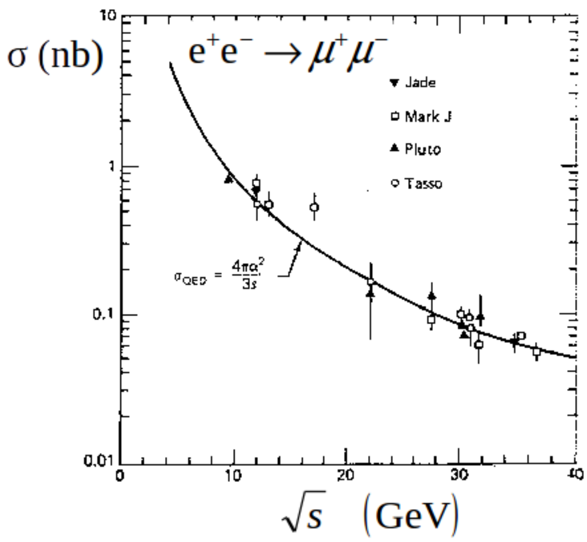
Integrating gives total cross-section: $\sigma = \dfrac{4\pi\alpha^2}{s}$

Scattering in QED 2. *"Spinless" e^+e^- Scattering*

... the actual cross-section (using the Dirac equation to take spin into account) is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$



Example: Cross-section at $\sqrt{s} = 22$ GeV
(i.e. 11 GeV electrons colliding with 11 GeV positrons)

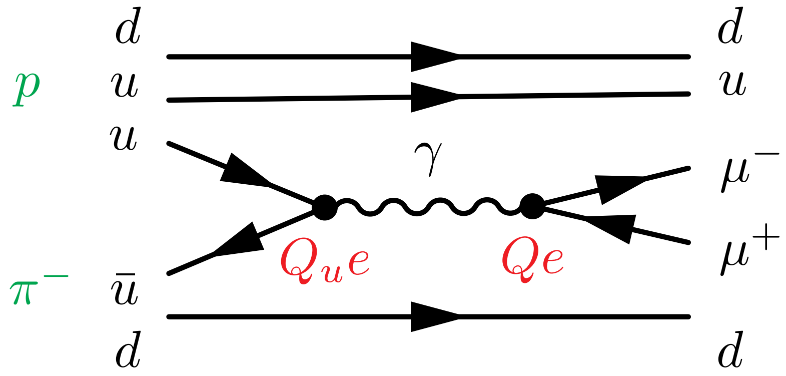
$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{4\pi}{(137)^2}\frac{1}{3\times 22^2}$$

$$= 4.6\times 10^{-7}\ \mathrm{GeV}^{-2} = 4.6\times 10^{-7}\times(0.197)^2\ \mathrm{fm}^2 = 1.8\times 10^{-8}\ \mathrm{fm}^2 = 0.18\ \mathrm{nb}$$

The Drell-Yan Process

Can also annihilate $q\bar{q}$ as in the "Drell-Yan" process.

Example: $\pi^- p \to \mu^+\mu^- +$ hadrons (See problem sheet q.13)



$$\sigma(\pi^- p \to \mu^+\mu^- + \text{ hadrons}) \propto Q_u^2\alpha^2 \propto Q_u^2 e^4$$

(Also need to account for presence of two u quarks in proton)

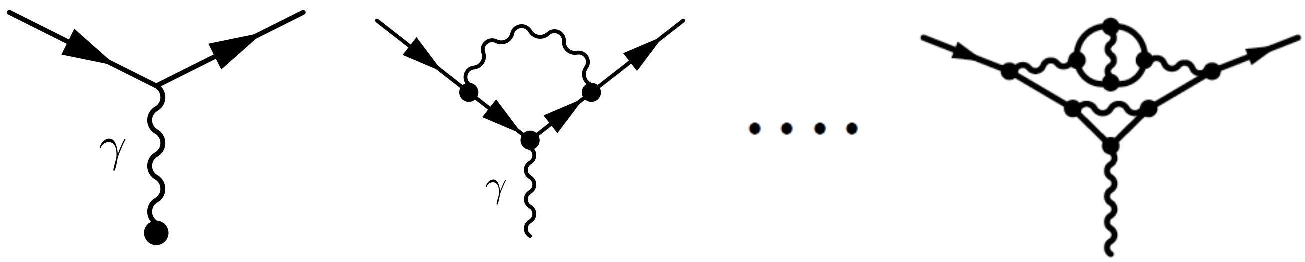
Experimental Tests of QED

QED is an extremely successful theory tested to very high precision.

Example:

- Magnetic moments of $e^\pm$, $\mu^\pm$: $\vec{\mu} = g\frac{e}{2m}\vec{s}$
- For a point-like spin 1/2 particle: $g = 2$ Dirac Equation

However, higher order terms in QED introduce an anomalous magnetic moment $\Rightarrow$ g is not quite equal to 2.

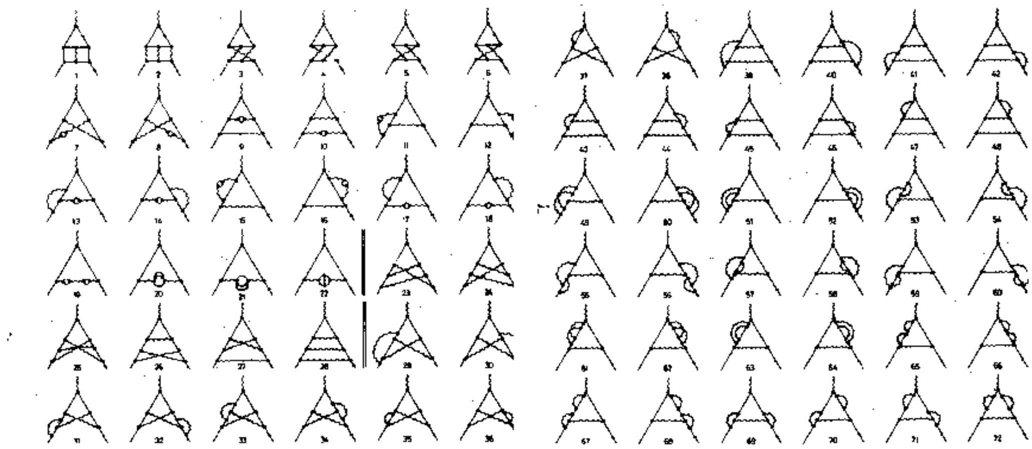


O(1) O(α) O(α^4)

12672 diagrams

Experimental Tests of QED

O(α^3)



$$\frac{g_e - 2}{2} = 11596521.811 \pm 0.007 \times 10^{-10} \quad \text{Experiment}$$

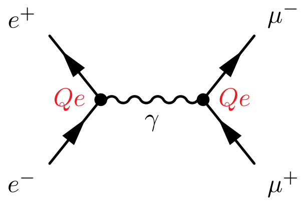
$$= 11596521.3 \pm 0.3 \times 10^{-10} \quad \text{Theory}$$

- Agreement at the level of 1 in 10^8
- QED provides a remarkably precise description of the electromagnetic interaction!

Higher Orders

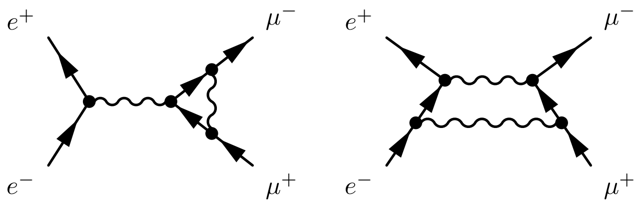
So far only considered lowest order term in the perturbation series.
Higher order terms also contribute (and also interfere with lower orders)

Lowest Order



$$|M|^2 \propto e^4 \propto \alpha^2 \sim \left(\frac{1}{137}\right)^2$$

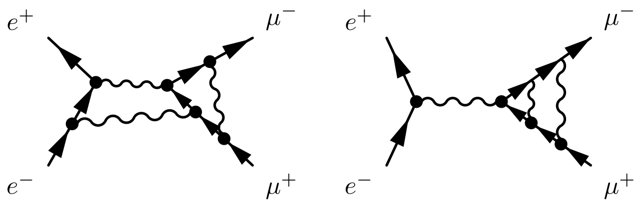
Second Order



$+ \ldots$

$$|M|^2 \propto \alpha^4 \sim \left(\frac{1}{137}\right)^4$$

Third Order



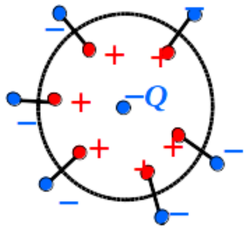
$+ \ldots$

$$|M|^2 \propto \alpha^6 \sim \left(\frac{1}{137}\right)^6$$

Second order suppressed by α^2 relative to first order.
Provided α is small, i.e. perturbation is small, lowest order dominates.

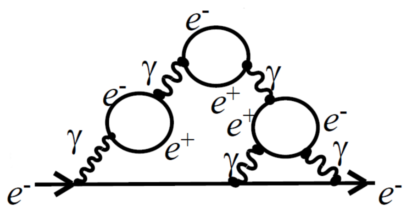
Running of α

- $\alpha = \frac{e^2}{4\pi}$ specifies the strength of the interaction between an electron and a photon.
- But α is not a constant



Consider an electric charge in a dielectric medium.
Charge Q appears screened by a halo of $+$ve charges.
Only see full value of charge Q at small distance.

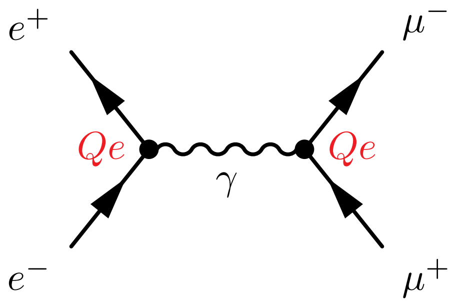
Consider a free electron.
The same effect can happen due to quantum fluctuations that lead to a cloud of virtual e^+e^- pairs.

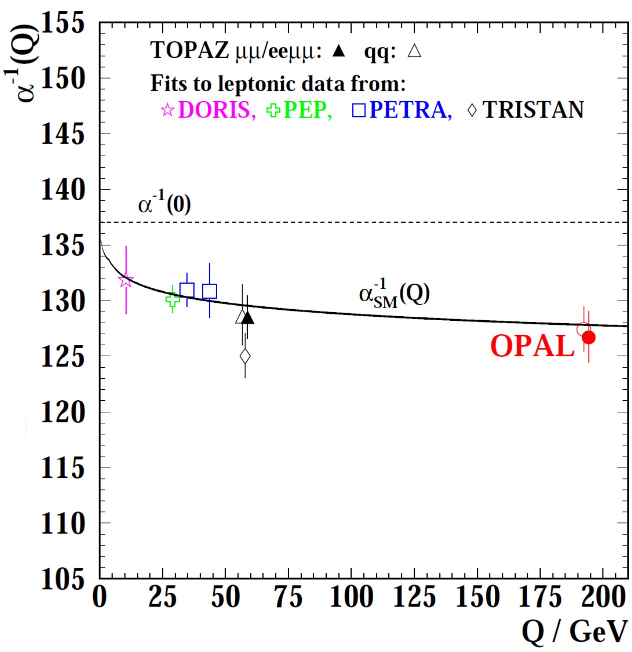


- The vacuum acts like a dielectric medium
- The virtual e^+e^- pairs are therefore polarised
- At large distances the bare electron charge is screened.
- At shorter distances, screening effect reduced and we see a larger effective charge i.e. a larger α.

Running of α

Can measure $\alpha(q^2)$ from $e^+e^- \to \mu^+\mu^-$ etc.

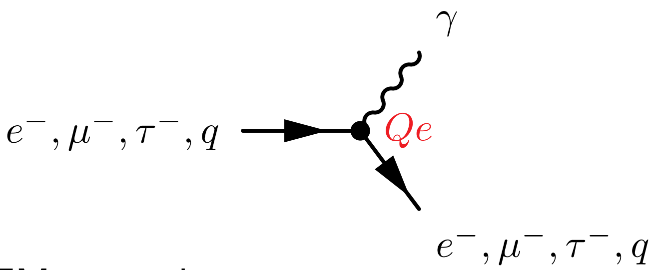




- α increases with increasing q^2 (i.e. closer to the bare charge)
- At $q^2 = 0$: $\alpha \sim 1/137$
- At $q^2 \sim (100\ \mathrm{GeV})^2$: $\alpha \sim 1/128$

Summary

- QED is the physics of the photon + "charged particle" vertex:



$$\alpha = \frac{e^2}{4\pi}$$

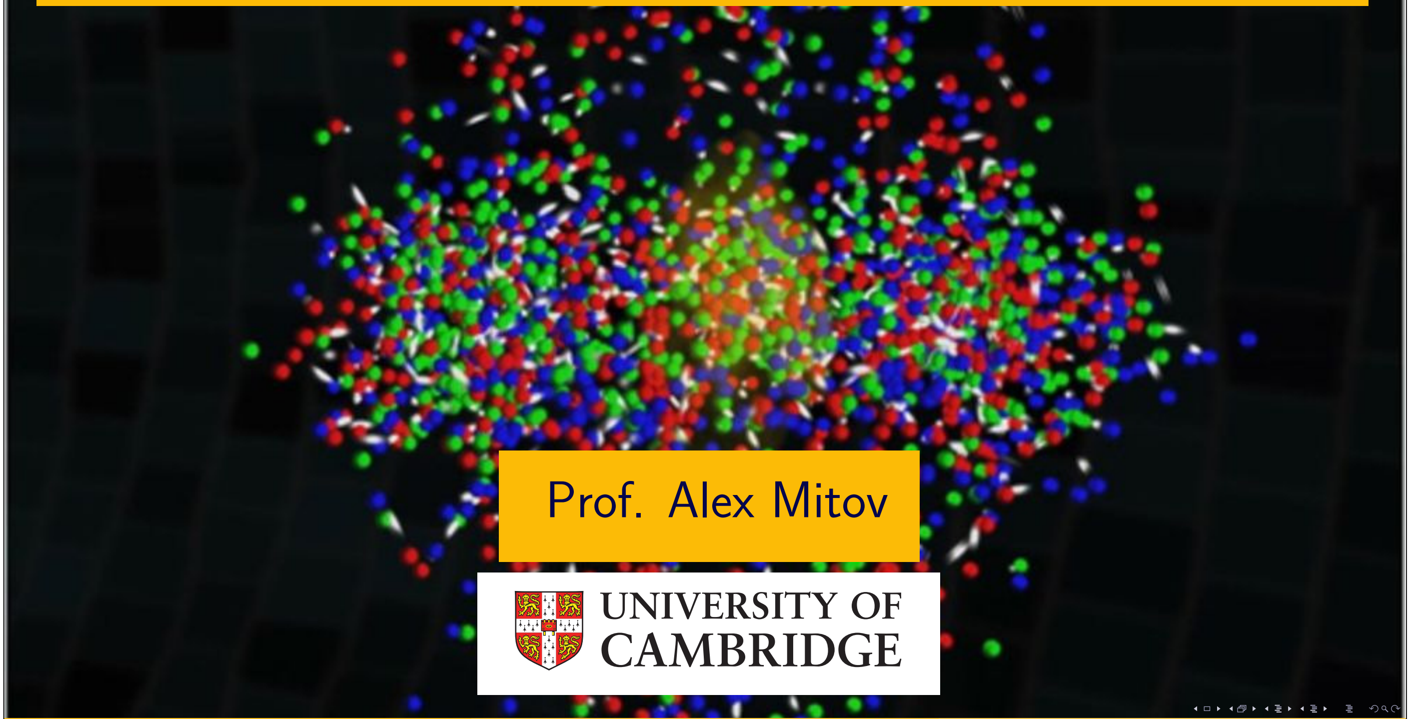
- Every EM vertex has:
  - has an arrow going in & out (lepton or quark), and a photon
  - does not change the type of lepton or quark "passing through"
  - conserves charge, energy and momentum
- The dimensionless coupling $\sqrt{\alpha}$ is proportional to the electric charge of the lepton or quark, and it "runs" with energy scale.
- QED has been tested at the level of 1 part in 10^8.

Problem Sheet: q.12-14

Up next... Section 7: QCD

7. QCD

Particle and Nuclear Physics



Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- The strong vertex
- Colour, gluons and self-interactions
- QCD potential, confinement
- Hadronisation, jets
- Running of α_s
- Experimental tests of QCD

QCD

Quantum Electrodynamics is the quantum theory of the electromagnetic interaction.

- mediated by massless photons
- photon couples to electric charge
- strength of interaction: $\langle\psi_f|\hat{H}|\psi_i\rangle \propto \sqrt{\alpha}$ $\qquad \alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

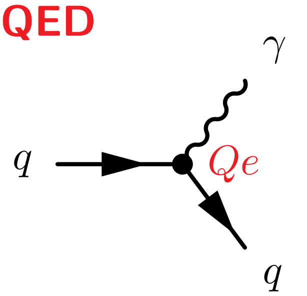
Quantum Chromodynamics is the quantum theory of the strong interaction.

- mediated by massless gluons
- gluon couples to "strong" charge
- only quarks have non-zero "strong" charge, therefore only quarks feel the strong interaction.
- strength of interaction: $\langle\psi_f|\hat{H}|\psi_i\rangle \propto \sqrt{\alpha_s}$ $\qquad \alpha_s = \frac{g_s^2}{4\pi} \sim 1$

The Strong Vertex

Basic QCD interaction looks like a stronger version of QED:

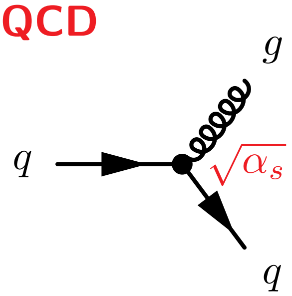
QED



+ antiquarks

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

QCD



+ antiquarks

$$\alpha_s = \frac{g_s^2}{4\pi} \sim 1$$

- The coupling of the gluon, g_s, is to the "strong" charge.
- Energy, momentum, angular momentum and charge always conserved.
- QCD vertex never changes quark flavour
- QCD vertex always conserves parity

Colour

QED:

- Charge of QED is electric charge, a conserved quantum number

QCD:

- Charge of QCD is called " colour "
- colour is a conserved quantum number with 3 values labelled red, green and blue.

Quarks carry	colour	r b g
Antiquarks carry	anti- colour	$\bar{r}$ $\bar{b}$ $\bar{g}$

- Colorless particles either have
  - no colour at all e.g. leptons, γ, W, Z and do not interact via the strong interaction
  - or equal parts r, b, g e.g. meson $q\bar{q}$ with $\frac{1}{\sqrt{3}}(r\bar{r}+b\bar{b}+g\bar{g})$, baryon qqq with rgb
- gluons do not have equal parts r, b, g, so carry colour (e.g. $r\bar{r}$, see later)

QCD as a gauge theory

- Recall QED was invariant under gauge symmetry

$$\psi \rightarrow \psi' = e^{iq\alpha(\vec{r},t)}\psi$$

- The equivalent symmetry for QCD is invariance under *(non-examinable)*

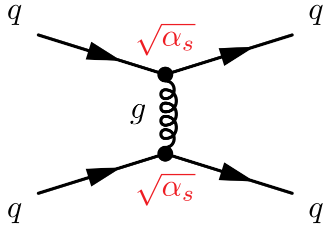
$$\psi \rightarrow \psi' = e^{ig\vec{\lambda}.\vec{\Lambda}(\vec{r},t)}\psi$$

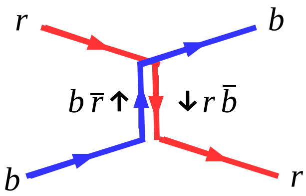
an "SU(3)" transformation (λ are eight 3x3 matrices).
- Operates on the colour state of the quark field – a "rotation" of the colour state which can be different at each point of space and time.
- Invariance under SU(3) transformations $\rightarrow$ eight massless gauge bosons, gluons (eight in this case). Gluon couplings are well specified.
- Gluons also have self-couplings, i.e. they carry colour themselves...

Gluons

Gluons are massless spin-1 bosons, which carry the colour quantum number (unlike γ in QED which is charge neutral).

Consider a red quark scattering off a blue quark. Colour is exchanged, but always conserved (overall and at each vertex).





Expect 9 gluons (3x3): $r\bar{b}$ $r\bar{g}$ $g\bar{r}$ $g\bar{b}$ $b\bar{g}$ $b\bar{r}$ $r\bar{r}$ $b\bar{b}$ $g\bar{g}$

However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\bar{r}+b\bar{b}+g\bar{g})$ is colourless and does not participate in the strong interaction. $\Rightarrow$ 8 coloured gluons

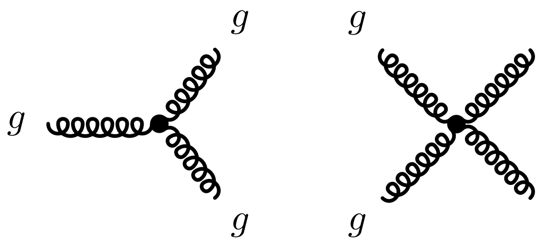
Conventionally chosen to be (all orthogonal):

$$r\bar{b} \quad r\bar{g} \quad g\bar{r} \quad g\bar{b} \quad b\bar{g} \quad b\bar{r} \quad \frac{1}{\sqrt{2}}(r\bar{r}-b\bar{b}) \quad \frac{1}{\sqrt{6}}(r\bar{r}+b\bar{b}-2g\bar{g})$$

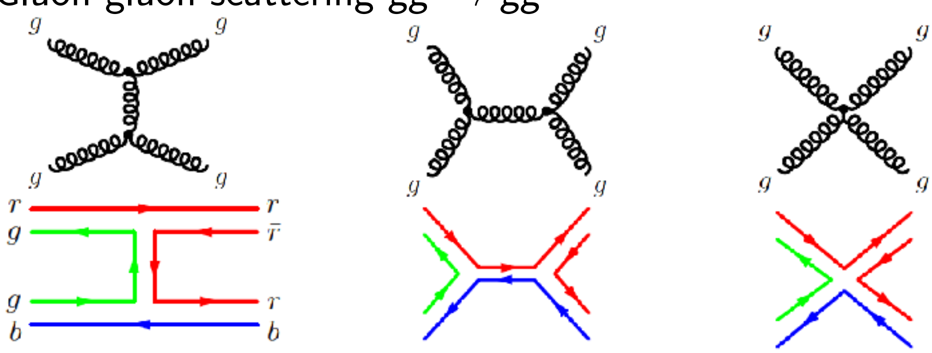
Gluon Self-Interactions

QCD looks like a stronger version of QED. However, there is one big difference and that is gluons carry colour charge.

$\Rightarrow$ Gluons can interact with other gluons



Example: Gluon-gluon scattering $gg \to gg$



Same colour flow in each case: $r\bar{g}+g\bar{b} \to r\bar{r}+r\bar{b}$

QCD Potential

QED Potential: $V_{\mathrm{QED}} = -\frac{\alpha}{r}$

QCD Potential: $V_{\mathrm{QCD}} = -C\frac{\alpha_s}{r}$

At short distances, QCD potential looks similar, apart from the "colour factor" C.

For $q\bar{q}$ in a colourless state in a meson, $C = 4/3$
For qq in a colourless state in baryon, $C = 2/3$

Note: the colour factor C arises because more than one gluon can participate in the process $q \to qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states.

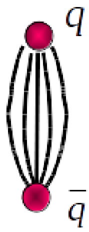
Confinement

Never observe single free quarks or gluons

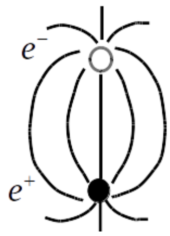
- Quarks are always confined within hadrons
- This is a consequence of the strong interaction of gluons.

Qualitatively, compare QCD with QED:

QCD
Colour field



QED
Electric field



Self interactions of the gluons squeezes the lines of force into a narrow tube or string. The string has a "tension" and as the quarks separate the string stores potential energy.

Energy stored per unit length in field $\sim$ constant $V(r) \propto r$

Energy required to separate two quarks is infinite. Quarks always come in combinations with zero net colour charge $\Rightarrow$ confinement.

How Strong is Strong?

QCD potential between quark and antiquark has two components:

- Short range, Coulomb-like term: $-\frac{4}{3}\frac{\alpha_s}{r}$
- Long range, linear term: $+kr$

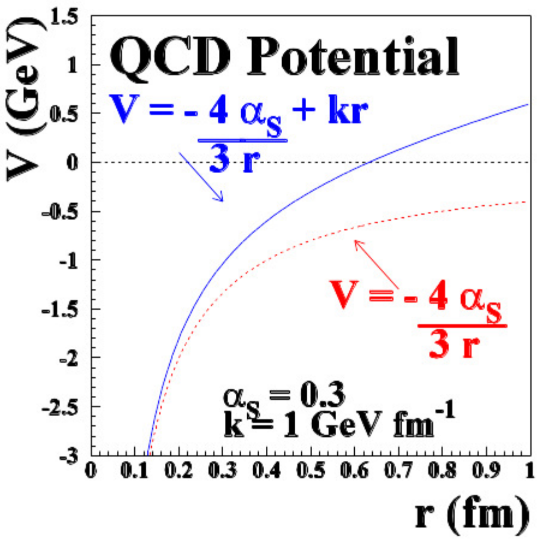
$$V_{\mathrm{QCD}} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

with $k \sim 1\ \mathrm{GeV/fm}$

$$F = -\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{4}{3}\frac{\alpha_s}{r^2} + k$$

at large r

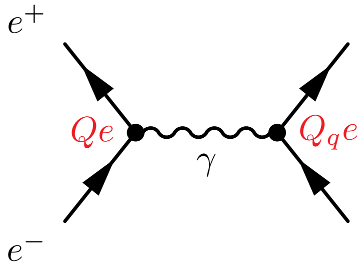
$$F = k \sim \frac{1.6 \times 10^{-10}}{10^{-15}}\,\mathrm{N} \;=\; 160,000\,\mathrm{N}$$

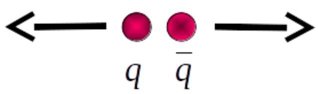


Equivalent to weight of ~150 people

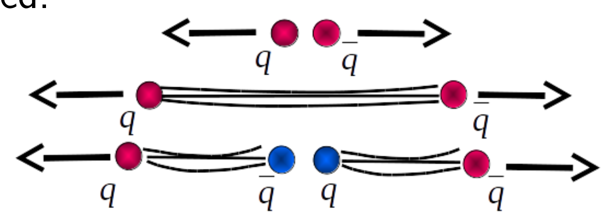
Jets

Consider the $q\bar{q}$ pair produced in $e^+e^- \to q\bar{q}$

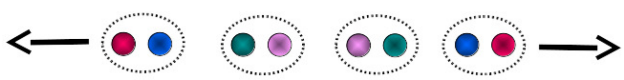




As the quarks separate, the potential energy in the colour field ("string") starts to increase linearly with separation. When the energy stored exceeds $2m_q$, new $q\bar{q}$ pairs can be created.

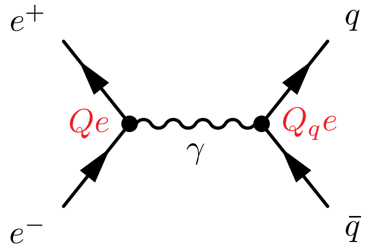


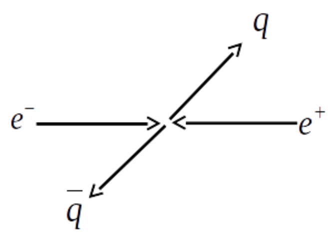
As energy decreases, hadrons (mainly mesons) freeze out

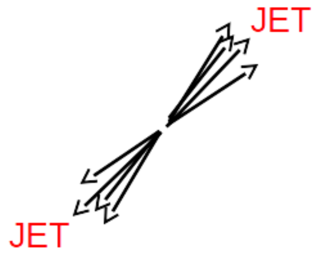


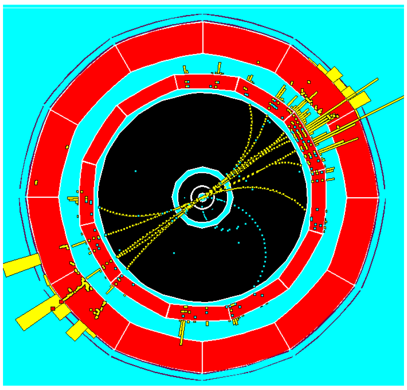
Jets

As quarks separate, more $q\bar{q}$ pairs are produced. This process is called hadronisation. Start out with quarks and end up with narrowly collimated jets of hadrons.







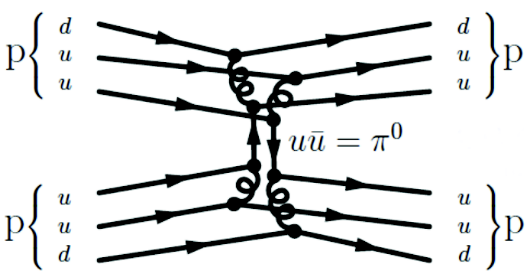


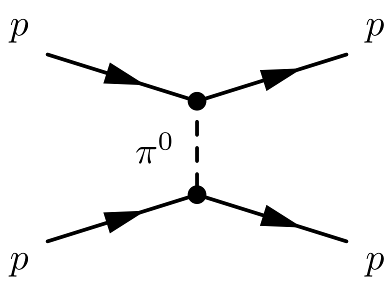
Typical $e^+e^- \to q\bar{q}$ event

The hadrons in a quark(antiquark) jet follow the direction of the original quark(antiquark). Consequently, $e^+e^- \to q\bar{q}$ is observed as a pair of back-to-back jets.

Nucleon-Nucleon Interactions

- Bound qqq states (e.g. protons and neutrons) are colourless (colour singlets)
- They can only emit and absorb another colour singlet state, i.e. not single gluons (conservation of colour charge).
- Interact by exchange of pions.
  Example: pp scattering (One possible diagram)



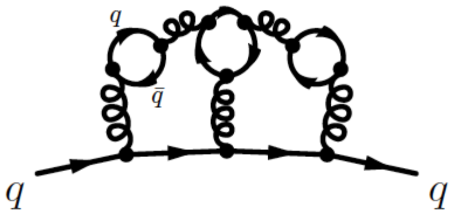


- Nuclear potential is Yukawa potential with $V(r) = -\dfrac{g^2}{4\pi}\dfrac{e^{-m_\pi r}}{r}$
- Short range force:

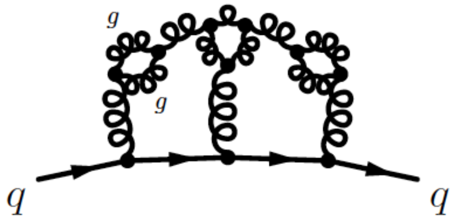
$$\text{Range} = \frac{1}{m_\pi} = (0.140\ \text{GeV})^{-1} = 7\ \text{GeV}^{-1} = 7 \times (\hbar c)\,\text{fm} = 1.4\,\text{fm}$$

Running of α_s

- α_s specifies the strength of the strong interaction.
- But, just as in QED, α_s is not a constant. It "runs" (i.e. depends on energy).
- In QED, the bare electron charge is screened by a cloud of virtual electron-positron pairs.
- In QCD, a similar "colour screening" effect occurs.



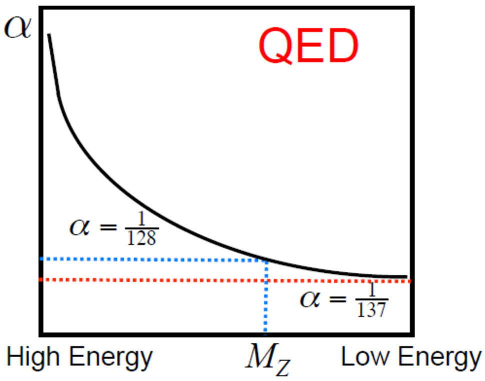
In QCD, quantum fluctuations lead to a cloud of virtual $q\bar{q}$ pairs.
One of many (an infinite set) of such diagrams analogous to those for QED.

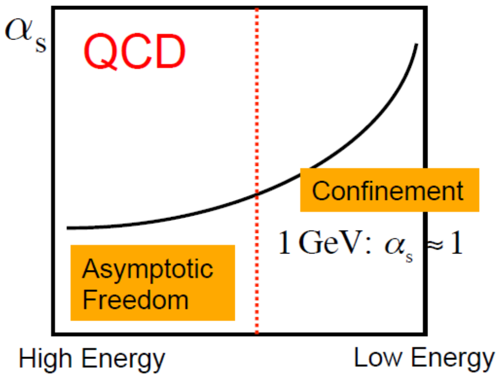


In QCD, the gluon self-interactions also lead to a cloud of virtual gluons.
One of many (an infinite set) of such diagrams.
No analogy in QED, photons do not carry the charge of the interaction.

Colour Anti-Screening

- Due to gluon self-interactions bare colour charge is screened by both virtual quarks and gluons.
- The cloud of virtual gluons carries colour charge and the effective colour charge decreases at smaller distances (high energy)!
- Hence, at low energies, α_s is large $\rightarrow$ cannot use perturbation theory.
- But at high energies, α_s is small. In this regime, can treat quarks as free particles and use perturbation theory $\rightarrow$ Asymptotic Freedom.

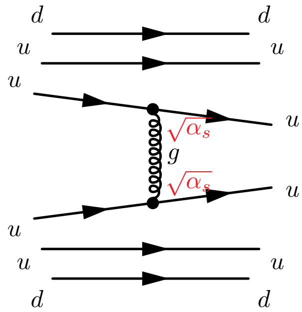


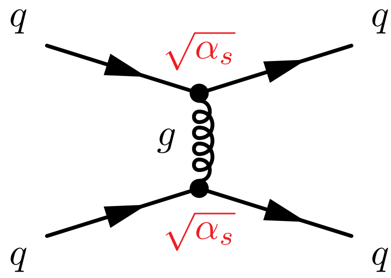


$\sqrt{s} = 100$ GeV, $\alpha_s = 0.12$

Scattering in QCD

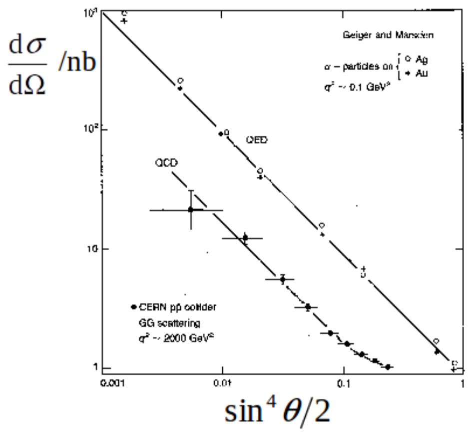
Example: High energy proton-proton scattering.





$$M \sim \frac{1}{q^2}\sqrt{\alpha_s}\sqrt{\alpha_s}$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_s)^2}{\sin^4\theta/2}$$

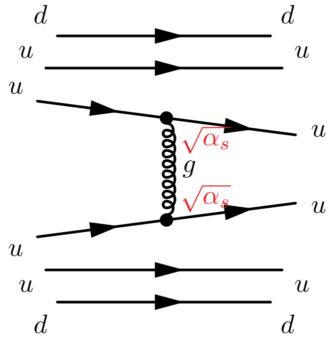


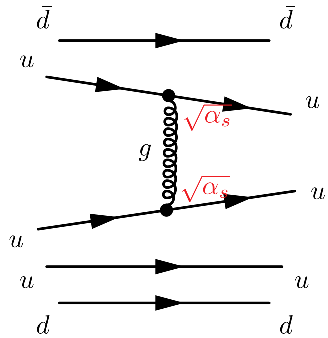
Upper points: Geiger and Marsden data (1911) for the elastic scattering of a particles from gold and silver foils.
Lower points: angular distribution of quark jets observed in *pp* scattering at $q^2 = 2000$ GeV^2.

Both follow the Rutherford formula for elastic scattering.

Scattering in QCD

Example: *pp* vs $\pi^+ p$ scattering





Calculate ratio of $\sigma(pp)_{\rm total}$ to $\sigma(\pi^+ p)_{\rm total}$

QCD does not distinguish between quark flavours, only colour charge of quarks matters.

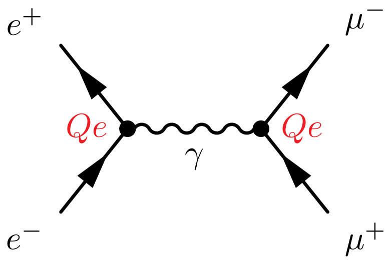
At high energy ($E \gg$ binding energy of quarks within hadrons), ratio of $\sigma(pp)_{\rm total}$ and $\sigma(\pi^+ p)_{\rm total}$ depends on number of possible quark-quark combinations.

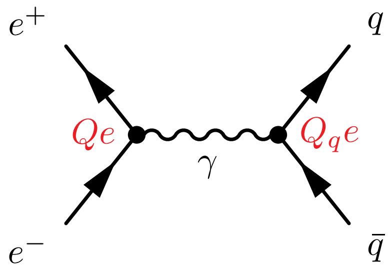
Predict: $\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2\times 3}{3\times 3} = \frac{2}{3}$ Experiment: $\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{24\,\mathrm{mb}}{38\,\mathrm{mb}} \sim \frac{2}{3}$

QCD in e^+e^- Annihilation

e^+e^- annihilation at high energies provides direct experimental evidence for colour and for gluons.

Start by comparing the cross-sections for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q}$





$$M \sim \frac{1}{q^2}\sqrt{\alpha}\sqrt{\alpha} \qquad\qquad M \sim \frac{1}{q^2}Q_q\sqrt{\alpha}\sqrt{\alpha}$$

$$\Rightarrow \sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

If we neglect the mass of the final state quarks/muons then the only difference is the charge of the final state particles:

$$Q_\mu = -1 \qquad\qquad Q_q = +\frac{2}{3}, \quad -\frac{1}{3}$$

Evidence for Colour

Consider the ratio $$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

For a single quark of a given colour $\quad R = Q_q^2$

However, we measure $\sigma(e^+e^- \to \text{ hadrons})$ not just $\sigma(e^+e^- \to u\bar{u})$.
A jet from a u-quark looks just like a jet from a d-quark etc.
Thus, we need to sum over all available flavours (u, d, c, s, t, b) and colours (r, g, b):

$$R = 3\sum_i Q_i^2 \qquad \text{(3 colours)}$$

where the sum is over all quark flavours (i) that are kinematically accessible at centre-of-mass energy, $\sqrt{s}$, of the collider.

Evidence for Colour

Expect to see steps in R as energy is increased.

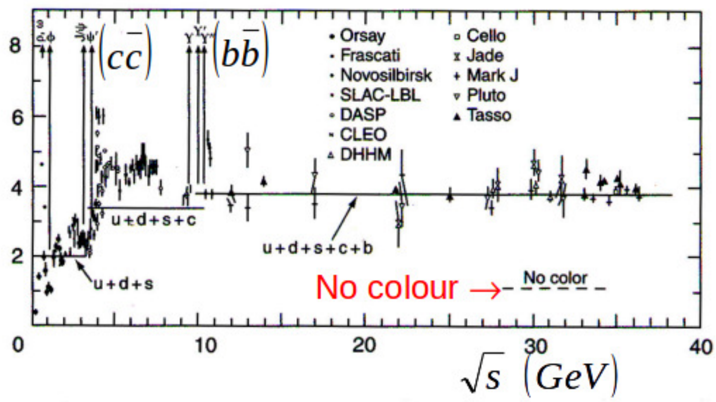
$$R = 3 \sum_i Q_i^2$$

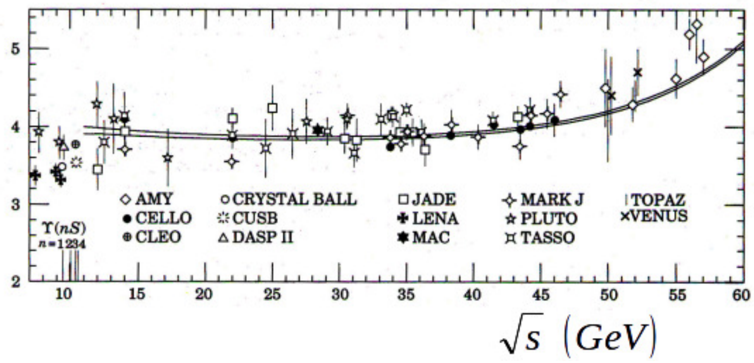
Energy	Expected ratio R
$\sqrt{s} > 2m_s, \quad \sim 1\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = 2$ uds
$\sqrt{s} > 2m_c, \quad \sim 4\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right) = 3\frac{1}{3}$ $udsc$
$\sqrt{s} > 2m_b, \quad \sim 10\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right) = 3\frac{2}{3}$ $udscb$
$\sqrt{s} > 2m_t, \quad \sim 350\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}+\frac{4}{9}\right) = 5$ $udscbt$

Evidence for Colour

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- R increases in steps with $\sqrt{s}$ **Strong evidence for colour**
- $\sqrt{s} < 11\ \mathrm{GeV}$ region observe bound state resonances: charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$)
- $\sqrt{s} > 50\ \mathrm{GeV}$ region observe low edge of Z resonance $\Gamma \sim 2.5\ \mathrm{GeV}$.





Experimental Evidence for Colour

- $$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- **The existence of Ω^- (sss)**

  The Ω^- (sss) is a ($L = 0$) spin-3/2 baryon consisting of three s-quarks.
  The wavefunction: $\psi = s \uparrow s \uparrow s \uparrow$
  is symmetric under particle interchange. However, quarks are fermions, therefore require an anti-symmetric wave-function, i.e. need another degree of freedom, namely colour, whose wavefunction must be antisymmetric.

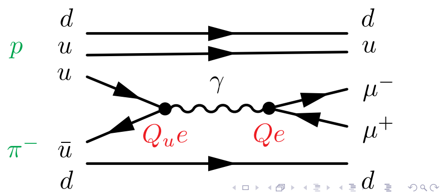
$$\psi = (s \uparrow s \uparrow s \uparrow)\psi_{\text{colour}}$$

$$\psi_{\text{colour}} = \frac{1}{\sqrt{6}}(rgb + gbr + brg - grb - rbg - bgr)$$

  i.e. need to introduce a new quantum number (colour) to distinguish the three quarks in Ω^- – avoids violation of Pauli's Exclusion Principle.

- **Drell-Yan process**

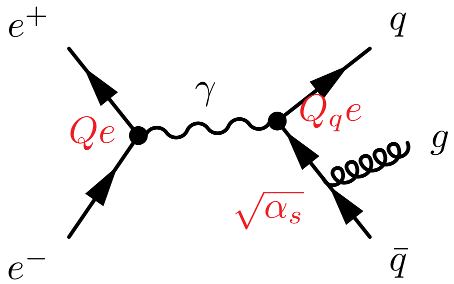
Need colour to explain cross-section; colours of the annihilating quarks must match to form a virtual photon. Cross-section suppressed by a factor N_{colour}^{-2}.



Evidence for Gluons

In QED, electrons can radiate photons. In QCD, quarks can radiate gluons.

Example: $e^-e^+ \to q\bar{q}g$



$$M \sim \frac{Q_q}{q^2}\sqrt{\alpha}\sqrt{\alpha}\sqrt{\alpha_s}$$

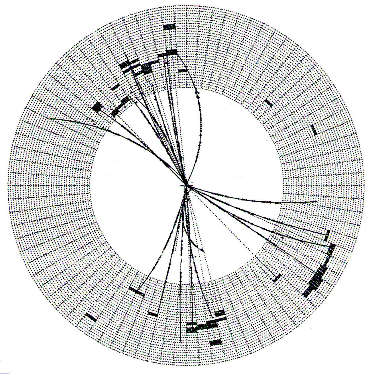
Giving an extra factor of $\sqrt{\alpha_s}$ in the matrix element, i.e. an extra factor of α_s in the cross-section.

In QED we can detect the photons. In QCD, we never see free gluons due to confinement.

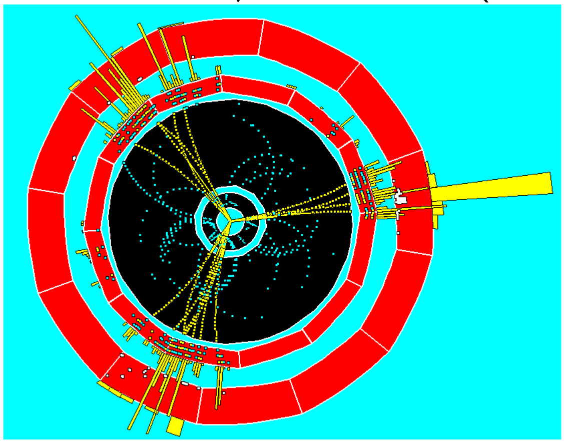
Experimentally, detect gluons as an additional jet: 3-jet events.
– Angular distribution of gluon jet depends on gluon spin.

Evidence for Gluons

JADE event $\sqrt{s} = 31$ GeV
First direct evidence of gluons (1978)

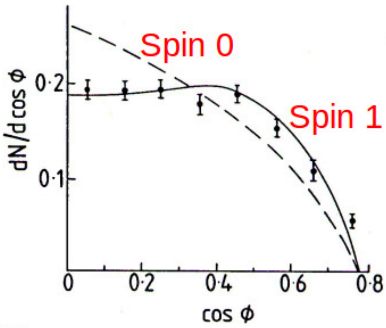


ALEPH event $\sqrt{s} = 91$ GeV (1990)



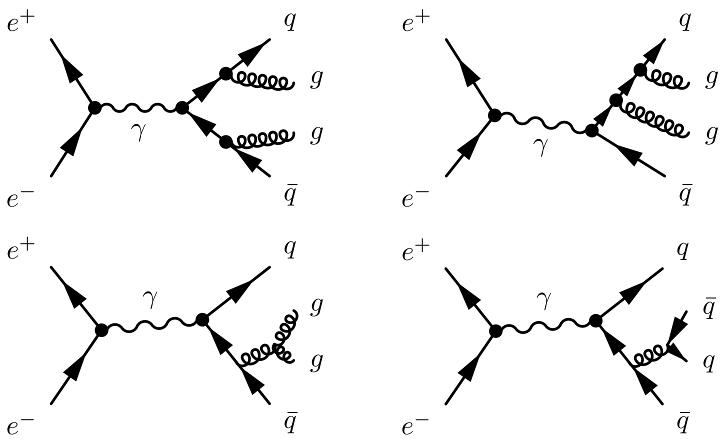
Distribution of the angle, ϕ , between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cm frame). ϕ distribution depends on the spin of the gluon.

$\Rightarrow$ Gluon is spin 1



Evidence for Gluon Self-Interactions

Direct evidence for the existence of the gluon self-interactions comes from 4-jet events:



The angular distribution of jets is sensitive to existence of triple gluon vertex (lower left diagram)

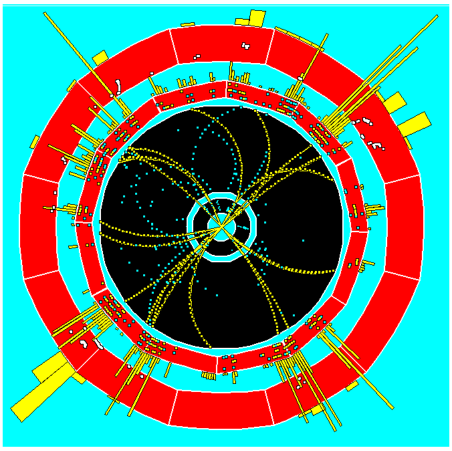
qqg vertex consists of two spin 1/2 quarks and one spin 1 gluon

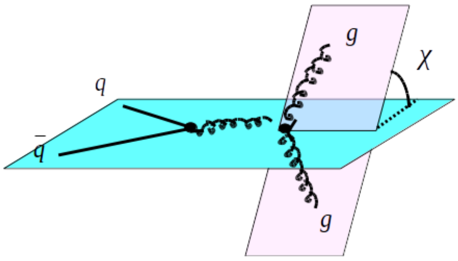
ggg vertex consists of three spin-1 gluons

$\Rightarrow$ Different angular distribution.

Evidence for Gluon Self-Interactions

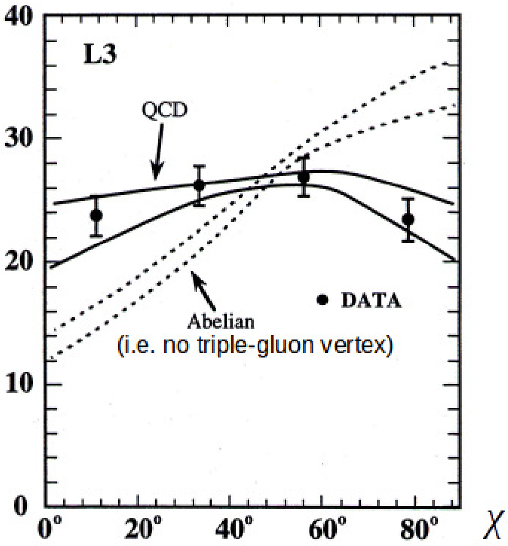
ALEPH 4-jet event





Experimental method:

- Define the two lowest energy jets as the gluons. (Gluon jets are more likely to be lower energy than quark jets).
- Measure angle χ between the plane containing the "quark" jets and the plane containing the "gluon" jets.



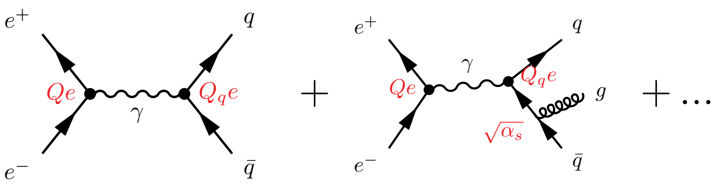
Gluon self-interactions are required to describe the experimental data.

Measurements of α_s

α_s can be measured in many ways.
The cleanest is from the ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

In practise, measure



i.e. don't distinguish between 2 and 3 jets

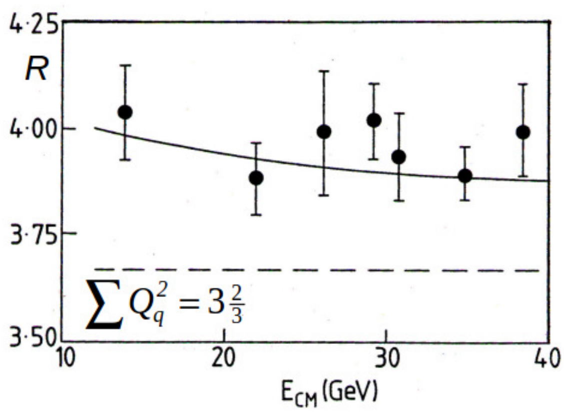
When gluon radiation is included:

$$R = 3\sum Q_q^2 \left(1 + \frac{\alpha_s}{\pi}\right)$$

Therefore,

$$\left(1 + \frac{\alpha_s}{\pi}\right) \sim \frac{3.9}{3.66}$$

$$\alpha_s(q^2 = 25^2) \sim 0.2$$

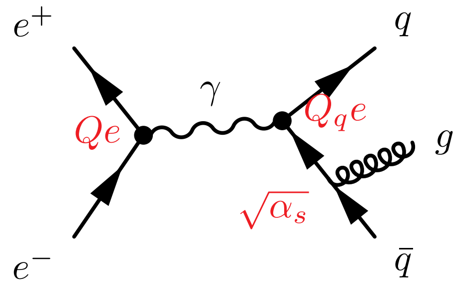


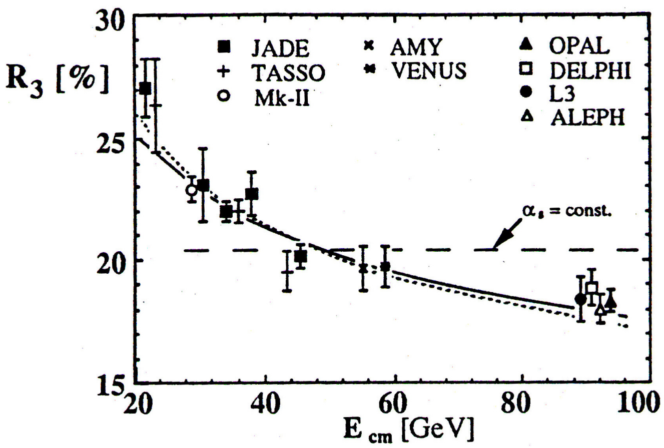
Measurements of α_s

Many other ways to measure α_s

Example: 3-jet rate $e^+e^- \to q\bar{q}g$

$$R_3 = \frac{\sigma(e^+e^- \to 3 \text{ jets})}{\sigma(e^+e^- \to 2 \text{ jets})} \propto \alpha_s$$



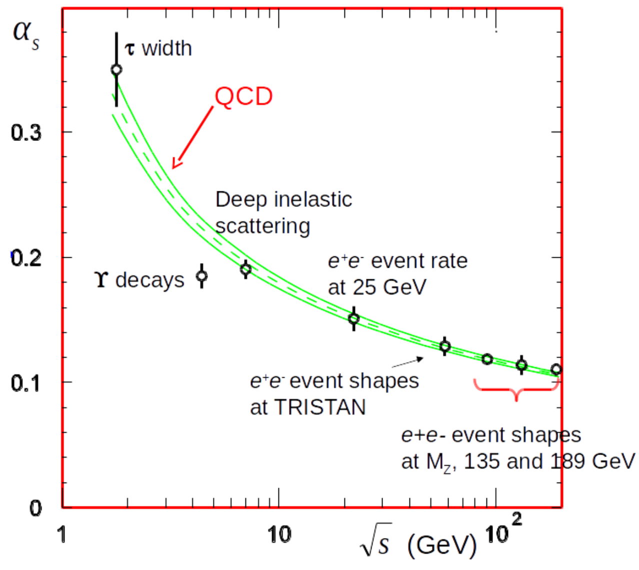


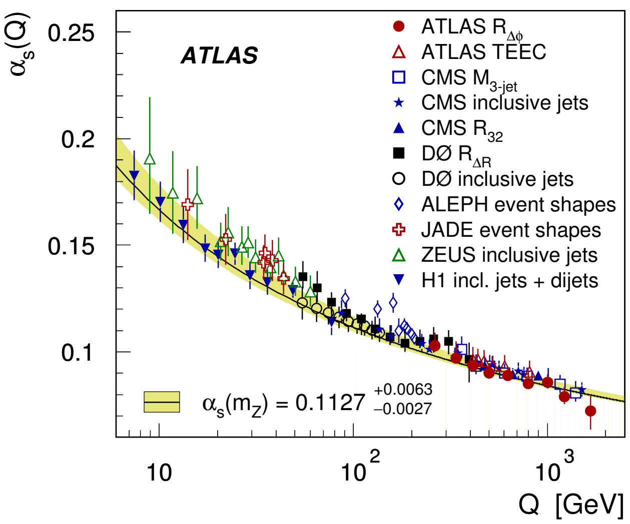
α_s decreases with energy

α_s runs!

in accordance with QCD

Observed running of α_s





Summary

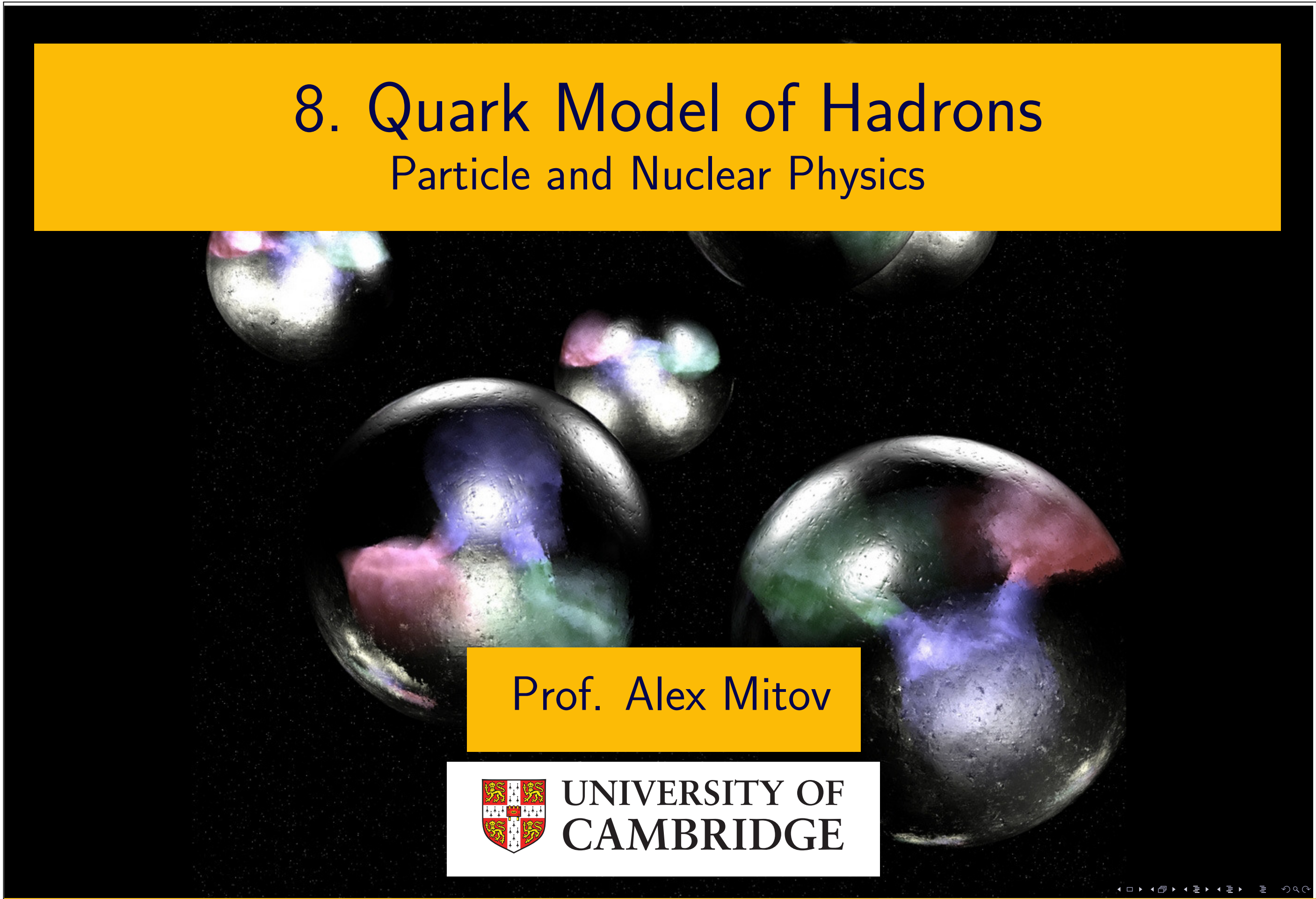
- QCD is a gauge theory, similar to QED, based on SU(3) symmetry
- Gluons are vector gauge bosons, which couple to (three types of) colour charge (r, b, g)
- Gluons themselves carry colour charge – hence they have self-interactions (unlike QED).
- Leads to running of α_s, in the opposite sense to QED. Force is weaker at high energies ("asymptotic freedom") and very strong at low energies.
- Quarks and gluons are confined. Seen as hadrons and jets of hadrons.
- Tests of QCD
  - Evidence for colour
  - Existence of gluons, test of their spin and self-interactions
  - Measurement of α_s and observation that it runs.

Problem Sheet: q.15-16

Up next... Section 8: Quark Model of Hadrons

8. Quark Model of Hadrons

Particle and Nuclear Physics



Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- Hadron wavefunctions and parity
- Light mesons
- Light baryons
- Charmonium
- Bottomonium

The Quark Model of Hadrons

Evidence for quarks

- The magnetic moments of proton and neutron are not $\mu_N = e\hbar/2m_p$ and 0 respectively $\Rightarrow$ not point-like
- Electron-proton scattering at high q^2 deviates from Rutherford scattering $\Rightarrow$ proton has substructure
- Hadron jets are observed in e^+e^- and pp collisions
- Symmetries (patterns) in masses and properties of hadron states, "quarky" periodic table $\Rightarrow$ sub-structure
- Steps in $R = \sigma(e^+e^- \to \mathrm{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$
- Observation of $c\bar{c}$ and $b\bar{b}$ bound states
- and much, much more...

Here, we will first consider the wave-functions for hadrons formed from light quarks (u, d, s) and deduce some of their static properties (mass and magnetic moments).

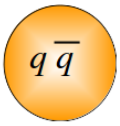
Then we will go on to discuss the heavy quarks (c, b).

We will cover the t quark later...

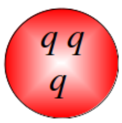
Hadron Wavefunctions

Quarks are always confined in hadrons (i.e. colourless states)

Mesons
Spin 0, 1, ...



Baryons
Spin 1/2, 3/2, ...



Treat quarks as identical fermions with states labelled with spatial, spin, flavour and colour.

$$\psi = \psi_{\mathrm{space}}\psi_{\mathrm{flavour}}\psi_{\mathrm{spin}}\psi_{\mathrm{colour}}$$

All hadrons are colour singlets, i.e. net colour zero

Mesons $$\psi^{q\bar{q}}_{\mathrm{colour}} = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

Baryons $$\psi^{qqq}_{\mathrm{colour}} = \frac{1}{\sqrt{6}}(rgb + gbr + brg - grb - rbg - bgr)$$

Parity

- The Parity operator, $\hat{P}$, performs spatial inversion
$$\hat{P}|\psi(\vec{r},t)\rangle = |\psi(-\vec{r},t)\rangle$$
- The eigenvalue of $\hat{P}$ is called Parity
$$\hat{P}|\psi\rangle = P|\psi\rangle, \qquad P = \pm 1$$
- Most particles are eigenstates of Parity and in this case P represents intrinsic Parity of a particle/antiparticle.
- Parity is a useful concept. If the Hamiltonian for an interaction commutes with $\hat{P}$
$$\left[\hat{P},\hat{H}\right] = 0$$
then Parity is conserved in the interaction:

Parity conserved in the **strong** and **EM** interactions, but **not** in the **weak** interaction.

Parity

- Composite system of two particles with orbital angular momentum L:
$$P = P_1 P_2 (-1)^L$$
where $P_{1,2}$ are the intrinsic parities of particles 1, 2.

Quantum Field Theory tells us that

Fermions and antifermions: opposite parity
Bosons and antibosons: same parity

Choose:

Quarks and leptons: $P_{q/\ell} = +1$
Antiquarks and antileptons: $P_{\bar{q},\bar{\ell}} = -1$

Gauge Bosons: (γ, g, W, Z) are vector fields which transform as
$$J^P = 1^-$$
$$P_\gamma = -1$$

Light Mesons

Mesons are bound $q\bar{q}$ states.
Consider ground state mesons consisting of light quarks (u, d, s).

$$m_u \sim 0.3\ \mathrm{GeV},\ \ m_d \sim 0.3\ \mathrm{GeV},\ \ m_s \sim 0.5\ \mathrm{GeV}$$

- **Ground State ($L = 0$):** Meson "spin" (total angular momentum) is given by the $q\bar{q}$ spin state.
  - Two possible $q\bar{q}$ total spin states: $S = 0, 1$
    - $S = 0$: pseudoscalar mesons
    - $S = 1$: vector mesons
- **Meson Parity:** (q and $\bar{q}$ have opposite parity)

$$P = P_q P_{\bar{q}} (-1)^L = (+1)(-1)(-1)^L = -1 \quad (\text{for } L = 0)$$

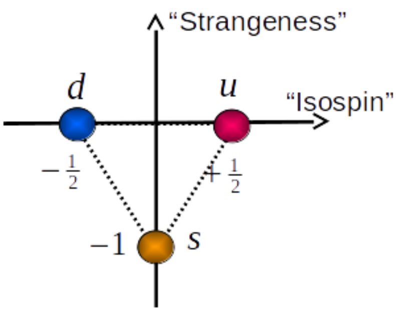
- **Flavour States:** $u\bar{d}, u\bar{s}, d\bar{u}, d\bar{s}, s\bar{u}, s\bar{d}$ and $u\bar{u}, d\bar{d}\ s\bar{s}$ mixtures
  - Expect: Nine $J^P = 0^-$ mesons: Pseudoscalar nonet
  - Nine $J^P = 1^-$ mesons: Vector nonet

uds Multiplets

Basic quark multiplet – plot the quantum numbers of (anti)quarks:

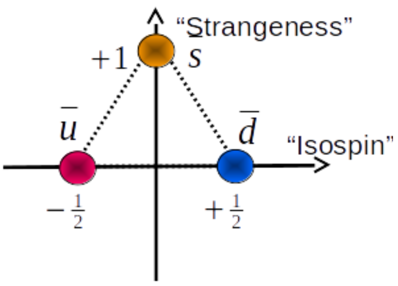
Quarks

$J^P = \frac{1}{2}^+$



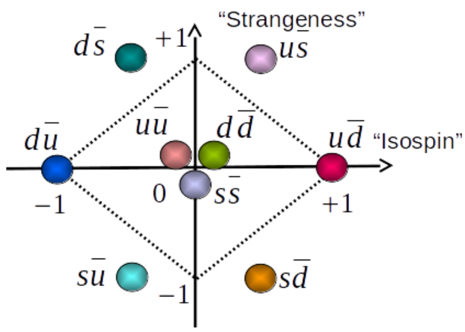
Antiquarks

$J^P = \frac{1}{2}^-$



Mesons

Spin $J = 0$ or 1



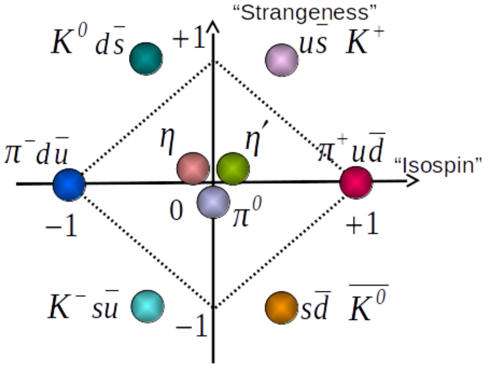
The ideas of strangeness and isospin are historical quantum numbers assigned to different states.
Essentially they count quark flavours (this was all before the formulation of the Quark Model).

$$\text{Isospin} = \tfrac{1}{2}(n_u - n_d - n_{\bar{u}} + n_{\bar{d}})$$
$$\text{Strangeness} = n_{\bar{s}} - n_s$$

Light Mesons

Pseudoscalar nonet

$J^P = 0^-$



π^0, η, η' are combinations of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$

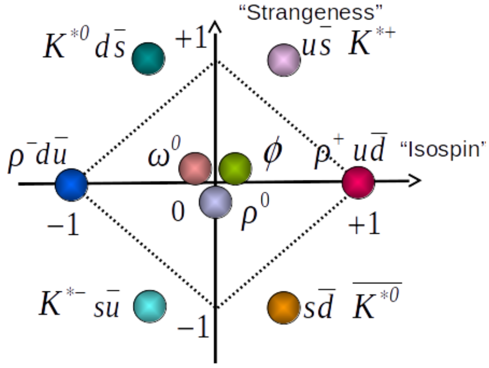
Masses / MeV

$\pi(140)$, $K(495)$

$\eta(550)$, $\eta'(960)$

Vector nonet

$J^P = 1^-$



ρ^0, ϕ, ω^0 are combinations of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$

Masses/ MeV

$\rho(770)$, $K^*(890)$

$\omega(780)$, $\phi(1020)$

$u\bar{u}$, $d\bar{d}$, $s\bar{s}$ States

The states $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ all have zero flavour quantum numbers and can mix

$J^P = 0^-$

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$\eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$J^P = 1^-$

$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\omega^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$\phi = s\bar{s}$$

Mixing coefficients determined experimentally from meson masses and decays.

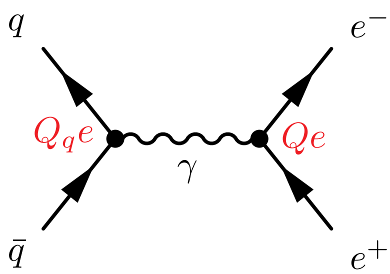
Example: Leptonic decays of vector mesons

$$M(\rho^0 \to e^+e^-) \sim \frac{e}{q^2}\left[\frac{1}{\sqrt{2}}(Q_u e - Q_d e)\right]$$

$$\Gamma(\rho^0 \to e^+e^-) \propto \left[\frac{1}{\sqrt{2}}(\frac{2}{3} - (-\frac{1}{3}))\right]^2 = \frac{1}{2}$$

$$\Gamma(\omega^0 \to e^+e^-) \propto \left[\frac{1}{\sqrt{2}}(\frac{2}{3} + (-\frac{1}{3}))\right]^2 = \frac{1}{18}$$

$$\Gamma(\phi \to e^+e^-) \propto \left[\frac{1}{3}\right]^2 = \frac{1}{9}$$



$M \sim Q_q\alpha \quad \Gamma \sim Q_q^2\alpha^2$

Predict: $\Gamma_{\rho^0} : \Gamma_{\omega^0} : \Gamma_\phi = 9 : 1 : 2$ **Experiment:** $(8.8 \pm 2.6) : 1 : (1.7 \pm 0.4)$

Meson Masses

Meson masses are only partly from constituent quark masses:

$m(K) > m(\pi)$ $\Rightarrow$ suggests $m_s > m_u, m_d$

495 MeV 140 MeV

Not the whole story...

$m(\rho) > m(\pi)$ $\Rightarrow$ although both are $u\bar{d}$

770 MeV 140 MeV

Only difference is the orientation of the quark spins ($\uparrow\uparrow$ vs $\uparrow\downarrow$)

$\Rightarrow$ **spin-spin interaction**

Meson Masses *Spin-spin Interaction*

QED: Hyperfine splitting in H_2 ($L = 0$)

Energy shift due to electron spin in magnetic field of proton

$$\Delta E = \vec{\mu}.\vec{B} = \frac{2}{3}\vec{\mu}_e.\vec{\mu}_p|\psi(0)|^2$$

and using $\vec{\mu} = \frac{e}{2m}\vec{S}$

$$\Delta E \propto \alpha \frac{\vec{S}_e}{m_e}\frac{\vec{S}_p}{m_p}$$

QCD: Colour Magnetic Interaction

Fundamental form of the interaction between a quark and a gluon is identical to that between an electron and a photon. Consequently, also have a colour magnetic interaction

$$\Delta E \propto \alpha_s \frac{\vec{S}_1}{m_1}\frac{\vec{S}_2}{m_2}$$

Meson Masses *Meson Mass Formula* $(L = 0)$

$$M_{q\bar{q}} = m_1 + m_2 + A\frac{\vec{S_1}\,\vec{S_2}}{m_1\, m_2} \quad \text{where } A \text{ is a constant}$$

For a state of spin $\quad \vec{S} = \vec{S_1} + \vec{S_2} \quad \vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2\vec{S_1}.\vec{S_2}$

$$\vec{S_1}.\vec{S_2} = \frac{1}{2}\left(\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2\right) \qquad \vec{S}_1^2 = \vec{S}_2^2 = \vec{S_1}(\vec{S_1}+1) = \frac{1}{2}\left(\frac{1}{2}+1\right) = \frac{3}{4}$$

giving $\quad \vec{S_1}.\vec{S_2} = \frac{1}{2}\vec{S}^2 - \frac{3}{4}$

For $J^P = 0^-$ mesons: $\quad \vec{S}^2 = 0 \quad \Rightarrow \vec{S_1}.\vec{S_2} = -3/4$

For $J^P = 1^-$ mesons: $\quad \vec{S}^2 = S(S+1) = 2 \quad \Rightarrow \vec{S_1}.\vec{S_2} = +1/4$

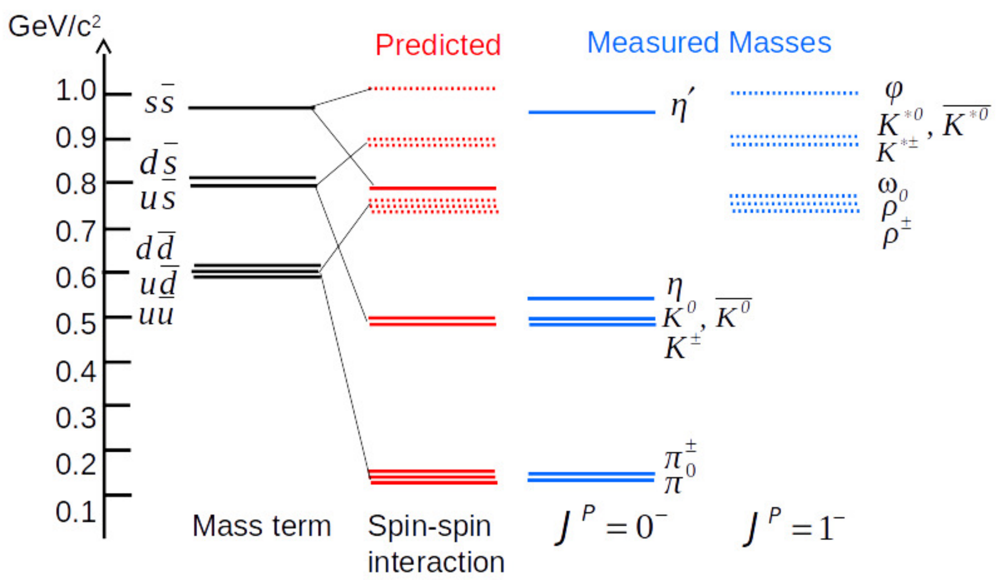
Giving the $(L = 0)$ Meson Mass formulae:

$$M_{q\bar{q}} = m_1 + m_2 - \frac{3A}{4m_1 m_2} \quad (J^P = 0^-)$$

$$M_{q\bar{q}} = m_1 + m_2 + \frac{A}{4m_1 m_2} \quad (J^P = 1^-)$$

So $J^P = 0^-$ mesons are lighter than $J^P = 1^-$ mesons

Meson Masses



Excellent fit obtained to masses of the different flavour pairs $(u\bar{d},\ u\bar{s},\ d\bar{u},\ d\bar{s},\ s\bar{u},\ s\bar{d})$ with

$$m_u = 0.305\ \text{GeV}, \quad m_d = 0.308\ \text{GeV}, \quad m_s = 0.487\ \text{GeV}, \quad A = 0.06\ \text{GeV}^3$$

η and η' are mixtures of states, e.g.

$$\eta = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right) \qquad M_\eta = \frac{1}{6}\left(2m_u - \frac{3A}{4m_u^2}\right) + \frac{1}{6}\left(2m_d - \frac{3A}{4m_d^2}\right) + \frac{4}{6}\left(2m_s - \frac{3A}{4m_s^2}\right)$$

Baryons

Baryons made from 3 indistinguishable quarks (flavour can be treated as another quantum number in the wave-function)

$$\psi_{\rm baryon} = \psi_{\rm space}\,\psi_{\rm flavour}\,\psi_{\rm spin}\,\psi_{\rm colour}$$

$\psi_{\rm baryon}$ must be anti-symmetric under interchange of any 2 quarks

Example: $\Omega^-(sss)$ wavefunction $(L = 0, J = 3/2)$

$\psi_{\rm spin}\,\psi_{\rm flavour} = s\uparrow s\uparrow s\uparrow$ is symmetric $\Rightarrow$ require antisymmetric $\psi_{\rm colour}$

Ground State $(L = 0)$

We will only consider the baryon ground states, which have zero orbital angular momentum

$\psi_{\rm space}$ symmetric

$\rightarrow$ All hadrons are colour singlets

$$\psi_{\rm colour} = \frac{1}{\sqrt{6}}(rgb + gbr + brg - grb - rbg - bgr)$$ antisymmetric

Therefore, $\psi_{\rm spin}\,\psi_{\rm flavour}$ must be symmetric

Baryon spin wavefunctions ($\psi_{\rm spin}$)

Combine 3 spin 1/2 quarks: Total spin $J = \frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2} = \frac{1}{2}$ or $\frac{3}{2}$

Consider $J = 3/2$

Trivial to write down the spin wave-function for the $\left|\frac{3}{2},\frac{3}{2}\right\rangle$ state: $\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$

Generate other states using the ladder operator $\hat{J}_-$

$$\hat{J}_-\left|\frac{3}{2},\frac{3}{2}\right\rangle = (\hat{J}_-\uparrow)\uparrow\uparrow + \uparrow(\hat{J}_-\uparrow)\uparrow + \uparrow\uparrow(\hat{J}_-\uparrow)$$

$$\sqrt{\frac{3}{2}\frac{5}{2} - \frac{3}{2}\frac{1}{2}}\left|\frac{3}{2},\frac{1}{2}\right\rangle = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow$$

$$\hat{J}_-\,|j,m\rangle = \sqrt{j(j+1) - m(m-1)}\,|j,m-1\rangle$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow)$$

Giving the $J = 3/2$ states: $\longrightarrow$

All symmetric under interchange of any two spins

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow)$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$

$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

Baryon spin wavefunctions (ψ_{spin})

Consider $J = 1/2$

First consider the case where the first 2 quarks are in a $|0,0\rangle$ state:

$$|0,0\rangle_{(12)} = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle_{(123)} = |0,0\rangle_{(12)}\left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \qquad \left|\frac{1}{2},-\frac{1}{2}\right\rangle_{(123)} = |0,0\rangle_{(12)}\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$$

Antisymmetric under exchange $1 \leftrightarrow 2$.

Three-quark $J = 1/2$ states can also be formed from the state with the first two quarks in a symmetric spin wavefunction.

Can construct a three-particle state $\left|\frac{1}{2},\frac{1}{2}\right\rangle_{(123)}$ from

$$|1,0\rangle_{(12)}\left|\frac{1}{2},\frac{1}{2}\right\rangle_{(3)} \qquad \text{and} \qquad |1,1\rangle_{(12)}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_{(3)}$$

Baryon spin wavefunctions (ψ_{spin})

Taking the linear combination

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = a\,|1,1\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle + b\,|1,0\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle$$

with $a^2 + b^2 = 1$. Act upon both sides with $\hat{J}_+$

$$\hat{J}_+\left|\frac{1}{2},\frac{1}{2}\right\rangle = a\left[\left(\hat{J}_+|1,1\rangle\right)\left|\frac{1}{2},-\frac{1}{2}\right\rangle + |1,1\rangle\left(\hat{J}_+\left|\frac{1}{2},-\frac{1}{2}\right\rangle\right)\right] + b\left[\left(\hat{J}_+|1,0\rangle\right)\left|\frac{1}{2},\frac{1}{2}\right\rangle + |1,0\rangle\left(\hat{J}_+\left|\frac{1}{2},\frac{1}{2}\right\rangle\right)\right]$$

$$0 = a\,|1,1\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle + \sqrt{2}b\,|1,1\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle$$

$$a = -\sqrt{2}b \qquad \hat{J}_+|j,m\rangle = \sqrt{j(j+1) - m(m+1)}\,|j,m+1\rangle$$

which with $a^2 + b^2 = 1$ implies $a = \sqrt{\frac{2}{3}},\ b = -\sqrt{\frac{1}{3}}$

Giving

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|1,1\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}|1,0\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$|1,1\rangle = \uparrow\uparrow$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \qquad \left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow)$$

Symmetric under interchange $1 \leftrightarrow 2$

Three-quark spin wavefunctions

$J = 3/2$

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$
$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow)$$
$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$
$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

Symmetric under interchange of any 2 quarks

$J = 1/2$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$$

Antisymmetric under interchange of $1 \leftrightarrow 2$

$J = 1/2$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow)$$

Symmetric under interchange of $1 \leftrightarrow 2$

$\psi_{\text{spin}}\,\psi_{\text{flavour}}$ must be symmetric under interchange of any 2 quarks.

Three-quark spin wavefunctions

Consider 3 cases:

1. **Quarks all same flavour: $uuu,\ ddd,\ sss$**
   - ψ_{flavour} is symmetric under interchange of any two quarks
   - Require ψ_{spin} to be symmetric under interchange of any two quarks
   - Only satisfied by $J = 3/2$ states
   - There are no $J = 1/2$ $uuu,\ ddd,\ sss$ baryons with $L = 0$.

   Three $J = 3/2$ states: $uuu,\ ddd,\ sss$

2. **Two quarks have same flavour: $uud,\ uus,\ ddu,\ dds,\ ssu,\ ssd$**
   - For the like quarks ψ_{flavour} is symmetric
   - Require ψ_{spin} to be symmetric under interchange of like quarks $1 \leftrightarrow 2$
   - Satisfied by $J = 3/2$ and $J = 1/2$ states

   Six $J = 3/2$ states and six $J = 1/2$ states: $uud,\ uus,\ ddu,\ dds,\ ssu,\ ssd$

Three-quark spin wavefunctions

3. **All quarks have different flavours:** uds

Two possibilities for the (ud) part:

- Flavour Symmetric $\frac{1}{\sqrt{2}}(ud + du)$
  - Require ψ_{spin} to be symmetric under interchange of ud
  - Satisfied by $J = 3/2$ and $J = 1/2$ states

One $J = 3/2$ and one $J = 1/2$ state: uds

- Flavour Antisymmetric $\frac{1}{\sqrt{2}}(ud - du)$
  - Require ψ_{spin} to be antisymmetric under interchange of ud
  - Only satisfied by $J = 1/2$ state

One $J = 1/2$ state: uds

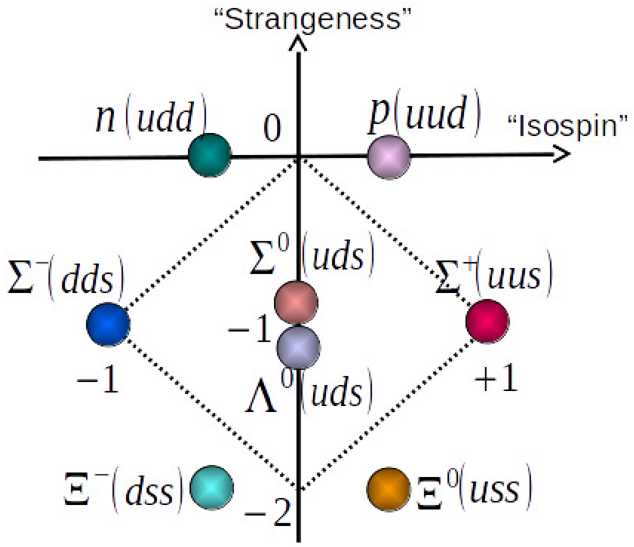
Quark Model predicts that light baryons appear in

Decuplets (10) of spin 3/2 states

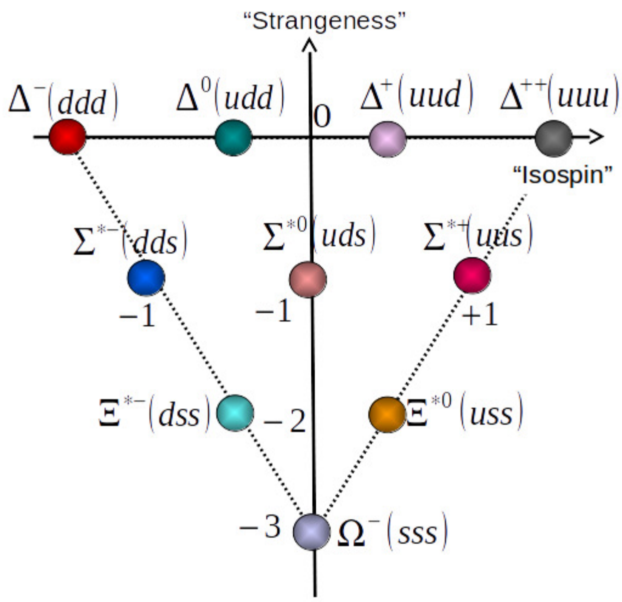
Octets (8) of spin 1/2 states

Baryon Multiplets

Octet $J^P = \frac{1}{2}^+$



Decuplet $J^P = \frac{3}{2}^+$



Antibaryons are in separate multiplets

Example:

Antiparticle of $\Sigma^+(uus)$ is $\bar{\Sigma}^-(\bar{u}\bar{u}\bar{s})$, $J^P = \frac{1}{2}^-$ and not $\Sigma^-(dds)$, $J^P = \frac{1}{2}^+$

Baryon Masses *Baryon Mass Formula ($L = 0$)*

$$M_{qqq} = m_1 + m_2 + m_3 + A'\left(\frac{\vec{S}_1}{m_1}.\frac{\vec{S}_2}{m_2} + \frac{\vec{S}_1}{m_1}.\frac{\vec{S}_3}{m_3} + \frac{\vec{S}_2}{m_2}.\frac{\vec{S}_3}{m_3}\right)$$

where A' is a constant

Example: All quarks have the same mass, $m_1 = m_2 = m_3 = m_q$

$$M_{qqq} = 3m_q + A'\sum_{i<j}\frac{\vec{S}_i.\vec{S}_j}{m_q^2}$$

$$\vec{S}^2 = \left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3\right)^2 = \vec{S}_1^2 + \vec{S}_2^2 + \vec{S}_3^2 + 2\sum_{i<j}\vec{S}_i.\vec{S}_j$$

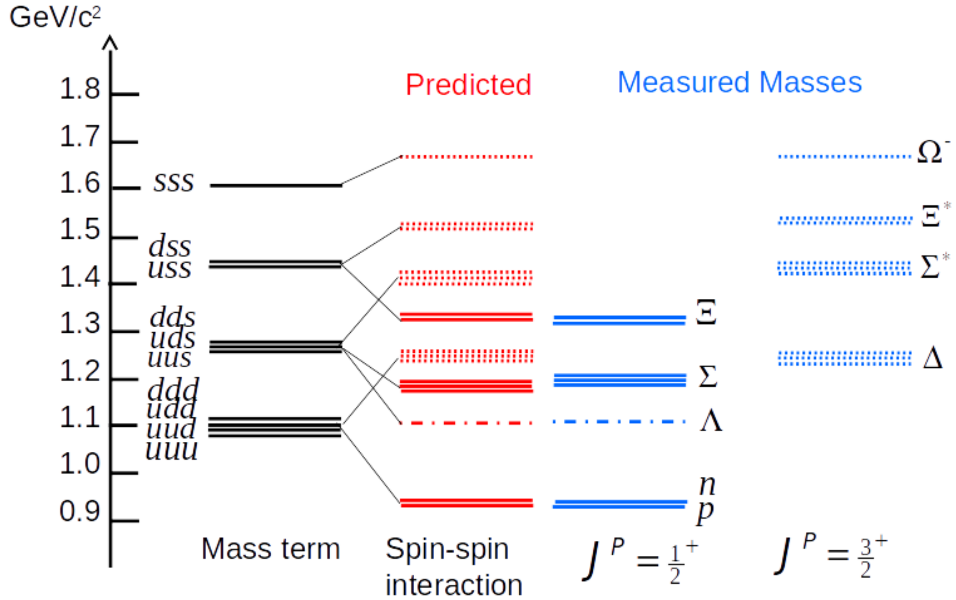
$$2\sum_{i<j}\vec{S}_i.\vec{S}_j = S(S+1) - 3\frac{1}{2}(\frac{1}{2}+1) = S(S+1) - \frac{9}{4}$$

$$\sum_{i<j}\vec{S}_i.\vec{S}_j = -\frac{3}{4}\ \left(J = \frac{1}{2}\right) \qquad \sum_{i<j}\vec{S}_i.\vec{S}_j = +\frac{3}{4}\ \left(J = \frac{3}{2}\right)$$

e.g. proton (uud) compared with Δ (uud) – same quark content

$$M_p = 3m_u - \frac{3A'}{4m_u^2}, \quad M_\Delta = 3m_u + \frac{3A'}{4m_u^2}$$

Baryon Masses



Excellent agreement using

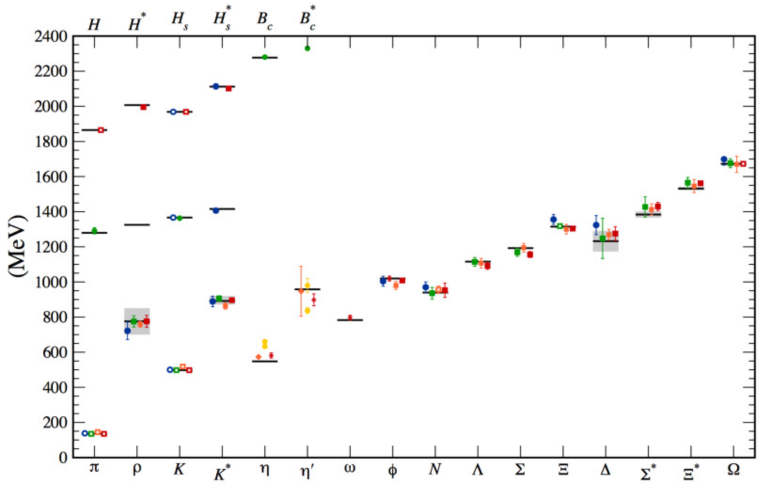
Colour factor of 2

$m_u = 0.362\ \mathrm{GeV},\ m_d = 0.366\ \mathrm{GeV},\ m_s = 0.537\ \mathrm{GeV},\ A' = 0.026\ \mathrm{GeV}^3 \sim A/2$

Constituent quark mass depends on hadron wave-function and includes cloud of gluons and qq pairs ⇒ slightly different values for mesons and baryons.

Hadron masses in QCD

- Calculation of hadron masses in QCD is a hard problem – can't use perturbation theory.
- Need to solve field equations exactly – only feasible on a discrete lattice of space-time points.
- Needs specialised supercomputing (Pflops) + clever techniques.
- Current state of the art (after 40 years of work)...



Baryon Magnetic Moments

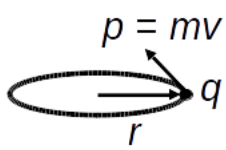
Magnetic dipole moments arise from

- the orbital motion of charged quarks
- the intrinsic spin-related magnetic moments of the quarks.

Orbital Motion

Classically, current loop

$$\mu = IA = \frac{qv}{2\pi r}\pi r^2 = \frac{qpr}{2m} = \frac{q}{2m}L_z$$



Quantum mechanically, get the same result

$$\hat{\mu} = g_L \frac{q}{2m}\hat{L}_z$$

g_L is the "g-factor"
$g_L = 1$ charged particles
$g_L = 0$ neutral particles

Intrinsic Spin

The magnetic moment operator due to the intrinsic spin of a particle is

$$\hat{\mu} = g_s \frac{q}{2m}\hat{S}_z$$

g_s is the "spin g-factor"
$g_s = 2$ for Dirac spin $1/2$ point-like particles.

Baryon Magnetic Moments

The magnetic dipole moment is the maximum measurable component of the magnetic dipole moment operator

$$\mu_L = \left\langle \psi_{\rm space} \left| g_L \frac{q}{2m} \hat{L}_z \right| \psi_{\rm space} \right\rangle \qquad \mu_s = \left\langle \psi_{\rm spin} \left| g_s \frac{q}{2m} \hat{S}_z \right| \psi_{\rm spin} \right\rangle$$

For an electron

$$\mu_L = -g_L \frac{e}{2m_e} \hbar L = -\mu_B L \qquad \mu_s = -g_s \frac{e}{2m_e} \frac{\hbar}{2} = -\mu_B$$

where $\mu_B = e\hbar/2m_e$ is the Bohr Magneton

Observed difference from $g_s = 2$ is due to higher order corrections in QED

$$\mu_s = -\mu_B \left[1 + \frac{\alpha}{2\pi} + O(\alpha^2) + \ldots \right] \qquad \alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

Baryon Magnetic Moments *Proton and Neutron*

If the proton and neutron were point-like particles,

$$\mu_L = g_L \frac{e}{2m_p} \hbar L \qquad \mu_s = g_s \frac{e}{2m_p} \frac{\hbar}{2} = \frac{1}{2} g_s \mu_N$$

where $\mu_N = e\hbar/2m_p$ is the Nuclear Magneton

Expect: p spin 1/2, charge $+e$ $\mu_s = \mu_N$
n spin 1/2, charge 0 $\mu_s = 0$

Observe: p $\mu_s = +2.793\mu_N \rightarrow g_s = +5.586$
n $\mu_s = -1.913\mu_N \rightarrow g_s = -3.826$

Observation shows that p and n are not point-like $\Rightarrow$ evidence for quarks.
$\Rightarrow$ use quark model to estimate baryon magnetic moments.

Baryon Magnetic Moments *in the Quark Model*

Assume that bound quarks within baryons behave as Dirac point-like spin 1/2 particles with fractional charge q_q.
Then quarks will have magnetic dipole moment operator and magnitude:

$$\vec{\mu}_q = \frac{q_q}{m_q}\hat{S}_z \qquad \mu_q = \left\langle \psi^q_{\rm spin} \left| \frac{q_q}{m_q}\hat{S}_z \right| \psi^q_{\rm spin} \right\rangle = \frac{q_q \hbar}{2m_q}$$

where m_q is the quark mass.

Therefore $\quad \mu_u = \frac{2}{3}\frac{e\hbar}{2m_u}, \quad \mu_d = -\frac{1}{3}\frac{e\hbar}{2m_d}, \quad \mu_s = -\frac{1}{3}\frac{e\hbar}{2m_s}$

For quarks bound within an $L = 0$ baryon, the baryon magnetic moment is the expectation value of the sum of the individual quark magnetic moment operators:

$$\hat{\mu}_{\rm baryon} = \frac{q_1}{m_1}\hat{S}_{1z} + \frac{q_2}{m_2}\hat{S}_{2z} + \frac{q_3}{m_3}\hat{S}_{3z}; \qquad \mu_{\rm baryon} = \langle \psi^B_{\rm spin} | \hat{\mu}_B | \psi^B_{\rm spin} \rangle$$

where $\psi^B_{\rm spin}$ is the baryon spin wavefunction.

Baryon Magnetic Moments *in the Quark Model*

Example: Magnetic moment of a proton

Baryon Magnetic Moments *in the Quark Model*

Repeat for the other $L = 0$ baryons. Predict $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$

compared to the experimentally measured value of -0.685

Baryon	μ_B in Quark Model	Predicted $[\mu_N]$	Observed $[\mu_N]$
p (uud)	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	$+2.79$	$+2.793$
n (ddu)	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.86	-1.913
Λ (uds)	μ_s	-0.61	-0.614 ± 0.005
Σ^+ (uus)	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	$+2.68$	$+2.46 \pm 0.01$
Ξ^0 (ssu)	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	-1.44	-1.25 ± 0.014
Ξ^- (ssd)	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	-0.51	-0.65 ± 0.01
Ω^- (sss)	$3\mu_s$	-1.84	-2.02 ± 0.05

Reasonable agreement with data using

$$m_u = m_d = 0.336\ \mathrm{GeV},\ \ m_s \sim 0.509\ \mathrm{GeV}$$

Hadron Decays

- Hadrons are eigenstates of the strong force.
- Hadrons will decay via the strong interaction to lighter mass states if energetically feasible (i.e. mass of parent > mass of daughters).
- And, angular momentum and parity must be conserved in strong decays.

Examples:

$$\rho^0 \rightarrow \pi^+\pi^-$$

$$\underset{769}{m(\rho^0)} > \underset{140}{m(\pi^+)} + \underset{140\ \mathrm{MeV}}{m(\pi^-)}$$

$$\Delta^{++} \rightarrow p\pi^+$$

$$\underset{1231}{m(\Delta^{++})} > \underset{938}{m(p)} + \underset{140\ \mathrm{MeV}}{m(\pi^+)}$$

Hadron Decays

Also need to check for identical particles in the final state.

Examples:

$$\omega^0 \to \pi^0\pi^0$$
$$\underset{782}{m(\omega^0)} > \underset{135}{m(\pi^0)} + \underset{135\ \mathrm{MeV}}{m(\pi^0)}$$

$$\omega^0 \to \pi^+\pi^-\pi^0$$
$$\underset{782}{m(\omega^0)} > \underset{140}{m(\pi^+)} + \underset{140}{m(\pi^-)} + \underset{135\ \mathrm{MeV}}{m(\pi^0)}$$

Hadron Decays

Hadrons can also decay via the electromagnetic interaction.

Examples:

$$\rho^0 \to \pi^0\gamma$$
$$\underset{769}{m(\rho^0)} > \underset{135\ \mathrm{MeV}}{m(\pi^0)} + m(\gamma)$$

$$\Sigma^0 \to \Lambda^0\gamma$$
$$\underset{1193}{m(\Sigma^0)} > \underset{1116\ \mathrm{MeV}}{m(\Lambda^0)} + m(\gamma)$$

The lightest mass states (p, $K^\pm$, K^0, $\bar{K}^0$, Λ, n) require a change of quark flavour in the decay and therefore decay via the weak interaction (see later).

Summary of light (*uds*) hadrons

- Baryons and mesons are composite particles (complicated).
- However, the naive Quark Model can be used to make predictions for masses/magnetic moments.
- The predictions give reasonably consistent values for the constituent quark masses:

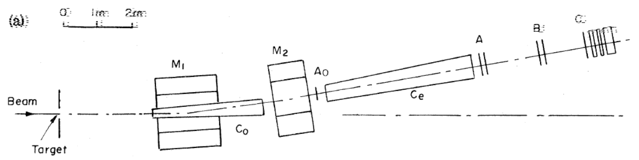
	$m_{u/d}$	m_s
Meson Masses	307 MeV	487 MeV
Baryon Masses	364 MeV	537 MeV
Baryon Magnetic Moments	336 MeV	509 MeV

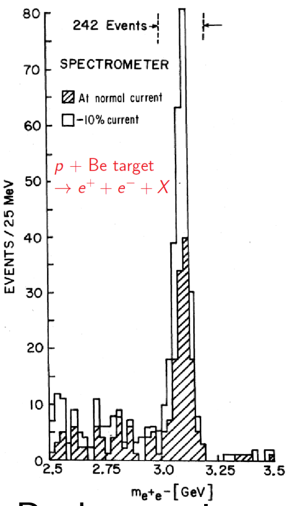
$$m_u \sim m_d \sim 335\ \mathrm{MeV}, \quad m_s \sim 510\ \mathrm{MeV}$$

- Hadrons will decay via the strong interaction to lighter mass states if energetically feasible.
- Hadrons can also decay via the EM interaction.
- The lightest mass states require a change of quark flavour to decay and therefore decay via the weak interaction (see later).

Heavy hadrons *The November Revolution*

Brookhaven National Laboratory
Led by Samuel Ting

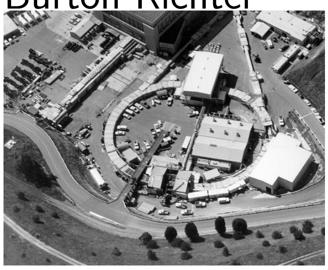




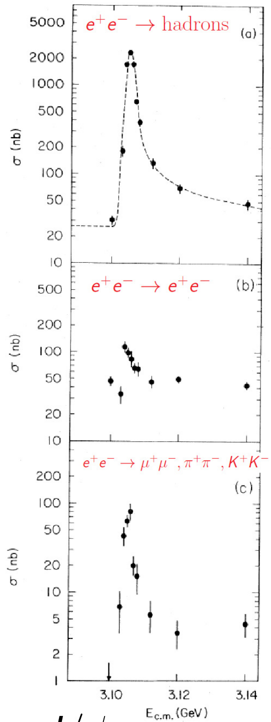
J particle:
PRL 33 (1974) 1404

Stanford Linear Accelerator Center, SPEAR

Led by
Burton Richter



ψ particle:
PRL 33 (1974) 1406



Both experiments announced discovery on 11 November 1974 $\Rightarrow J/\psi$
1976 Nobel Prize awarded to Ting and Richter.

Heavy hadrons *Charmonium*

1974: Discovery of a narrow resonance in e^+e^- collisions at $\sqrt{s} \sim 3.1$ GeV

$$J/\psi(3097)$$

Observed width ~ 3 MeV, all due to experimental resolution.
Actual Total Width, $\Gamma_{J/\psi} \sim 97$ keV

Branching fractions:

$$B(J/\psi \to \text{ hadrons}) \sim 88\%$$

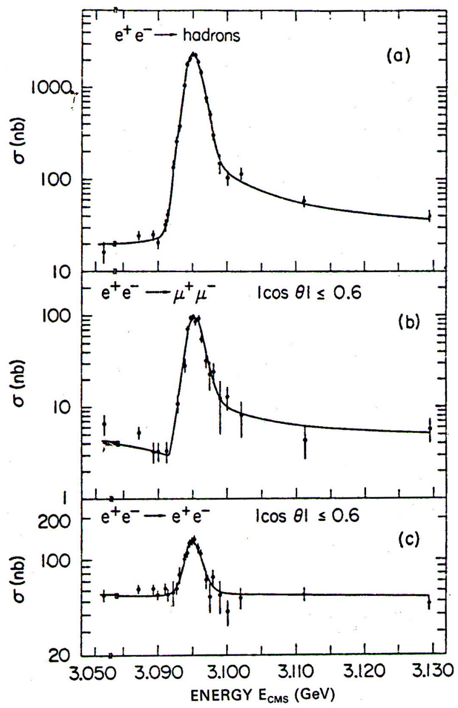
$$B(J/\psi \to \mu^+\mu^-) \sim (J/\psi \to e^+e^-) \sim 6\%$$

Partial widths:

$$\Gamma_{J/\psi \to \text{ hadrons}} \sim 87 \text{ keV}$$

$$\Gamma_{J/\psi \to \mu^+\mu^-} \sim \Gamma_{J/\psi \to e^+e^-} \sim 5 \text{ keV}$$

Mark II Experiment, SLAC, 1978



Heavy hadrons *Charmonium*

Resonance seen in

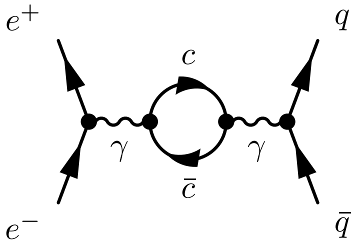
$$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

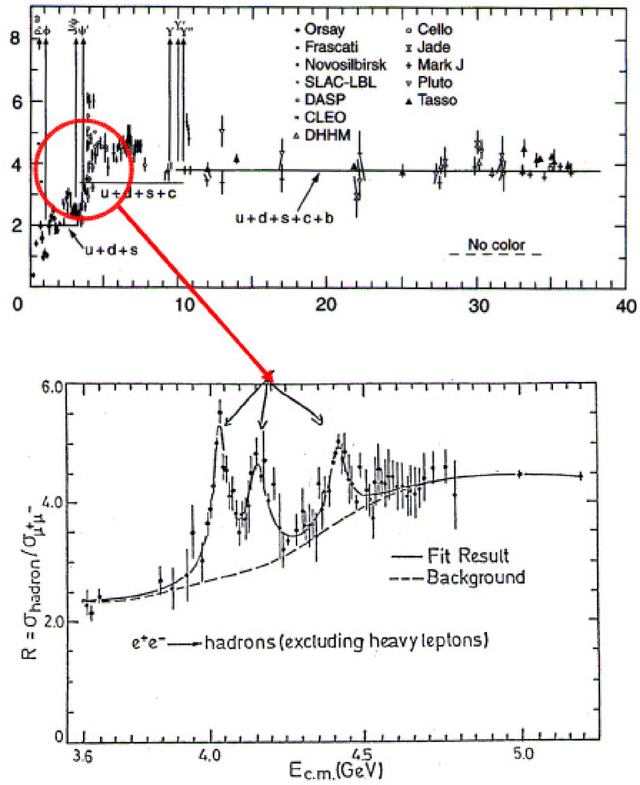
Zoom into the charmonium ($c\bar{c}$) region

$$\sqrt{s} \sim 2m_c$$

mass of charm quark, $m_c \sim 1.5$ GeV

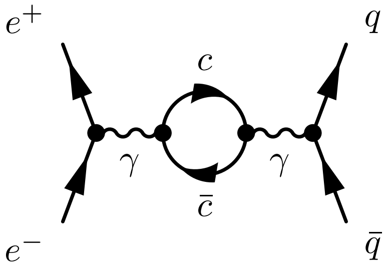
Resonances due to formation of bound unstable $c\bar{c}$ states. The lowest energy of these is the narrow J/ψ state.





Charmonium

$c\bar{c}$ bound states produced directly in e^+e^- collisions must have the same spin and parity as the photon



$J^P = 1^-$

However, expect that a whole spectrum of bound $c\bar{c}$ states should exist (analogous to e^+e^- bound states, positronium)

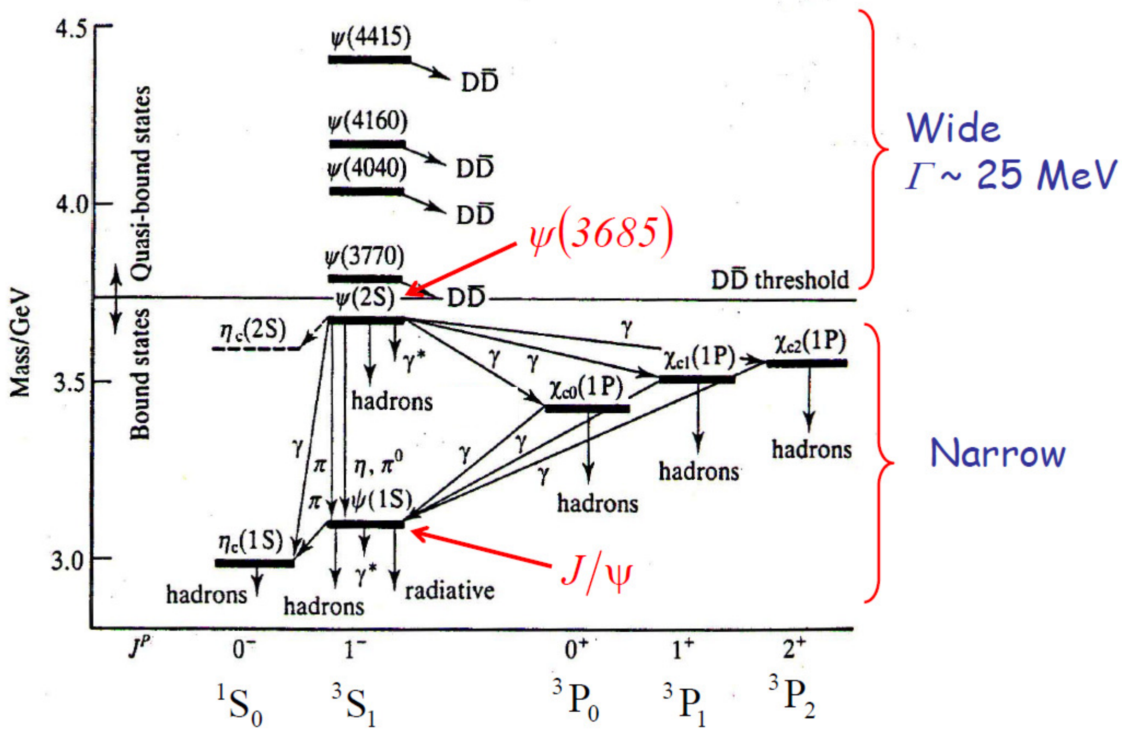
$$n = 1 \quad L = 0 \qquad S = 0, 1 \quad {}^1S_0, {}^3S_1$$

$$n = 2 \quad L = 0, 1 \qquad S = 0, 1 \quad {}^1S_0, {}^3S_1, {}^1P_1, {}^3P_{0,1,2}$$

... etc

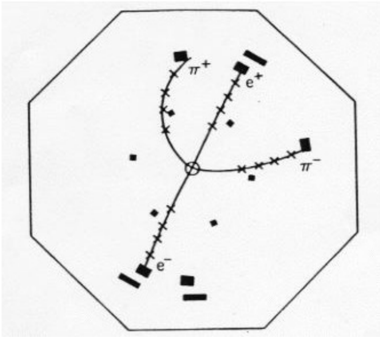
$$\text{Parity} = (-1)(-1)^L \qquad {}^{2S+1}L_J$$

The Charmonium System



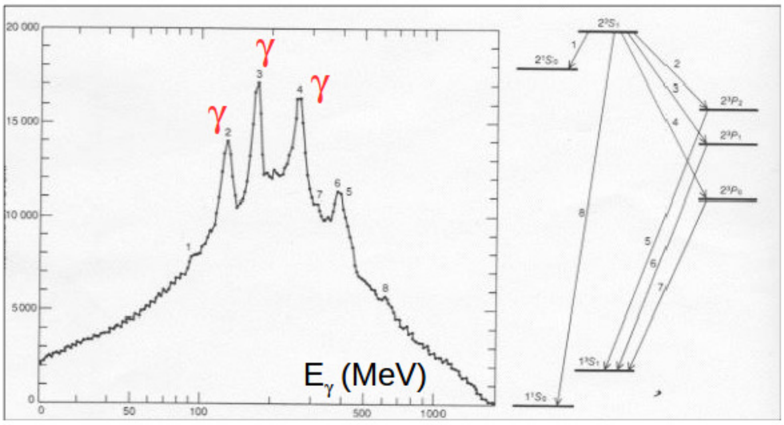
The Charmonium System

All $c\bar{c}$ bound states can be observed via their decay:



Example: Hadronic decay

$$\psi(3685) \to J/\psi\ \pi^+\pi^-$$



Example: Photonic decay

$$\psi(3685) \to \chi + \gamma$$

$$\chi \to J/\psi + \gamma$$

Peaks in γ spectrum

Charmonium Spectroscopy

The Charmonium System

Knowing the $c\bar{c}$ energy levels provides a probe of the QCD potential.

- Because QCD is a theory of a strong confining force (self-interacting gluons), it is **very** difficult to calculate the exact form of the QCD potential from first principles.
- However, it is possible to experimentally "determine" the QCD potential by finding an appropriate form which gives the observed charmonium states.
- In practise, the QCD potential

$$V_{\mathrm{QCD}} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

with $\alpha_s = 0.2$ and $k = 1\ \mathrm{GeVfm}^{-1}$ provides a good description of the experimentally observed levels in the charmonium system.

Why is the J/ψ so narrow?

Consider the charmonium $^3S_1$ states:

1^3S_1 $\psi(3097)$ $\Gamma \sim 0.09$ MeV

2^3S_1 $\psi(3685)$ $\Gamma \sim 0.24$ MeV

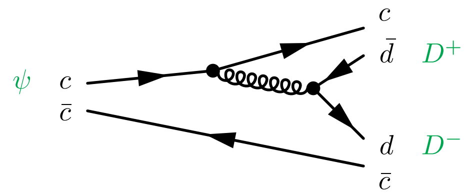
3^3S_1 $\psi(3767)$ $\Gamma \sim 25$ MeV

4^3S_1 $\psi(4040)$ $\Gamma \sim 50$ MeV

The width depends on whether the decay to lightest mesons containing c quarks, $D^-(d\bar{c})$, $D^+(c\bar{d})$, is kinematically possible or not:

$$m(D^{\pm}) = 1869.4 \pm 0.5 \text{ MeV}$$

$m(\psi) > 2m(D)$

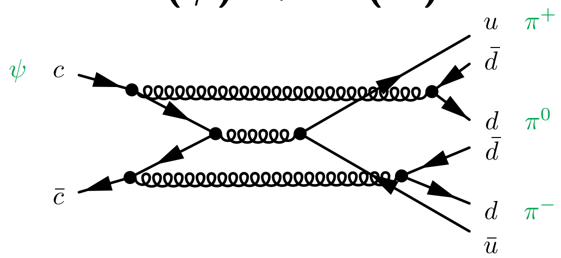


$\psi \to D^+D^-$ allowed

"ordinary" strong decay

$\Rightarrow$ large width

$m(\psi) < 2m(D)$



Zweig Rule: Unconnected lines in the Feynman diagram lead to suppression of the decay amplitude

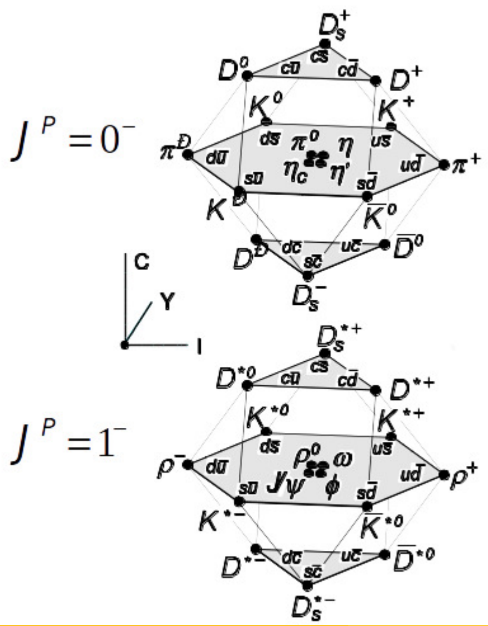
$\Rightarrow$ narrow width

Charmed Hadrons

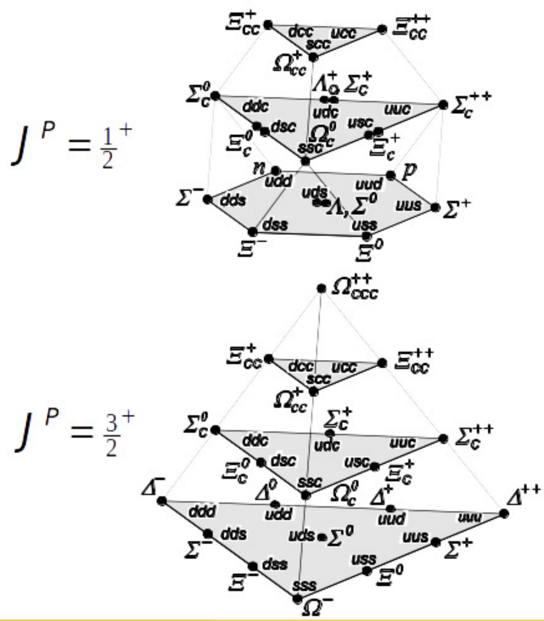
The existence of the c quark $\Rightarrow$ expect to see charmed mesons and baryons (i.e. containing a c quark).

Extend quark symmetries to 3 dimensions:

Mesons



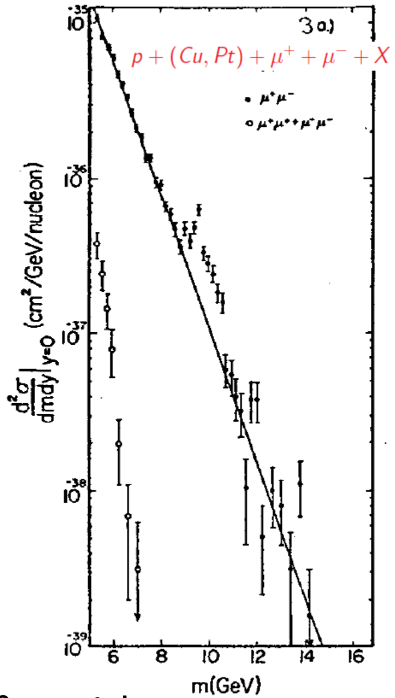
Baryons



Heavy hadrons *the Υ $(b\bar{b})$*

E288 collaboration, Fermilab

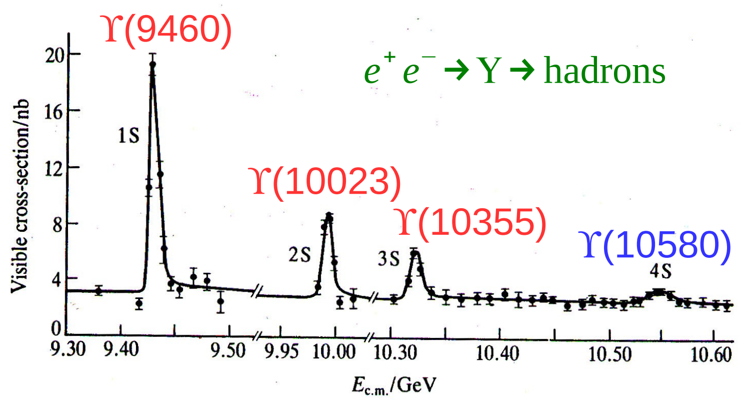
Led by Leon Lederman



Υ particle: PRL 39 (1977) 252-255

- 1977: Discovery of the $\Upsilon(9460)$ resonance state.
- Lowest energy $^3S_1$ bound $b\bar{b}$ state (bottomonium).
- $\Rightarrow m_b \sim 4.7$ GeV

Similar properties to the ψ

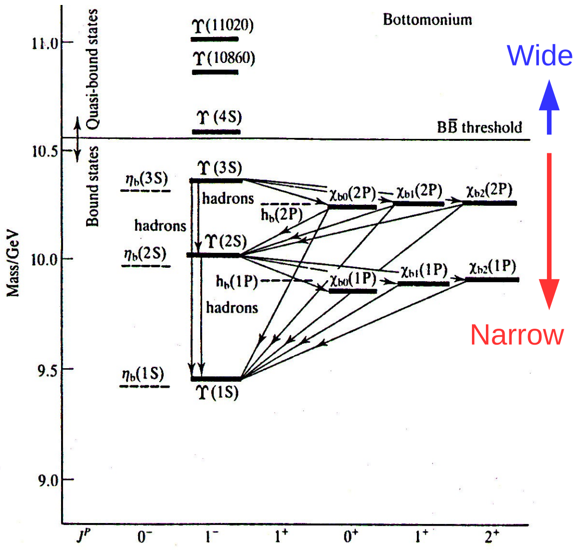


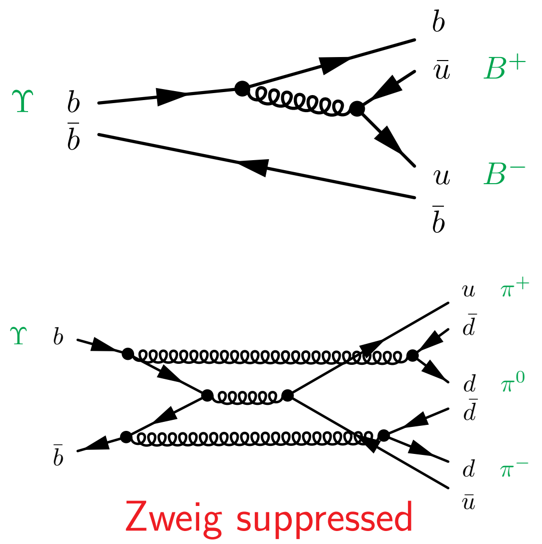
Full Width ~53 keV 44 keV 26 keV 14 MeV

Narrow Wide

Bottomonium

- Bottomonium is the analogue of charmonium for b quark.
- Bottomonium spectrum well described by same QCD potential as used for charmonium.
- Evidence that QCD potential does not depend on quark type.





Bottom Hadrons

Extend quark symmetries to 4 dimensions (difficult to draw!)

Examples:

Mesons $(J^P = 0^-)$: $B^-(b\bar{u})$; $B^0(\bar{b}d)$; $B_s^0(\bar{b}s)$; $B_c^-(b\bar{c})$

The B_c^- is the heaviest hadron discovered so far: $m(B_c^-) = 6.4 \pm 0.4\ \mathrm{GeV}$

$(J^P = 1^-)$: $B^{*-}(b\bar{u})$; $B^{*0}(\bar{b}d)$; $B_s^{*0}(\bar{b}s)$

The mass of the B^* mesons is only 50 MeV above the B meson mass. Expect only electromagnetic decays $B^* \to B\gamma$.

Baryons $\left(J^P = \frac{1}{2}^+\right)$: $\Lambda_b(bud)$; $\Sigma_b(buu)$; $\Xi_b(bus)$

Summary of heavy hadrons

- c and b quarks were first observed in bound state resonances ("onia").
- Consequences of the existence of c and b quarks are
  - Spectra of $c\bar{c}$ (charmonium) and $b\bar{b}$ (bottomonium) bound states
  - Peaks in $R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$
  - Existence of mesons and baryons containing c and b quarks
- The majority of charm and bottom hadrons decay via the weak interaction (strong and electromagnetic decays are forbidden by energy conservation).
- The t quark is very heavy and decays rapidly via the weak interaction before a $t\bar{t}$ bound state (or any other hadron) can be formed.

  $$\tau_t \sim 10^{-25}\,\mathrm{s} \quad t_{\mathrm{hadronisation}} \sim 10^{-22}\,\mathrm{s}$$

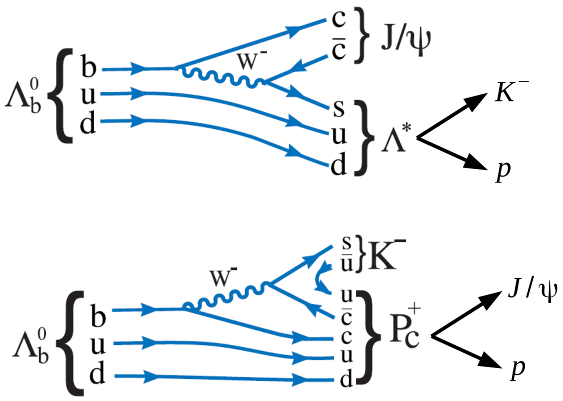
  Rapid decay because $m(t) > m(W)$ so weak interaction is no longer weak.

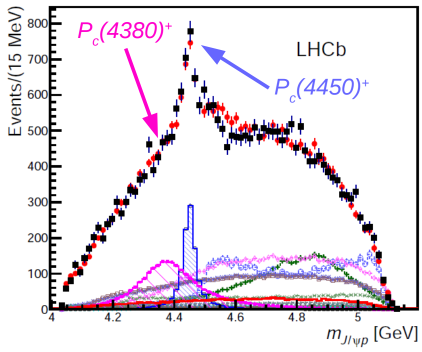
  $$\begin{pmatrix} m(u) = 335\ \mathrm{MeV} \\ m(d) = 335\ \mathrm{MeV} \end{pmatrix} \begin{pmatrix} m(c) = 1.5\ \mathrm{GeV} \\ m(s) = 510\ \mathrm{MeV} \end{pmatrix} \begin{pmatrix} m(t) = 175\ \mathrm{GeV} \\ m(b) = 4.5\ \mathrm{GeV} \end{pmatrix}$$

Tetraquarks and Pentaquarks *(non-examinable)*

Quark Model of Hadrons is not limited to $q\bar{q}$ or qqq content.

Recent observations from *LHCb* show unquestionable discovery of pentaquark states, PRL 115, 072001 (2015).





+ others more recently.

How are these quarks bound? $qqqqq$? $qq + qqq$? $qq + qq + q$?

A few tetraquarks discovered by *Belle* and *BESIII*

e.g. $Z(4430)^-$, $c\bar{c}d\bar{u}$ discovered by *Belle* and confirmed by *LHCb*

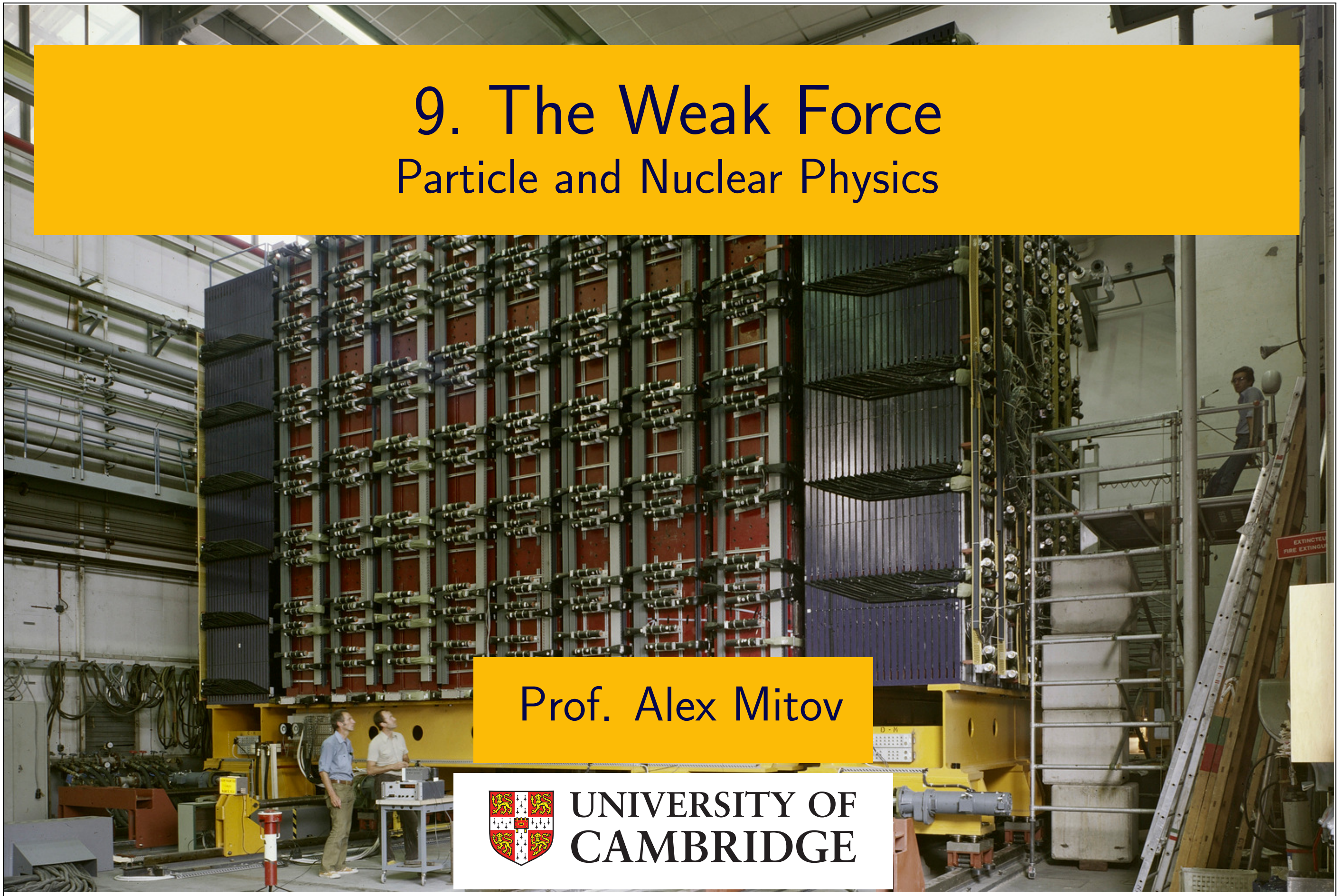
LHCb has discovered many more!

Summary

- Evidence for hadron sub-structure – quarks
- Hadron wavefunctions and allowed states
- Hadron masses and magnetic moments
- Hadron decays (strong, EM, weak)
- Heavy hadrons: charmonium and bottomonium
- Recent tetraquark and pentaquark discoveries

Problem Sheet: q.17-22

Up next...
Section 9: The Weak Force



9. The Weak Force

Particle and Nuclear Physics

Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- The charged current weak interaction
- Four-fermion interactions
- Massive propagators and the strength of the weak interaction
- C-symmetry and Parity violation
- Lepton universality
- Quark interactions and the CKM

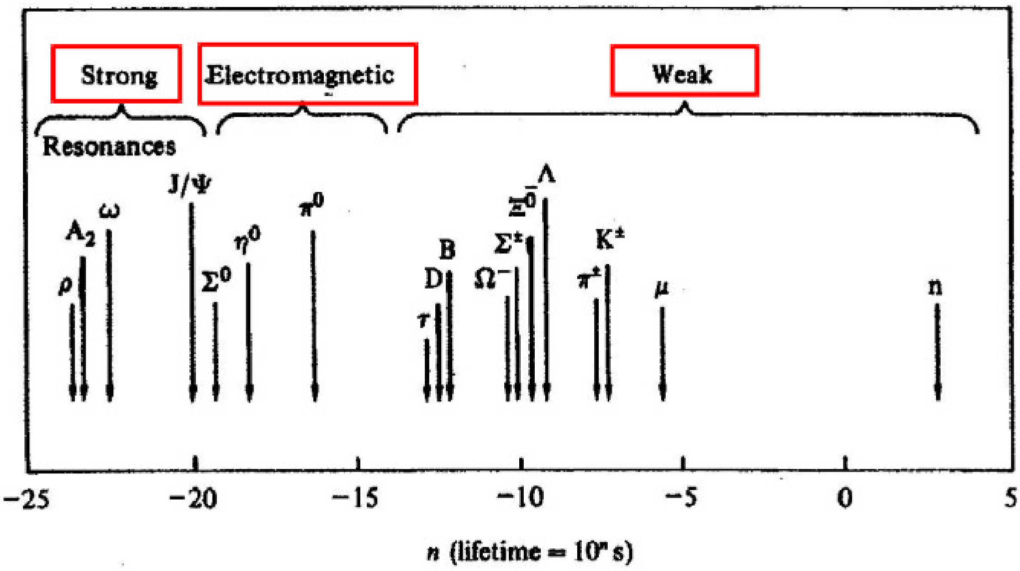
The Weak Interaction

The weak interaction accounts for many decays in particle physics, e.g.

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu \qquad \tau^- \to e^- \bar{\nu}_e \nu_\tau$$

$$\pi^+ \to \mu^- \bar{\nu}_\mu \qquad n \to p e^- \bar{\nu}_e$$

Characterised by long lifetimes and small interaction cross-sections



The Weak Interaction

- Two types of weak interaction

  Charged current (CC): $W^\pm$ bosons

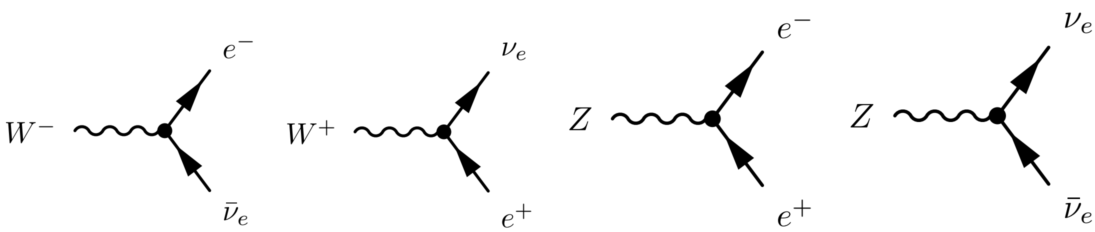
  Neutral current (NC): Z bosons See Chapter 10

- The weak force is mediated by massive vector bosons:

$$m_W = 80 \text{ GeV}$$
$$m_Z = 91 \text{ GeV}$$

Examples: (The list below is not complete, will see more vertices later!)

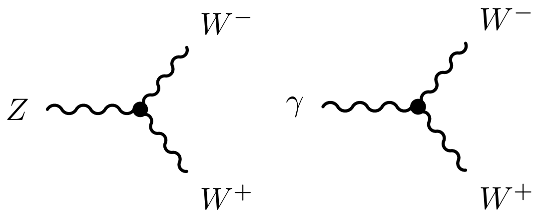
Weak interactions of electrons and neutrinos:

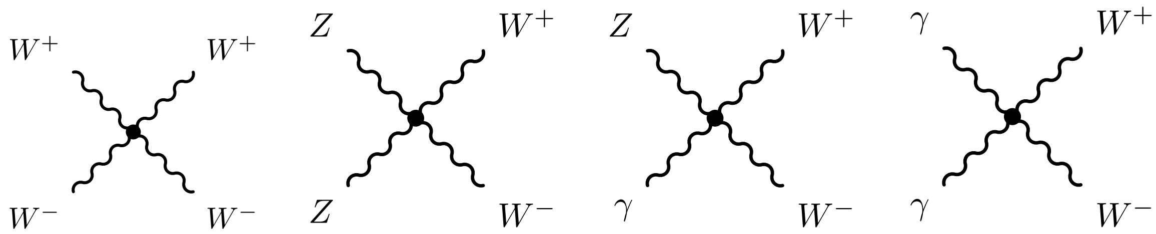


Boson Self-Interactions

- In QCD the gluons carry colour charge.
- In the weak interaction the $W^{\pm}$ and Z bosons carry the weak charge
- $W^{\pm}$ also carry the electric charge

$\Rightarrow$ boson self-interactions





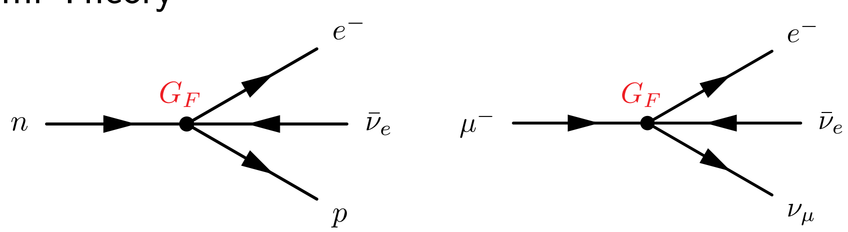
(The list above is complete as far as weak self-interactions are concerned, but we have still not seen all the weak vertices. Will see the rest later)

Fermi Theory *The old ("imperfect") idea*

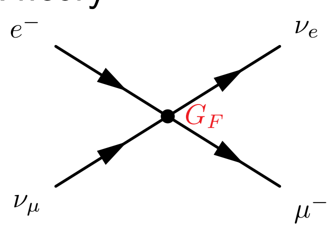
Weak interaction taken to be a "4-fermion contact interaction"

- No propagator
- Coupling strength given by the Fermi constant G_F
- $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

β-decay in Fermi Theory

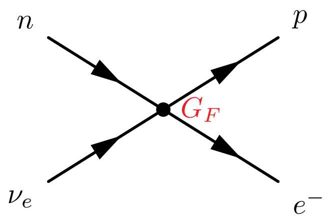


Neutrino scattering in Fermi Theory



Why must Fermi Theory be “Wrong”?

$$\nu_e + n \to p + e^-$$



$$\mathrm{d}\sigma = 2\pi |M_{\rm fi}|^2 \frac{\mathrm{d}N}{\mathrm{d}E} = 2\pi 4 G_F^2 \frac{E_e^2}{(2\pi)^3}\,\mathrm{d}\Omega$$

$$\sigma = \frac{G_F^2 s}{\pi}$$ See Appendix F

where E_e is the energy of the e^- in the centre-of-mass system and $\sqrt{s}$ is the energy in the centre-of-mass system.

In the laboratory frame: $s = 2E_\nu m_n$ (fixed target collision, see Chapter 3)

$$\Rightarrow \sigma \sim (E_\nu/\ \mathrm{MeV}) \times 10^{-43}\,\mathrm{cm}^{-2}$$

- ν's only interact weakly $\therefore$ have very small interaction cross-sections.
- Here weak implies that you need approximately 50 light-years of water to stop a 1 MeV neutrino!

However, as $E_\nu \to \infty$ the cross-section can become very large. Violates maximum value allowed by conservation of probability at $\sqrt{s} \sim 1$ TeV (“unitarity limit”). This is a big problem.

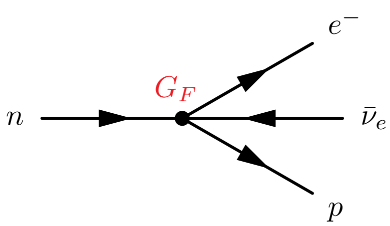
$\Rightarrow$ Fermi theory breaks down at high energies.

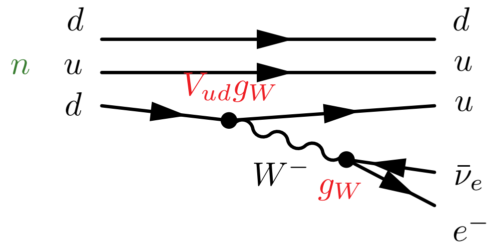
Weak Charged Current: $W^\pm$ Boson

- Fermi theory breaks down at high energy
- True interaction described by exchange of charged $W^\pm$ bosons
- Fermi theory is the low energy ($q^2 \ll m_W^2$) effective theory of the weak interaction

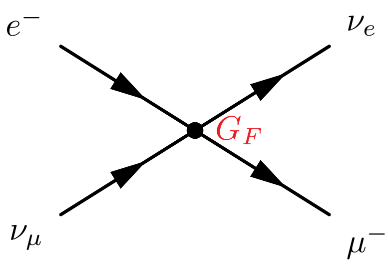
Old Fermi Theory **Standard Model**

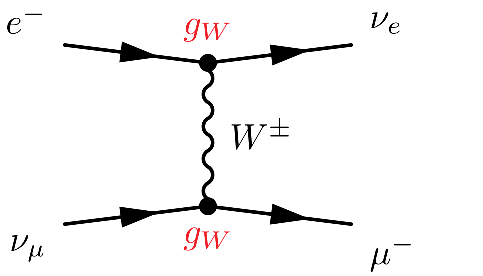
β decay





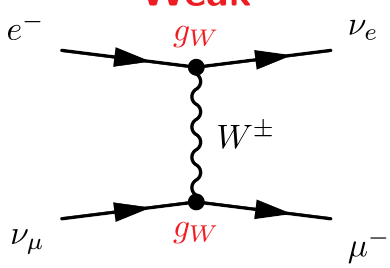
$\nu_e e^-$ scattering





Weak Charged Current: $W^{\pm}$ Boson

Weak

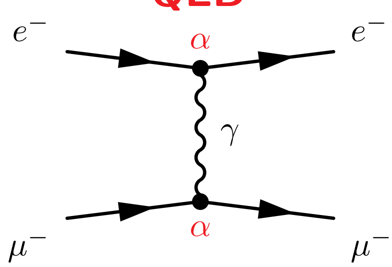


Propagator

$$\sim \frac{1}{q^2 - m_W^2}$$

$$\alpha_W = \frac{g_W^2}{4\pi}$$

QED



Propagator

$$\sim \frac{1}{q^2}$$

$$\alpha = \frac{e^2}{4\pi}$$

Charged Current Weak Interaction

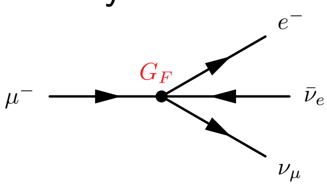
- At low energies, $q^2 \ll m_W^2$, propagator $\frac{1}{q^2 - m_W^2} \to \frac{1}{-m_W^2}$
  i.e. appears as the point-like interaction of Fermi theory.
- Massive propagator $\to$ short range

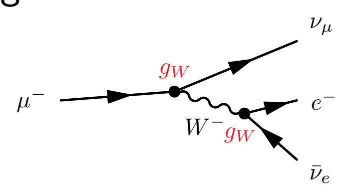
$$m_W = 80.4\ \mathrm{GeV} \quad \Rightarrow \ \mathrm{Range} \sim \frac{1}{m_W} \sim 0.002\,\mathrm{fm}$$

- Exchanged boson carries electromagnetic charge.
- Flavour changing - only the CC weak interaction changes flavour
- Parity violating - only the CC weak interaction can violate parity conservation

Weak Charged Current: $W^{\pm}$ Boson

Compare Fermi theory with a massive propagator





For $q^2 \ll m_W^2$ compare matrix elements $\frac{g_W^2}{m_W^2} \to G_F$

The precise relationship is: $\dfrac{g_W^2}{8m_W^2} \to \dfrac{G_F}{\sqrt{2}}$

G_F is small
because m_W is large

The numerical factors are partly of historical origin (see Perkins 4^{th} ed., page 210).

$m_W = 80.4\ \mathrm{GeV}$ and $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ measured in muon β decay

$$g_W = 0.65 \quad \text{and} \quad \alpha_W = \frac{g_W^2}{4\pi} \sim \frac{1}{30}$$

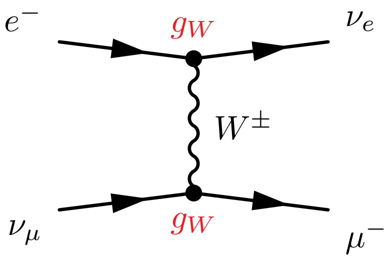
Compare to EM $\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$

The intrinsic strength of the weak interaction is actually greater than that of the electromagnetic interaction. At low energies (low q^2), it appears weak owing to the massive propagator.

Weak Charged Current: $W^{\pm}$ Boson

Neutrino Scattering with a Massive W Boson

Replace contact interaction by massive boson exchange diagram:



Fermi theory $$\frac{d\sigma}{d\Omega} = 2\pi G_F^2 \frac{E_e^2}{(2\pi)^3}$$

Standard Model
$$\frac{d\sigma}{d\Omega} = 2\pi G_F^2 \frac{E_e^2}{(2\pi)^3}\left(\frac{m_W^2}{m_W^2 - q^2}\right)^2$$

with $|\vec{q}^2| = 4E_e^2 \sin^2\theta/2$, where θ is the scattering angle.

Integrate to give
$$\sigma = \frac{G_F^2 s}{\pi} \quad s \ll m_W^2$$

$$\sigma = \frac{G_F^2 m_W^2}{\pi} \quad s \gg m_W^2$$

see Appendix G

Cross-section is now well behaved at high energies.

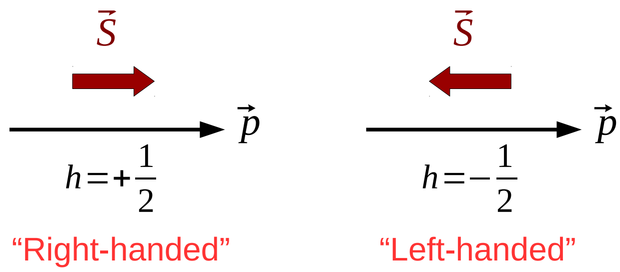
Spin and helicity

Consider a free particle of constant momentum, $\vec{p}$

- Total angular momentum, $\vec{J} = \vec{L} + \vec{S}$ is always conserved
- The orbital angular momentum, $\vec{L} = \vec{r} \times \vec{p}$ is perpendicular to $\vec{p}$
- The spin angular momentum, $\vec{S}$ can be in any direction relative to $\vec{p}$
- The value of spin $\vec{S}$ along $\vec{p}$ is always constant

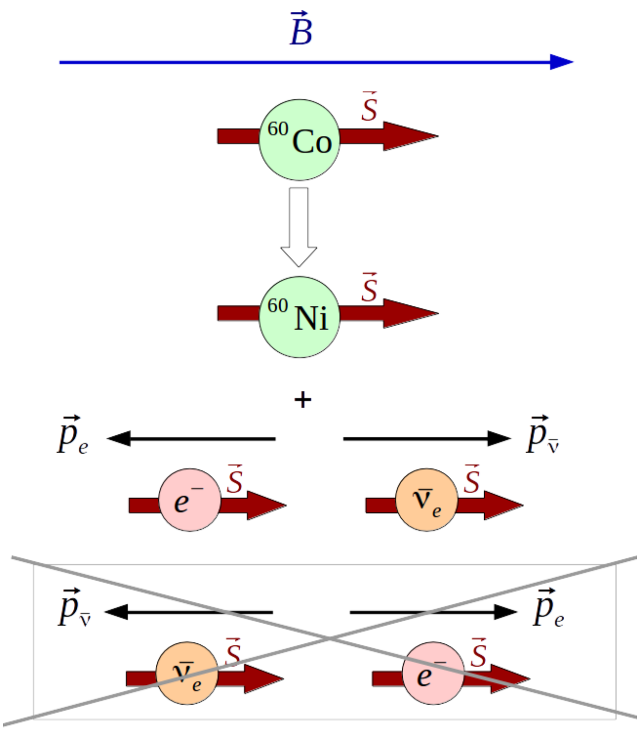
The sign of the component of spin along the direction of motion is known as the "helicity",
$$h = \frac{\vec{S}.\vec{p}}{|\vec{p}|}$$

Taking spin 1/2 as an example:



The Wu Experiment

β decay of ${}^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e$



1956
Chien-Shiung Wu

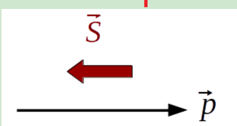
Align cooled ^{60}Co nuclei with $\vec{B}$ field and look at direction of emission of electrons

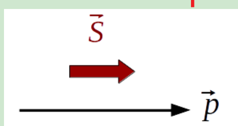
- e^- always observed in direction opposite to spin – left-handed.
- $\vec{p}$ conservation: $\bar{\nu}$ must be emitted in opposite direction – right-handed.
- Right-handed e^- not observed here $\Rightarrow$ Parity Violation

The Weak Interaction and Helicity

The weak interaction distinguishes between left- and right-handed states. This is an experimental observation, which we need to build into the Standard Model.

The weak interaction couples preferentially to
left-handed particles and right-handed antiparticles





To be precise, the probability for weak coupling to the $\pm$ helicity state is

$\frac{1}{2}\left[1 \mp \frac{v}{c}\right]$ for a lepton $\rightarrow$ coupling to RH particles vanishes

$\frac{1}{2}\left[1 \pm \frac{v}{c}\right]$ for an antilepton $\rightarrow$ coupling to LH antiparticles vanishes

$\Rightarrow$ right-handed ν's do not exist

left-handed $\bar{\nu}$'s do not exist

Even if they did exist, they would be unobservable.

Charge Conjugation

C-symmetry: the physics for $+Q$ should be the same as for $-Q$.

This is true for QED and QCD, but not the Weak force...

$$\begin{array}{ccc} \text{LH } e^- & \xrightarrow{\text{Charge Conjugation}} & \text{LH } e^+ \\ \textit{EM, Weak} & & \textit{EM, } \cancel{\textit{Weak}} \end{array}$$

$$\begin{array}{ccc} \text{RH } e^- & \xrightarrow{\text{Charge Conjugation}} & \text{RH } e^+ \\ \textit{EM, } \cancel{\textit{Weak}} & & \textit{EM, Weak} \end{array}$$

$$\begin{array}{ccc} \text{LH } \nu_e & \xrightarrow{\text{Charge Conjugation}} & \text{LH } \bar{\nu}_e \\ \textit{Weak} & & \cancel{\textit{Weak}} \end{array}$$

C-symmetry is maximally violated in the weak interaction.

Parity Violation

Parity is always conserved in the strong and EM interactions

$\eta \to \pi^0\pi^0\pi^0$ $\eta \to \pi^+\pi^-$

Parity Violation

Parity is often conserved in the weak interaction, but not always

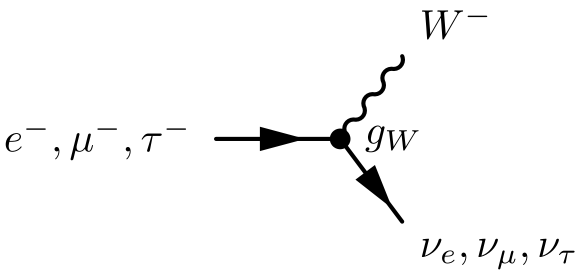
The weak interaction treats LH and RH states differently and therefore can violate parity (because the interaction Hamiltonian does not commute with $\hat{P}$).

$K^+ \to \pi^+\pi^-\pi^+$

$K^+ \to \pi^+\pi^0$

Weak interactions of leptons

All weak charged current lepton interactions can be described by the W boson propagator and the weak vertex:



The Standard Model Weak CC Lepton Vertex

+ antiparticles

- W bosons only "couple" to the (left-handed) lepton and neutrino within the same generation

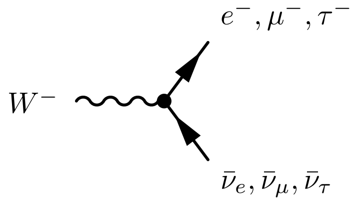
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

e.g. no $W^\pm e^- \nu_\mu$ coupling

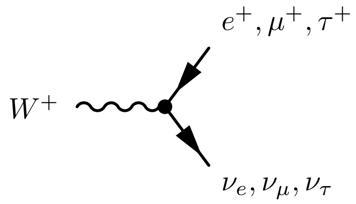
- Coupling constant g_W $\alpha_W = \frac{g_W^2}{4\pi}$

Weak interactions of leptons *Examples*

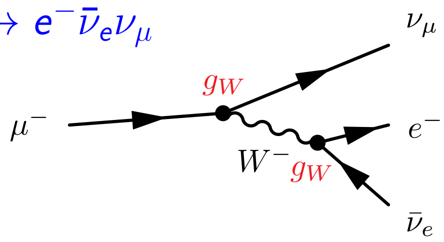
$W^- \to e^-\bar{\nu}_e,\ \mu^-\bar{\nu}_\mu,\ \tau^-\bar{\nu}_\tau$



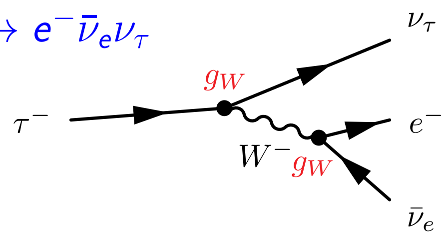
$W^+ \to e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau$



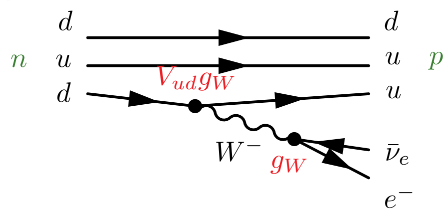
$\mu^- \to e^-\bar{\nu}_e\nu_\mu$



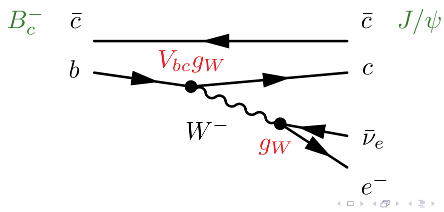
$\tau^- \to e^-\bar{\nu}_e\nu_\tau$



$n \to pe^-\bar{\nu}_e$

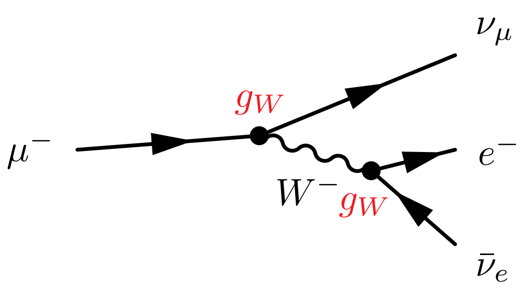


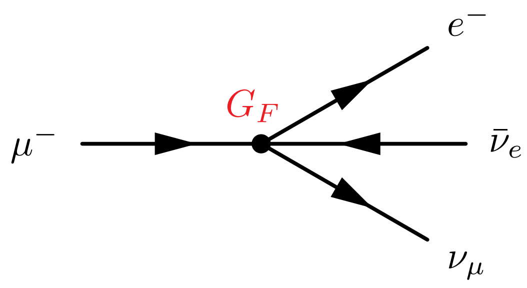
$B_c^- \to J/\psi e^-\bar{\nu}_e$



μ Decay

- Muons are fundamental leptons ($m_\mu \sim 206 m_e$)
- Electromagnetic decay $\mu^- \to e^-\gamma$ is not observed (branching ratio $< 2.4 \times 10^{-12}$) $\Rightarrow$ the EM interaction does not change flavour.
- Only the weak CC interaction changes lepton type, but only within a generation. "Lepton number conservation" for each lepton generation.
- Muons decay weakly: $\mu^- \to e^-\bar{\nu}_e\nu_\mu$





As $m_\mu \ll m_W$ can safely use Fermi theory to calculate decay width (analogous to nuclear β decay).

μ Decay

Fermi theory gives decay width $\propto m_\mu^5$ (Sargent Rule)

However, more complicated phase space integration (previously neglected kinetic energy of recoiling nucleus) and taking account of helicity/spin gives different constants

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

- Muon mass and lifetime known with high precision.

$$m_\mu = 105.6583715 \pm 0.0000035 \text{ MeV}$$

$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6}\,\text{s}$$

- Use muon decay to fix strength of weak interaction G_F

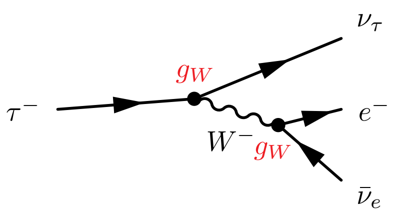
$$G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

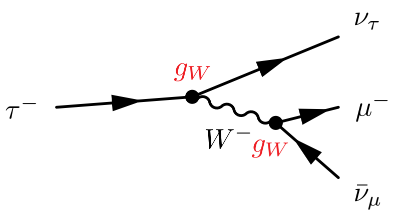
- G_F is one of the best determined fundamental quantities in particle physics.

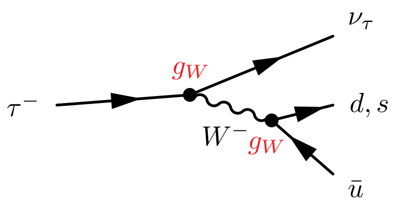
τ Decay

The τ mass is relatively large $m_\tau = 1.77686 \pm 0.00012$ GeV

Since $m_\tau > m_\mu, m_\pi, m_p, \ldots$ there are a number of possible decay modes







Measure the τ branching fractions as:

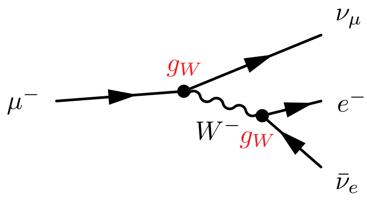
$$\tau^- \to e^- \bar{\nu}_e \nu_\tau \quad 17.83 \pm 0.04\%$$

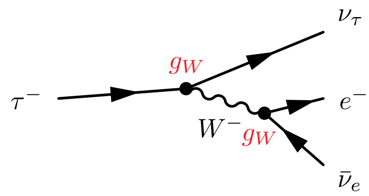
$$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \quad 17.41 \pm 0.04\%$$

$$\tau^- \to \text{hadrons} \quad 64.76 \pm 0.06\%$$

Lepton Universality

Do all leptons have the same weak coupling?
Look at measurements of the decay rates and branching fractions.





$$\frac{1}{\tau_\mu} = \Gamma_{\mu\to e} = \frac{G_F^2}{192\pi^3} m_\mu^5 \qquad \frac{1}{\tau_\tau} = \frac{\Gamma_{\tau\to e}}{B(\tau\to e)} = \frac{1}{0.178}\frac{G_F^2}{192\pi^3} m_\tau^5$$

If weak interaction strength is universal, expect: $\dfrac{\tau_\tau}{\tau_\mu} = 0.178\,\dfrac{m_\mu^5}{m_\tau^5}$

Measure m_μ, m_τ, τ_μ to high precision:

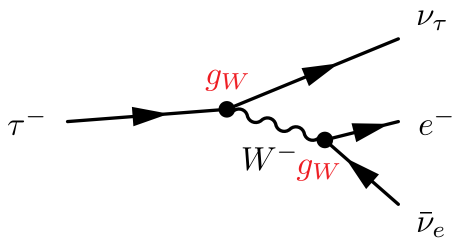
$$m_\mu = 105.6583715 \pm 0.0000035\ \mathrm{MeV}$$
$$m_\tau = 1.77686 \pm 0.00012\ \mathrm{GeV}$$
$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6}\,\mathrm{s}$$

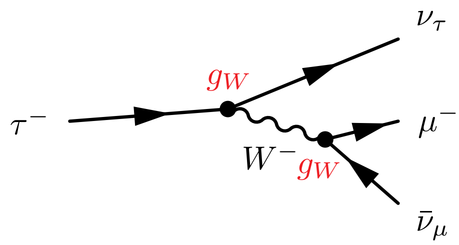
Predict $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}\,\mathrm{s}$ **Measure** $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}\,\mathrm{s}$

$\Rightarrow$ same weak CC coupling for μ and τ

Lepton Universality

We can also compare





If the couplings are the same, expect: $\dfrac{B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = 0.9726$

(the small difference is due to the slight reduction in phase space due to the non-negligible muon mass).

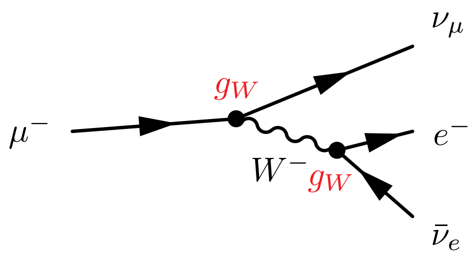
Measured $\dfrac{B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = 0.974 \pm 0.005$ consistent with prediction.

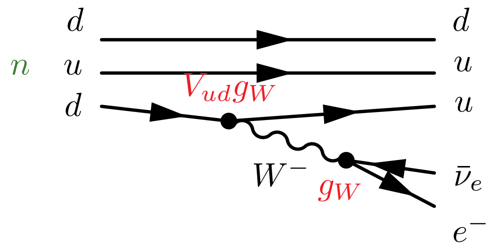
$\Rightarrow$ same weak CC coupling for e, μ and τ

$\Rightarrow$ Lepton Universality

Universality of Weak Coupling

Compare G_F measured from μ^- decay with that from nuclear β decay





$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$ $G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$

Ratio $\dfrac{G_F^\beta}{G_F^\mu} = 0.974 \pm 0.003$

Conclude that the strength of the weak interaction is almost the same for leptons as for quarks. But the difference is significant, and has to be explained.

Weak Interactions of Quarks

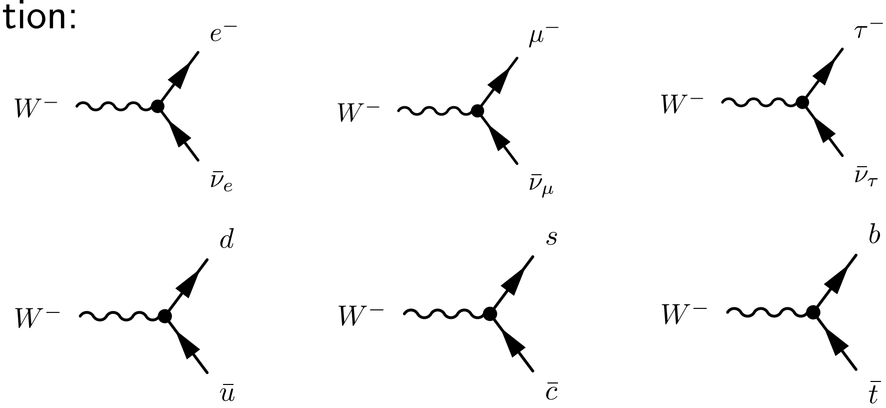
Impose a symmetry between leptons and quarks, so weak CC couplings take place within one generation:

Leptons

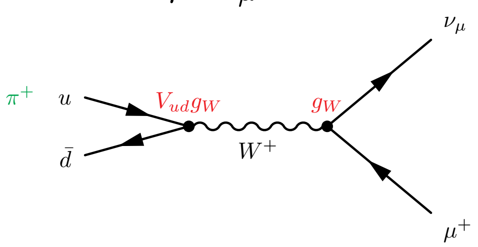
$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$

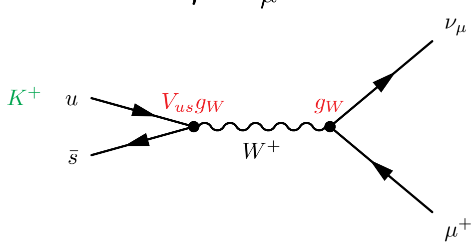
Quarks

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$



So $\pi^+ \to \mu^+\nu_\mu$ would be allowed but $K^+ \to \mu^+\nu_\mu$ would not





But we have observed $K^+ \to \mu^+\nu_\mu$! (much smaller rate than π^+ decay.)

Quark Mixing

Instead, alter the lepton-quark symmetry to: (only considering 1^{st} and 2^{nd} gen. here)

Leptons	Quarks
$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$	$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$

where $d' = d\cos\theta_C + s\sin\theta_C$
$s' = -d\sin\theta_C + s\cos\theta_C$

Now, the down type quarks in the weak interaction are actually linear superpositions of the down type quarks

i.e. weak eigenstates (d',s') are superpositions of the mass eigenstates (d,s)

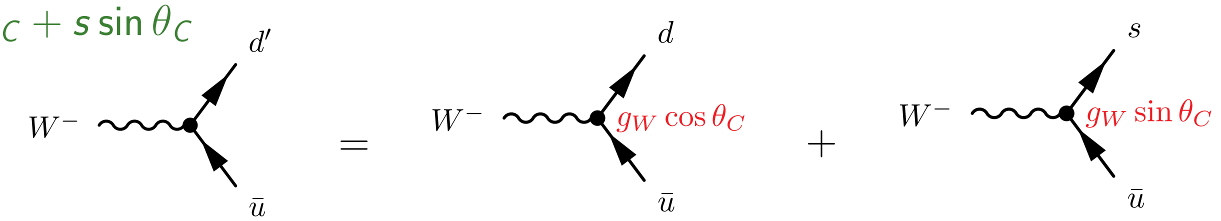
Weak Eigenstates $\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$ Mass Eigenstates

$\Rightarrow$ Cabibbo angle $\theta_C \sim 13^\circ$ (from experiment)

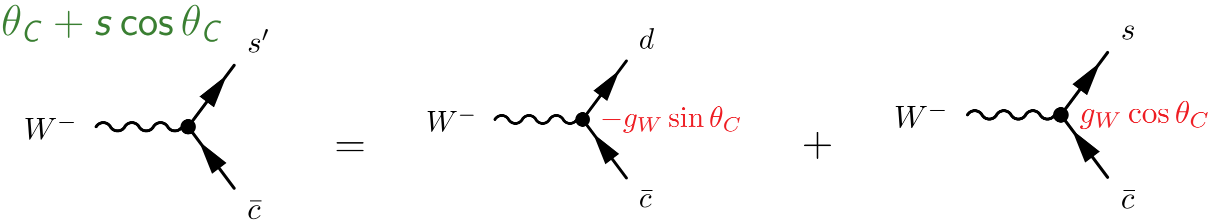
Quark Mixing

Now, the weak coupling to quarks is:

$d\cos\theta_C + s\sin\theta_C$



$-d\sin\theta_C + s\cos\theta_C$



Quark mixing explains the lower rate of $K^+ \to \mu^+\nu_\mu$ compared to $\pi^+ \to \mu^+\nu_\mu$

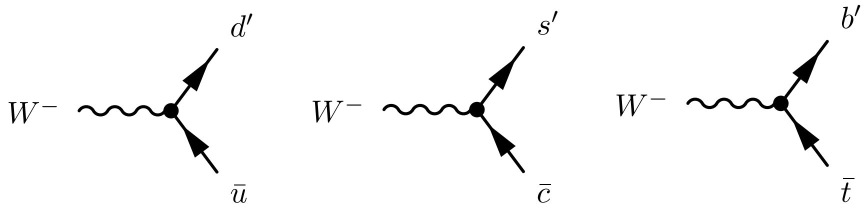
and the ratio $\frac{G_F^\beta}{G_F^\mu} = 0.974 \pm 0.003$

Difference in couplings affects $|M|^2 \propto (G_F^\beta)^2 \propto (\cos\theta_C)^2$

Now expect $\frac{G_F^\beta}{G_F^\mu} = \cos\theta_C$ which holds for $\theta_C \sim 13^\circ$

CKM matrix *Cabibbo-Kobayashi-Maskawa Matrix*

Extend quark mixing to three generations



Weak Eigenstates $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$ Mass Eigenstates

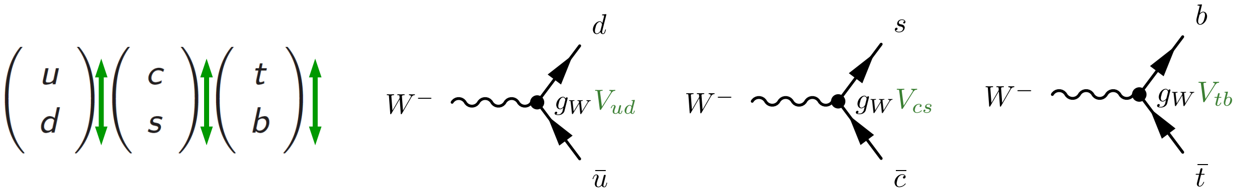
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} \cos\theta_C & \sin\theta_C & \sin^3\theta_C e^{-i\delta} \\ -\sin\theta_C & \cos\theta_C & \sin^2\theta_C \\ \sin^3\theta_C e^{i\delta} & -\sin^2\theta_C & 1 \end{pmatrix}$$

Unitary matrix.
Mixing angle $\theta_C \sim 13^\circ$
Charge-Parity violating phase $\delta \sim 69^\circ$
(full CKM matrix includes 3 mixing angles)

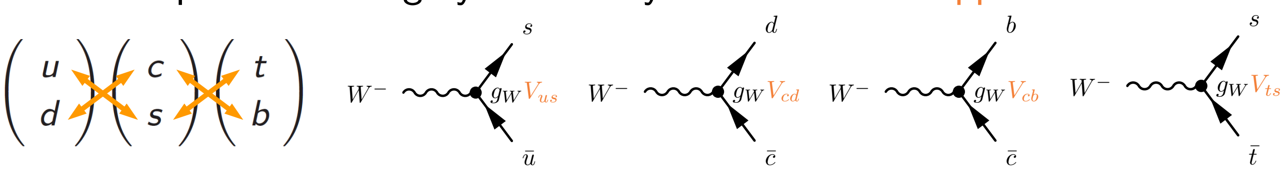
$$\sim \begin{pmatrix} 0.975 & 0.220 & 0.01 \\ -0.220 & 0.975 & 0.05 \\ 0.01 & -0.05 & 1 \end{pmatrix}$$

Quark Mixing

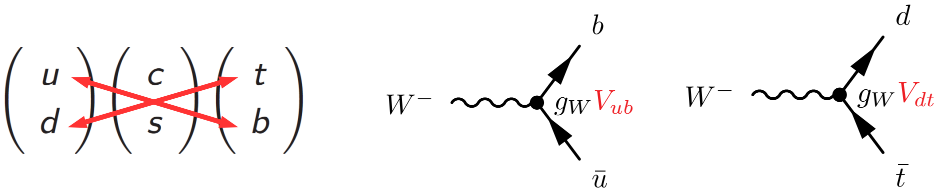
Weak interactions between quarks of the same family are "Cabibbo Allowed"



between quarks differing by one family are "Cabibbo Suppressed"

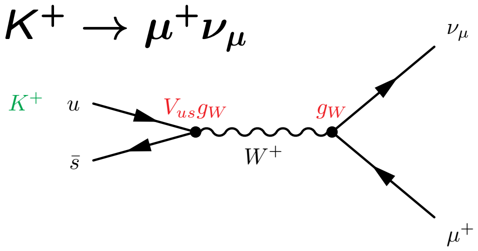


between quarks differing by two families are "Doubly Cabbibo Suppressed"



Quark Mixing Examples

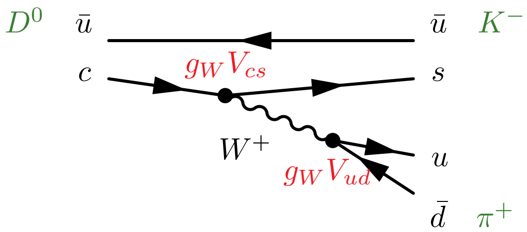
$K^+ \to \mu^+\nu_\mu$



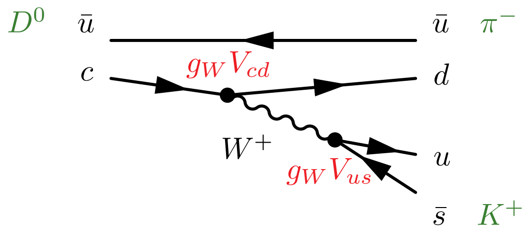
$u\bar{s}$ coupling $\Rightarrow$ Cabbibo suppressed

$$|M|^2 \propto g_W^4 V_{us}^2 = g_W^4 \sin^2\theta_C$$

$D^0 \to K^-\pi^+$



$D^0 \to K^+\pi^-$



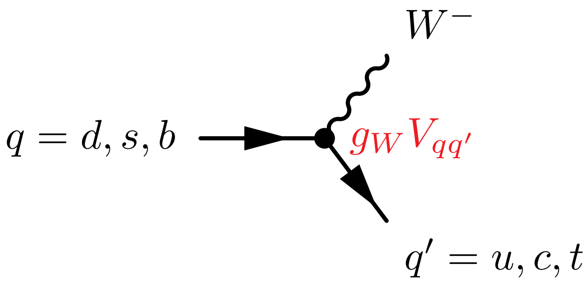
Expect $\quad \dfrac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} \sim \dfrac{(g_W^2 V_{cd} V_{us})^2}{(g_W^2 V_{cs} V_{ud})^2} = \dfrac{\sin^4\theta_C}{\cos^4\theta_C} \sim 0.0028$

Measure $\quad 0.0038 \pm 0.0008$

$D^0 \to K^+\pi^-$ is Doubly Cabibbo suppressed (two Cabibbo suppressed vertices)

Summary of the Weak CC Vertex

All weak charged current quark interactions can be described by the W boson propagator and the weak vertex:



The Standard Model
Weak CC Quark Vertex

+ antiparticles

- $W^\pm$ bosons always change quark flavour
- $W^\pm$ prefers to couple to quarks in the same generation, but quark mixing means that cross-generation coupling can occur.
  Crossing two generations is less probable than one.
  W-lepton coupling constant $\longrightarrow g_W$
  W-quark coupling constant $\longrightarrow g_W V_{\mathrm{CKM}}$

Summary

Weak interaction (charged current)

- Weak force mediated by massive W bosons $m_W = 80.385 \pm 0.015\ \mathrm{GeV}$
- Weak force intrinsically stronger than EM interaction

$$\alpha_W \sim \frac{1}{30} \qquad \alpha_{EM} \sim \frac{1}{137}$$

- Universal coupling to quarks and leptons, but...
- Quarks take part in the interaction as mixtures of the mass eigenstates
- Parity & C-symmetry can be violated due to the helicity structure of the interaction
- Strength of the weak interaction given by

$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

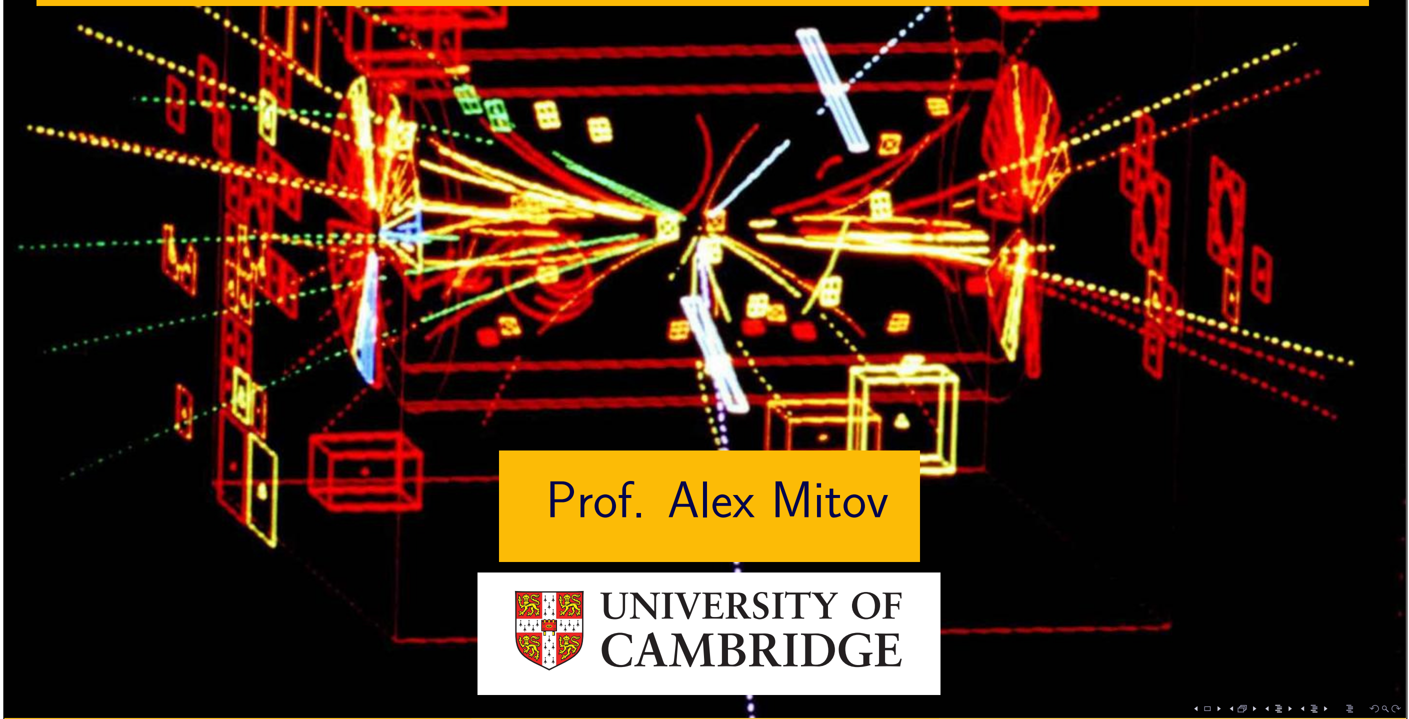
from μ decay.

Problem Sheet: q.23-25

Up next... Section 10: Electroweak Unification

10. Electroweak Unification

Particle and Nuclear Physics



Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

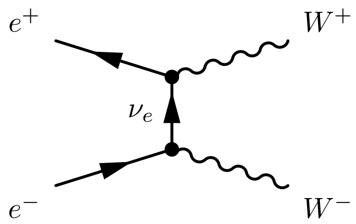
In this section...

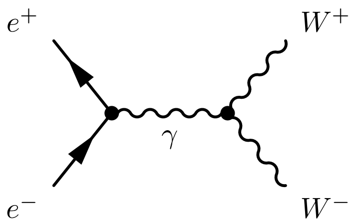
- GWS model
- Allowed vertices
- Revisit Feynman diagrams
- Experimental tests of Electroweak theory

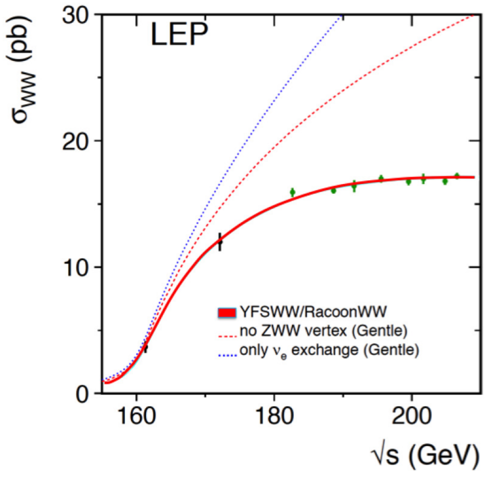
Electroweak Unification

- Weak CC interactions explained by $W^\pm$ boson exchange
- $W^\pm$ bosons are charged, thus they couple to the γ

Consider $e^-e^+ \to W^+W^-$: 2 diagrams (+interference)

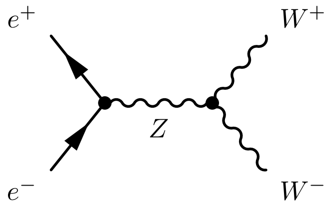






- Cross-section diverges at high energy
- Divergence cured by introducing Z boson
- Extra diagram for $e^-e^+ \to W^+W^-$
- Idea only works if γ, $W^\pm$, Z couplings are related

$\Rightarrow$ **Electroweak Unification**



Electroweak gauge theory *(non-examinable)*

- Postulate invariance under a gauge transformation like:

$$\psi \to \psi' = e^{ig\vec{\sigma}.\vec{\Lambda}(\vec{r},t)}\psi$$

  an "SU(2)" transformation (σ are 2x2 matrices).
- Operates on the state of "weak isospin" – a "rotation" of the isospin state.
- Invariance under SU(2) transformations $\Rightarrow$ three massless gauge bosons (W_1, W_2, W_3) whose couplings are well specified.
- They also have self-couplings.

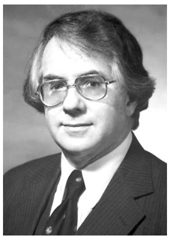
But this doesn't quite work...
Predicts W and Z have the same couplings – not seen experimentally!

Electroweak gauge theory

The solution...

- Unify QED and the weak force $\Rightarrow$ electroweak model
- "SU(2)xU(1)" transformation
  U(1) operates on the "weak hypercharge" $Y = 2(Q - I_3)$
  SU(2) operates on the state of "weak isospin, I"
- Invariance under SU(2)xU(1) transformations $\Rightarrow$ four massless gauge bosons W^+, W^-, W_3, B
- The two neutral bosons W_3 and B then **mix** to produce the physical bosons Z and γ
- Photon properties must be the same as QED $\Rightarrow$ predictions of the couplings of the Z in terms of those of the W and γ
- Still need to account for the masses of the W and Z. This is the job of the Higgs mechanism (later).

The GWS Model



The Glashow, Weinberg and Salam model treats EM and weak interactions as different manifestations of a single unified electroweak force (Nobel Prize 1979)

Start with 4 massless bosons W^+, W_3, W^- and B. The neutral bosons mix to give physical bosons (the particles we see), i.e. the $W^{\pm}$, Z, and γ.

$$\begin{pmatrix} W^+ \\ W_3 \\ W^- \end{pmatrix} ; \quad B \quad \rightarrow \quad \begin{pmatrix} W^+ \\ Z \\ W^- \end{pmatrix} ; \quad \gamma$$

Physical fields: W^+, Z, W^- and A (photon).

$$Z = W_3 \cos\theta_W - B \sin\theta_W$$

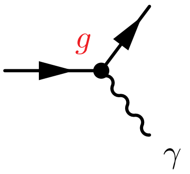
$$A = W_3 \sin\theta_W + B \cos\theta_W \qquad \theta_W \text{ Weak Mixing Angle}$$

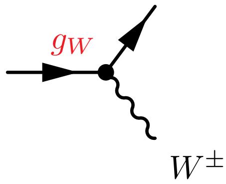
$W^{\pm}$, Z "acquire" mass via the Higgs mechanism.

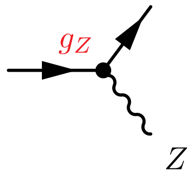
The GWS Model

The beauty of the GWS model is that it makes exact predictions of the $W^{\pm}$ and Z masses and of their couplings with only 3 free parameters.

Couplings given by α_{EM} and θ_W







$$\alpha_{EM} = \frac{e^2}{4\pi} \quad g \sim e \qquad g_W = \frac{e}{\sin\theta_W} \qquad g_Z = \frac{e}{\sin\theta_W \cos\theta_W} = \frac{g_W}{\cos\theta_W}$$

Masses also given by G_F and θ_W

From Fermi theory

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2\theta_W} \qquad m_{W^{\pm}} = \left(\frac{\sqrt{2}e^2}{8G_F\sin^2\theta_W}\right)^{1/2} \qquad m_Z = \frac{m_W}{\cos\theta_W}$$

If we know α_{EM}, G_F, $\sin\theta_W$ (from experiment), everything else is defined.

Example — mass relation *(non-examinable)*

- As a result of the mixing, we require that the mass eigenstates should be the Z and γ, and the mass of the photon be zero.
- We then compute the matrix elements of the mass operator:

$$m_Z^2 = \langle W_3\cos\theta_W - B\sin\theta_W|\hat{M}^2|W_3\cos\theta_W - B\sin\theta_W\rangle$$
$$= m_W^2\cos^2\theta_W + m_B^2\sin^2\theta_W - 2m_{WB}^2\cos\theta_W\sin\theta_W$$
$$m_\gamma^2 = \langle W_3\sin\theta_W + B\cos\theta_W|\hat{M}^2|W_3\sin\theta_W + B\cos\theta_W\rangle$$
$$= m_W^2\sin^2\theta_W + m_B^2\cos^2\theta_W + 2m_{WB}^2\cos\theta_W\sin\theta_W \quad = 0$$
$$m_{Z\gamma}^2 = \langle W_3\cos\theta_W - B\sin\theta_W|\hat{M}^2|W_3\sin\theta_W + B\cos\theta_W\rangle$$
$$= (m_W^2 - m_B^2)\sin\theta_W\cos\theta_W + m_{WB}^2(\cos^2\theta_W - \sin^2\theta_W) \quad = 0$$

- Solving these three equations gives

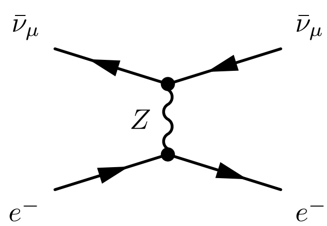
$$m_Z = \frac{m_W}{\cos\theta_W}$$

Couplings

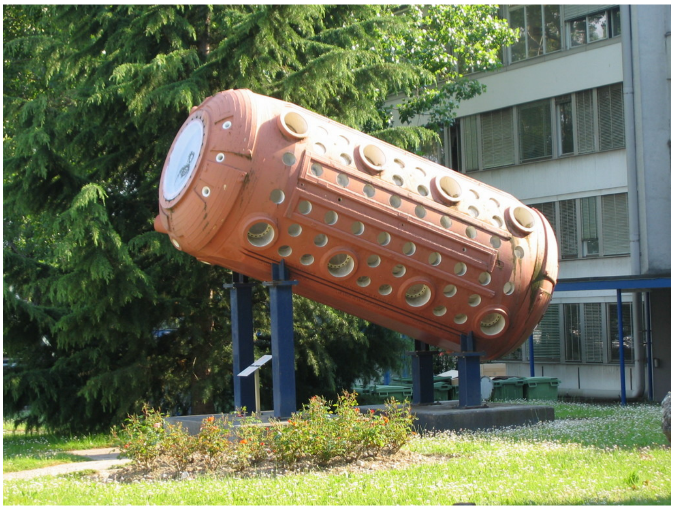
- Slightly simplified – see Part III for better treatment. Starting from
$$Z = W_3 \cos\theta_W - B \sin\theta_W$$
$$A = W_3 \sin\theta_W + B \cos\theta_W$$
- W_3 couples to I_3 with strength g_W and B couples to $Y = 2(Q - I_3)$ with g'
- So, coupling of A (photon) is
$$g_W I_3 \sin\theta_W + g' 2(Q - I_3) \cos\theta_W = Qe \qquad \text{for all } I_3$$
$$\Rightarrow g' = \frac{g_W \tan\theta_W}{2} \quad \text{and} \quad g' \cos\theta_W = \frac{e}{2} \qquad \Rightarrow g_W = \frac{e}{\sin\theta_W}$$
- The couplings of the Z are therefore
$$g_W I_3 \cos\theta_W - g' 2(Q - I_3) \sin\theta_W = \frac{e}{\sin\theta_W \cos\theta_W}\left[I_3 - Q\sin^2\theta_W\right]$$
$$= g_Z \left[I_3 - Q\sin^2\theta_W\right]$$
- For right-handed fermions, $I_3 = 0$, while for left-handed fermions $I_3 = +1/2(\nu, u, c, t)$ or $I_3 = -1/2(e^-, \mu^-, \tau^-, d', s', b')$; Q is charge in units of e

Evidence for GWS Model

- **Discovery of Neutral Currents (1973)**
  The process $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ was observed.
  Only possible Feynman diagram (no $W^\pm$ diagram).
  Indirect evidence for Z.

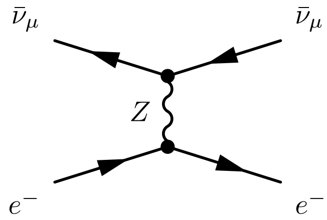


Gargamelle Bubble Chamber at CERN



Evidence for GWS Model

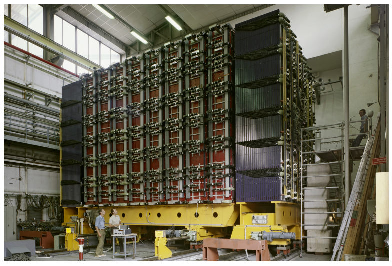
- **Discovery of Neutral Currents (1973)**
  The process $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ was observed.
  Only possible Feynman diagram (no $W^\pm$ diagram).
  Indirect evidence for Z.



- **Direct Observation of $W^\pm$ and Z (1983)**
  First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540$ GeV via decays into leptons

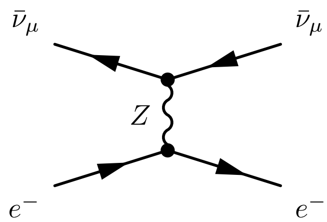
$$p\bar{p} \to W^\pm + X \qquad\qquad p\bar{p} \to Z + X$$
$$\hookrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu \qquad\qquad \hookrightarrow e^+e^-, \mu^+\mu^-$$

UA1 Experiment at CERN
Used Super Proton Synchrotron
(now part of LHC!)



Evidence for GWS Model

- **Discovery of Neutral Currents (1973)**
  The process $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ was observed.
  Only possible Feynman diagram (no $W^\pm$ diagram).
  Indirect evidence for Z.



- **Direct Observation of $W^\pm$ and Z (1983)**
  First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540$ GeV via decays into leptons

$$p\bar{p} \to W^\pm + X \qquad\qquad p\bar{p} \to Z + X$$
$$\hookrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu \qquad\qquad \hookrightarrow e^+e^-, \mu^+\mu^-$$

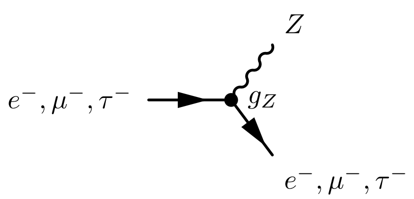
- **Precision Measurements of the Standard Model (1989-2000)**
  LEP e^+e^- collider provided many precision measurements of the Standard Model.

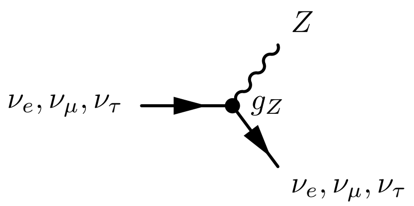
- Wide variety of different processes consistent with GWS model predictions and measure same value of

$$\sin^2\theta_W = 0.23113 \pm 0.00015 \qquad\qquad \theta_W \sim 29^\circ$$

The Weak NC Vertex

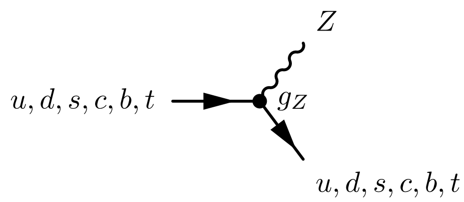
All weak neutral current interactions can be described by the Z boson propagator and the weak vertices:





The Standard Model Weak NC Lepton Vertex

+ antiparticles



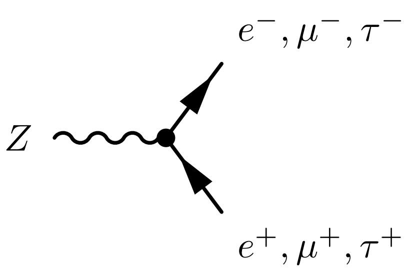
The Standard Model Weak NC Quark Vertex

+ antiparticles

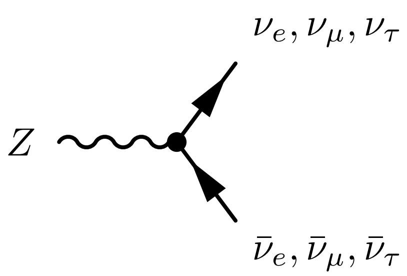
- Z never changes type of particle
- Z never changes quark or lepton flavour
- Z couplings are a mixture of EM and weak couplings, and therefore depend on $\sin^2\theta_W$.

Examples

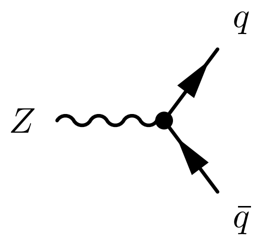
$Z \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$



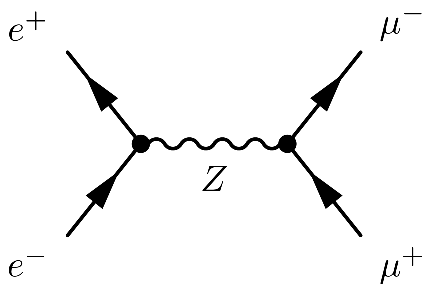
$Z \to \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$



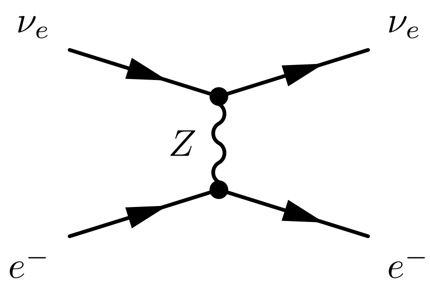
$Z \to q\bar{q}$



$e^+e^- \to \mu^+\mu^-$



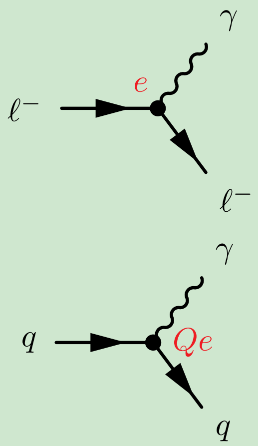
$\nu_e e^- \to \nu_e e^-$



Summary of Standard Model (matter) Vertices

Electromagnetic (QED)	Strong (QCD)	Weak CC	Weak NC

Electromagnetic (QED)

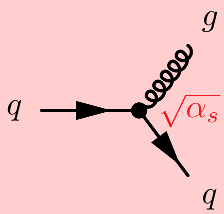


$$\alpha = \frac{e^2}{4\pi}$$

$q = u, d, s, c, b, t$

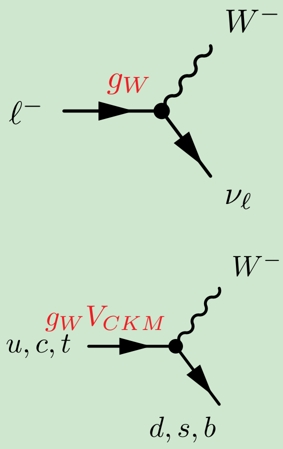
$+$ antiparticles

Strong (QCD)



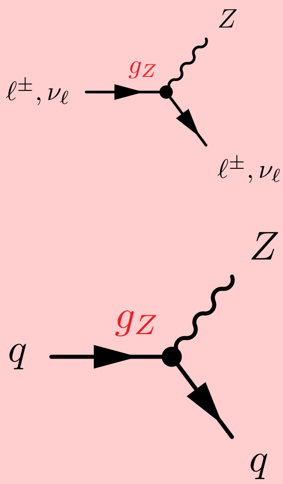
$$\alpha_s = \frac{g_s^2}{4\pi}$$

Weak CC



$$\alpha_W = \frac{g_W^2}{4\pi}$$

Weak NC



$$g_Z = \frac{g_W}{\cos\theta_W}$$

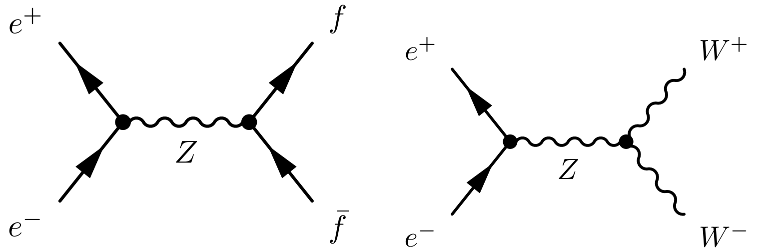
Feynman Diagrams *a reminder*

1. $\pi^- + p \to K^0 + \Lambda$
2. $\nu_\tau + e^- \to \nu_\tau + e^-$
3. $\bar{\nu}_\tau + \tau^- \to \bar{\nu}_\tau + \tau^-$
4. $D^+ \to K^- \pi^+ \pi^+$

Experimental Tests of the Electroweak model at LEP

The Large Electron Positron (LEP) collider at CERN provided high precision measurements of the Standard Model (1989-2000).

Designed as a Z and $W^{\pm}$ boson factory

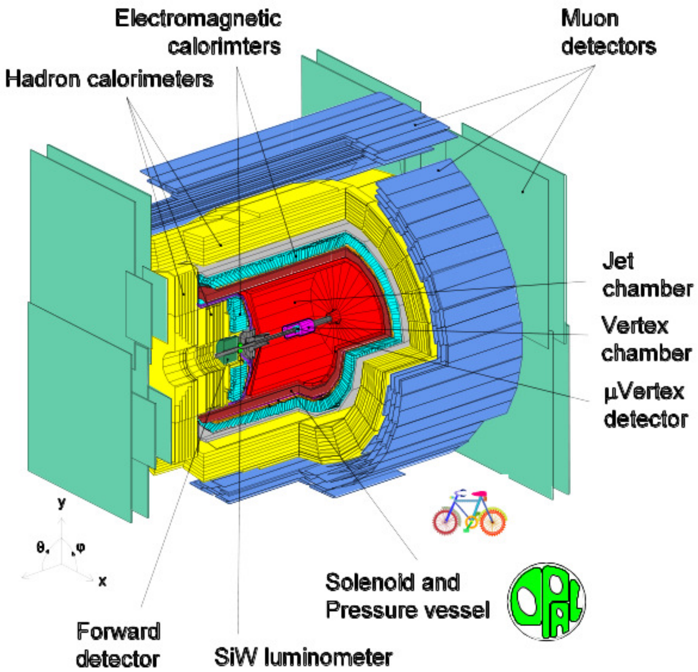


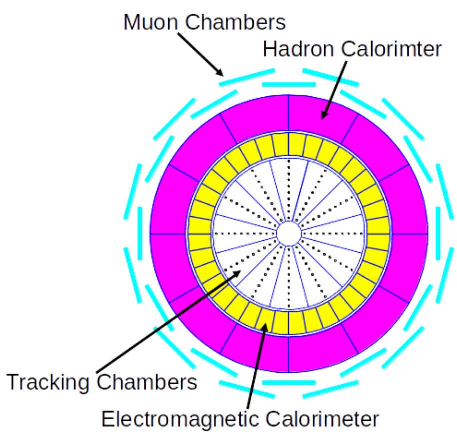
Precise measurements of the properties of Z and $W^{\pm}$ bosons provide the most stringent test of our current understanding of particle physics.

- LEP is the highest energy e^+e^- collider ever built $\sqrt{s} = 90 - 209\ \mathrm{GeV}$
- Large circumference, 27 km
- 4 experiments combined saw 16×10^6 Z events, 30×10^3 $W^{\pm}$ events

OPAL: a LEP detector

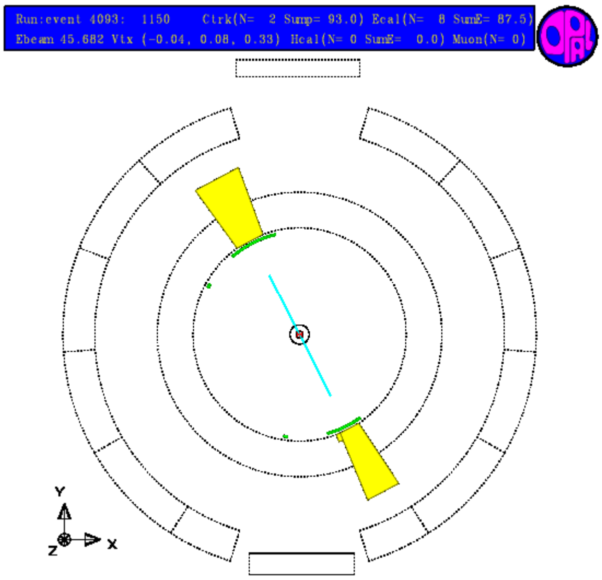
OPAL was one of the 4 experiments at LEP. Size: 12 m $\times$ 12 m $\times$ 15 m.



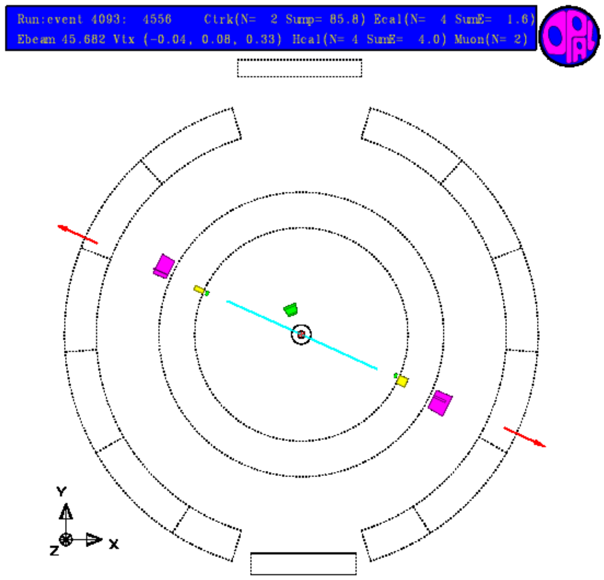


Typical $e^+e^- \to Z$ events

$e^+e^- \to Z \to e^+e^-$

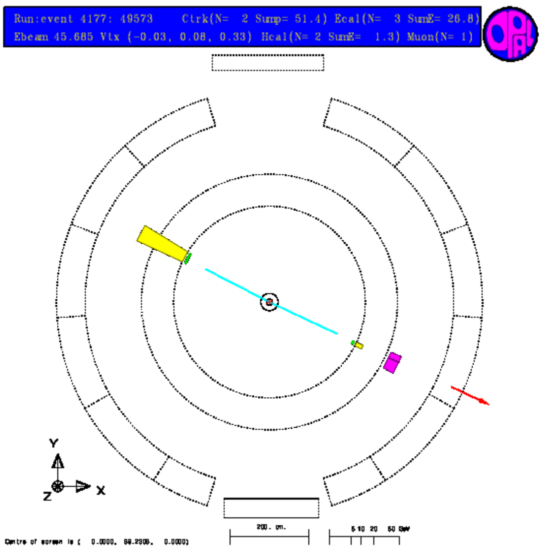


$e^+e^- \to Z \to \mu^+\mu^-$



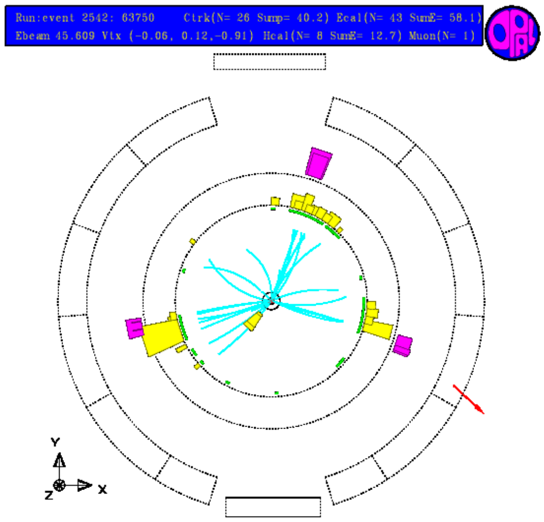
Typical $e^+e^- \to Z$ events

$e^+e^- \to Z \to \tau^+\tau^-$



Taus decay within the detector (lifetime $\sim 10^{-13}$ s).
Here $\tau^- \to e^-\bar{\nu}_e\nu_\tau$, $\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau$

$e^+e^- \to Z \to q\bar{q}$

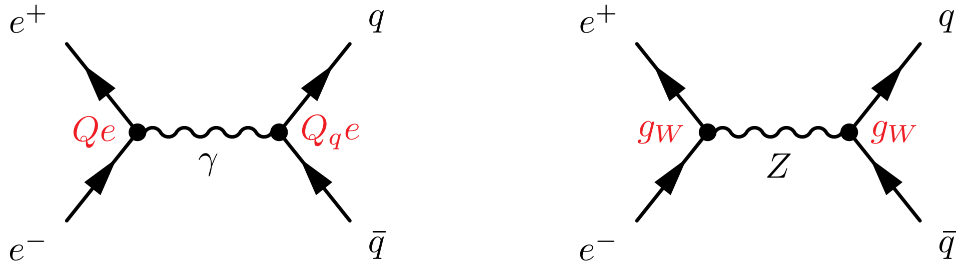


3-jet event (gluon emitted by $q/\bar{q}$)

The Z Resonance

Consider the process $e^+e^- \to q\bar{q}$

- At small $\sqrt{s}(< 50\ \mathrm{GeV})$, we only considered an intermediate photon
- At higher energies, the Z exchange diagram contributes ($+Z\gamma$ interference)

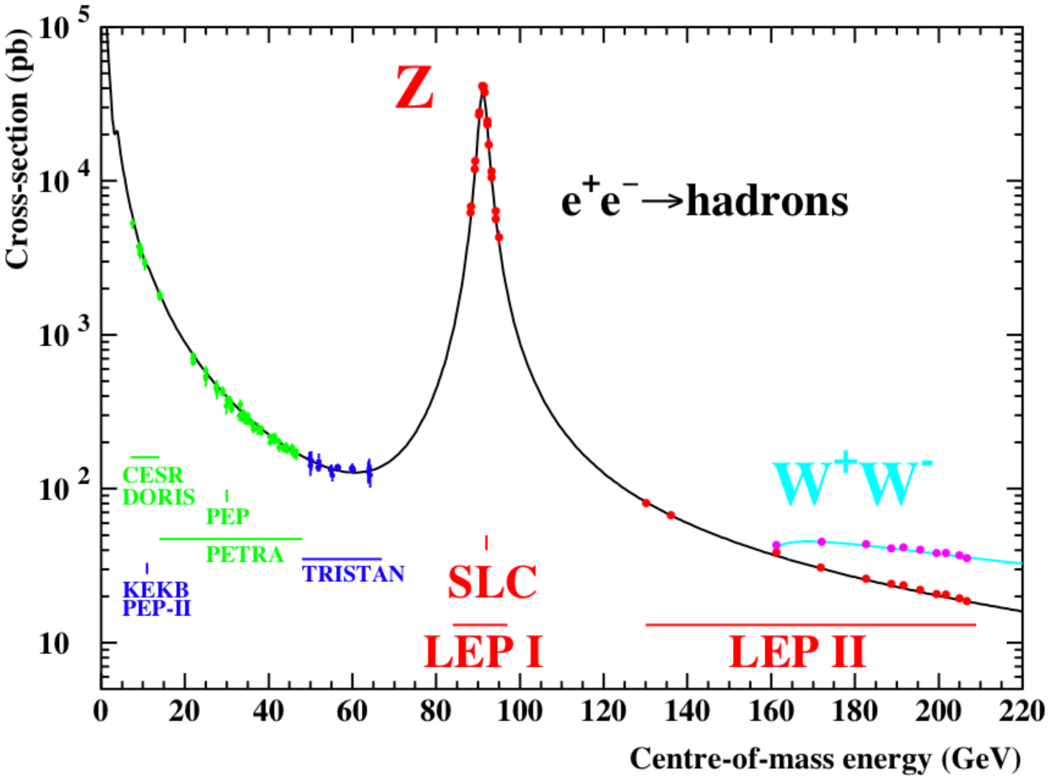


$$\sigma(e^+e^- \to \gamma \to q\bar{q}) = \frac{4\pi\alpha^2}{3s}\sum 3Q_q^2$$

- The Z is a decaying intermediate massive state (lifetime $\sim 10^{-25}$ s)

  $\Rightarrow$ Breit-Wigner resonance
- Around $\sqrt{s} \sim m_Z$, the Z diagram dominates

The Z Resonance



The Z Resonance

Breit-Wigner cross-section for $e^+e^- \to Z \to f\bar{f}$ (where $f\bar{f}$ is any fermion-antifermion pair)

Centre-of-mass energy $\sqrt{s} = E_{CM} = E_{e^+} + E_{e^-}$

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{g\pi}{E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - m_Z)^2 + \frac{\Gamma_Z^2}{4}}$$

with $\quad g = \dfrac{2J_Z + 1}{(2J_{e^-}+1)(2J_{e^+}+1)} = \dfrac{3}{4} \qquad J_Z = 1;\ J_{e^\pm} = \dfrac{1}{2}$

giving

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{3\pi}{4E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - m_Z)^2 + \frac{\Gamma_Z^2}{4}} = \frac{3\pi}{s} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(\sqrt{s} - m_Z)^2 + \frac{\Gamma_Z^2}{4}}$$

Γ_Z is the total decay width, i.e. the sum over the partial widths for different decay modes

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$$

The Z Resonance

At the peak of the resonance $\sqrt{s} = m_Z$:

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Hence, for all fermion/antifermion pairs in the final state

$$\sigma(e^+e^- \to Z \to \text{anything}) = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}}{\Gamma_Z} \qquad \Gamma_{f\bar{f}} = \Gamma_Z$$

Compare to the QED cross-section at $\sqrt{s} = m_Z$

$$\sigma_{\text{QED}} = \frac{4\pi\alpha^2}{3s}$$

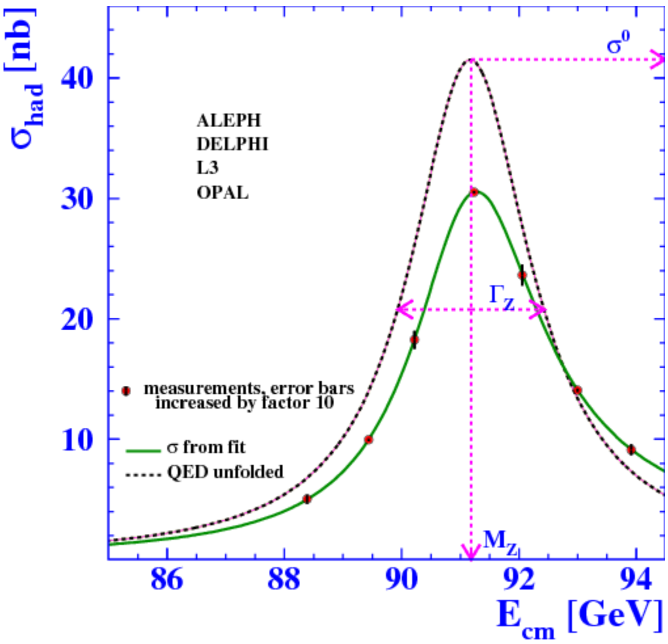
$$\frac{\sigma(e^+e^- \to Z \to \text{anything})}{\sigma_{\text{QED}}} = \frac{9}{\alpha^2}\frac{\Gamma_{ee}}{\Gamma_Z} \sim 5700$$

$\Gamma_{ee} = 85\ \text{MeV}, \quad \Gamma_Z = 2.5\ \text{GeV}, \quad \alpha = 1/137$

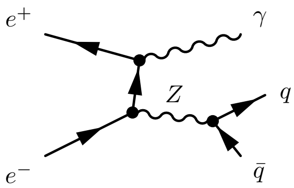
Measurement of m_Z and Γ_Z

- Run LEP at various centre-of-mass energies ($\sqrt{s}$) close to the peak of the Z resonance and measure $\sigma(e^+e^- \to q\bar{q})$
- Determine the parameters of the resonance:

  Mass of the Z, m_Z
  Total decay width, Γ_Z
  Peak cross-section, σ^0



One subtle feature: need to correct measurements for QED effects due to radiation from the e^+e^- beams. This radiation has the effect of reducing the centre-of-mass energy of the e^+e^- collision which smears out the resonance.

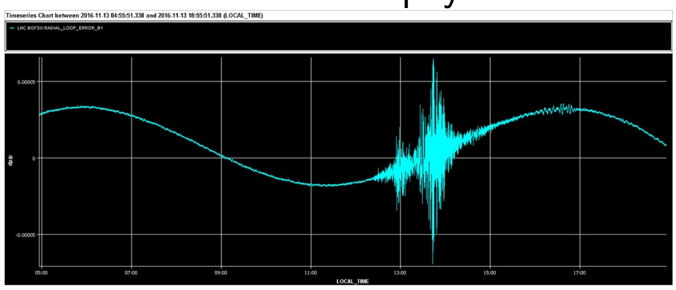


Measurement of m_Z and Γ_Z

m_Z was measured with precision 2 parts in 10^5

- Need a detailed understanding of the accelerator and astrophysics.

Tidal distortions of the Earth by the Moon cause the rock surrounding LEP to be distorted – changing the radius by 0.15 mm (total 4.3 km). This is enough to change the centre-of-mass energy.



LHC ring is stretched by 0.1mm by the 7.5 magnitude earthquake in New Zealand, Nov 2016. Tidal forces can also be seen.

- Also need a train timetable.

  Leakage currents from the TGV rail via Lake Geneva follow the path of least resistance... using LEP as a conductor.

Accounting for these effects (and many others):

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\sigma^0_{q\bar{q}} = 41.450 \pm 0.037 \text{ nb}$$

Number of Generations

- Currently know of three generations of fermions. Masses of quarks and leptons increase with generation. Neutrinos are approximately massless (or are they?)

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \qquad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

- Could there be more generations? e.g. $\begin{pmatrix} t' \\ b' \end{pmatrix} \quad \begin{pmatrix} L \\ \nu_L \end{pmatrix}$
- The Z boson couples to all fermions, including neutrinos. Therefore, the total decay width, Γ_Z, has contributions from all fermions with $m_f < m_Z/2$

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$$

$$\text{with} \quad \Gamma_{\nu\bar{\nu}} = \Gamma_{\nu_e\bar{\nu}_e} + \Gamma_{\nu_\mu\bar{\nu}_\mu} + \Gamma_{\nu_\tau\bar{\nu}_\tau}$$

- If there were a fourth generation, it seems likely that the neutrino would be light, and, if so would be produced at LEP $\quad e^+e^- \to Z \to \nu_L\bar{\nu}_L$
- The neutrinos would not be observed directly, but could infer their presence from the effect on the Z resonance curve.

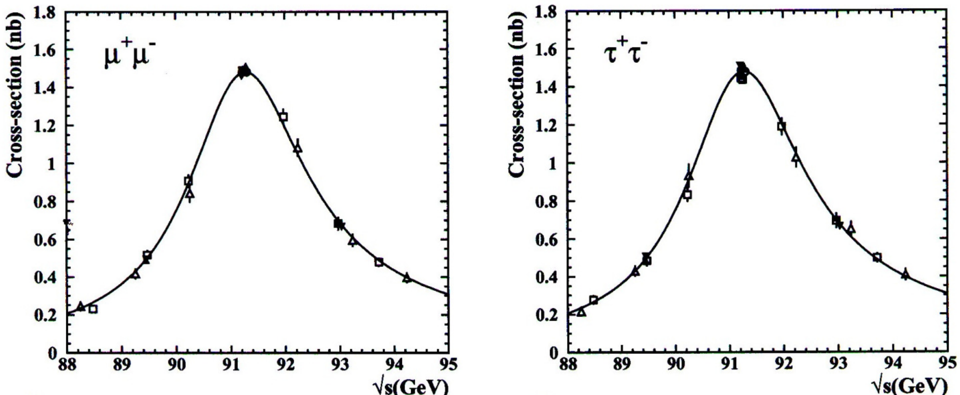
Number of Generations

At the peak of the Z resonance, $\sqrt{s} = m_Z \qquad \sigma^0_{f\bar{f}} = \dfrac{12\pi}{m_Z^2}\dfrac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$

A fourth generation neutrino would increase the Z decay rate and thus increase Γ_Z. As a result, a decrease in the measured peak cross-sections for the visible final states would be observed.

Measure the $e^+e^- \to Z \to f\bar{f}$ cross-sections for all visible decay models (i.e. all fermions apart from $\nu\bar{\nu}$)

Examples: $e^+e^- \to \mu^+\mu^- \qquad e^+e^- \to \tau^+\tau^-$



Number of Generations

- Have already measured m_Z and Γ_Z from the shape of the Breit-Wigner resonance. Therefore, obtain $\Gamma_{f\bar{f}}$ from the peak cross-sections in each decay mode using
$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$
Note, obtain Γ_{ee} from $\sigma^0_{ee} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}^2}{\Gamma_Z^2}$

- Can relate the partial widths to the measured total width (from the resonance curve)
$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + N_\nu \Gamma_{\nu\nu}$$
where N_ν is the number of neutrino species and $\Gamma_{\nu\nu}$ is the partial width for a single neutrino species.

Number of Generations

The difference between the measured value of Γ_Z and the sum of the partial widths for visible final states gives the invisible width $N_\nu \Gamma_{\nu\nu}$

Γ_Z	**2495.2±2.3 MeV**
Γ_{ee}	83.91±0.12 MeV
$\Gamma_{\mu\mu}$	83.99±0.18 MeV
$\Gamma_{\tau\tau}$	84.08±0.22 MeV
Γ_{qq}	1744.4±2.0 MeV
$N_\nu\Gamma_{\nu\nu}$	499.0±1.5 MeV

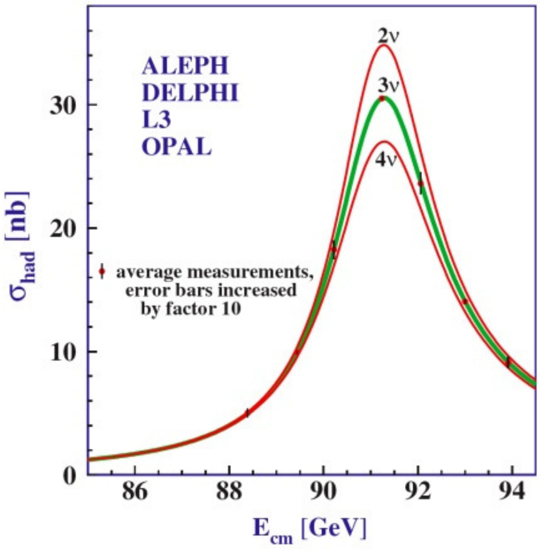
In the Standard Model, calculate $\Gamma_{\nu\nu} \sim 167\ \mathrm{MeV}$

Therefore
$$N_\nu = \frac{\Gamma_{\nu\nu}^{\mathrm{measured}}}{\Gamma_{\nu\nu}^{\mathrm{SM}}} = 2.984 \pm 0.008$$

$\Rightarrow$ three generations of light neutrinos for $m_\nu < m_Z/2$

Number of Generations

Most likely that only 3 generations of quarks and leptons exist

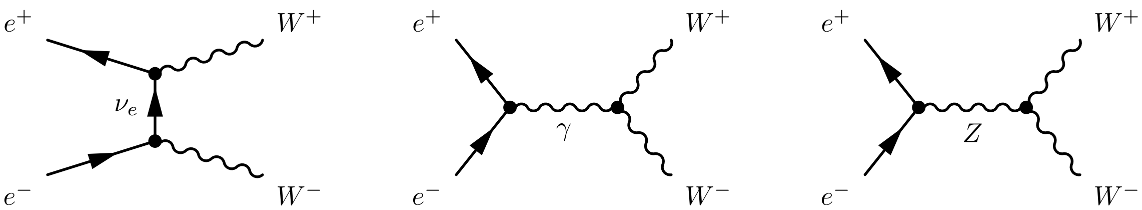


In addition

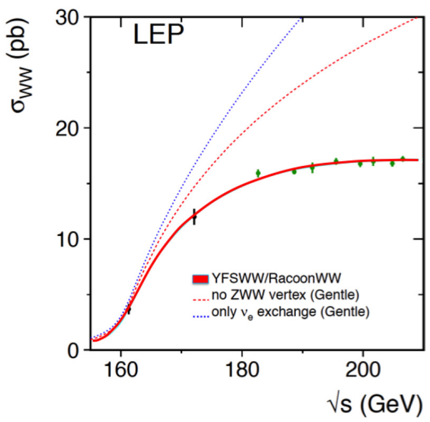
- Γ_{ee}, $\Gamma_{\mu\mu}$, $\Gamma_{\tau\tau}$ are consistent $\Rightarrow$ tests universality of the lepton couplings to the Z boson.
- Γ_{qq} is consistent with the expected value which assumes 3 colours – further evidence for colour

W^+W^- at LEP

- In e^+e^- collisions W bosons are produced in pairs.
- Standard Model: 3 possible diagrams:

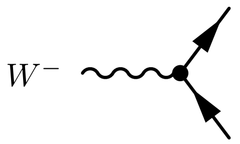


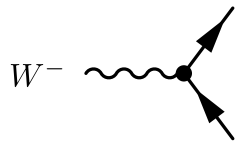
- LEP operated above the threshold for W^+W^- production (1996-2000) $\sqrt{s} > 2m_W$
- Cross-section sensitive to the presence of the Triple Gauge Boson vertex



W^+W^- at LEP

In the Standard Model $W\ell\nu$ and $Wq\bar{q}$ couplings are $\sim$ equal.





$m_W < m_t$

$\times 3$ for colour

Expect (assuming 3 colours)

$$B(W^\pm \to q\bar{q}) = \frac{6}{9} = \frac{2}{3}$$

$$B(W^\pm \to \ell\nu) = \frac{3}{9} = \frac{1}{3}$$

QCD corrections $\sim \left(1 + \frac{\alpha_s}{\pi}\right)$

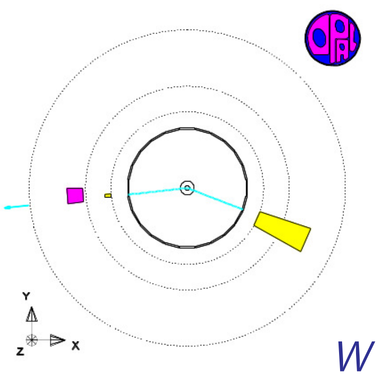
$$\Rightarrow B(W^\pm \to q\bar{q}) = 0.675$$

Measured BR

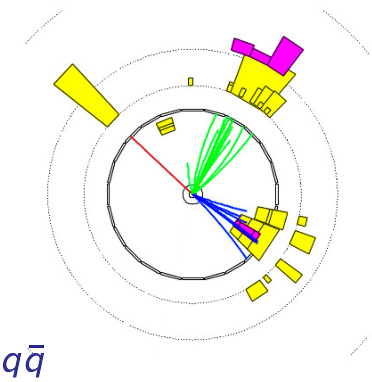
$W^+W^- \to \ell\nu\ell\nu$	10.5%
$W^+W^- \to q\bar{q}\ell\nu$	43.9%
$W^+W^- \to q\bar{q}q\bar{q}$	45.6%

W^+W^- events in OPAL

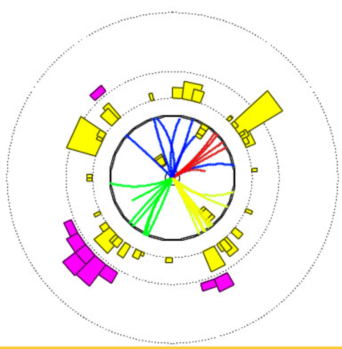
$W^+W^- \to e\nu\mu\nu$



$W^+W^- \to q\bar{q}e\nu$



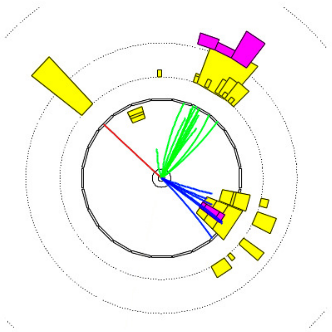
$W^+W^- \to q\bar{q}q\bar{q}$

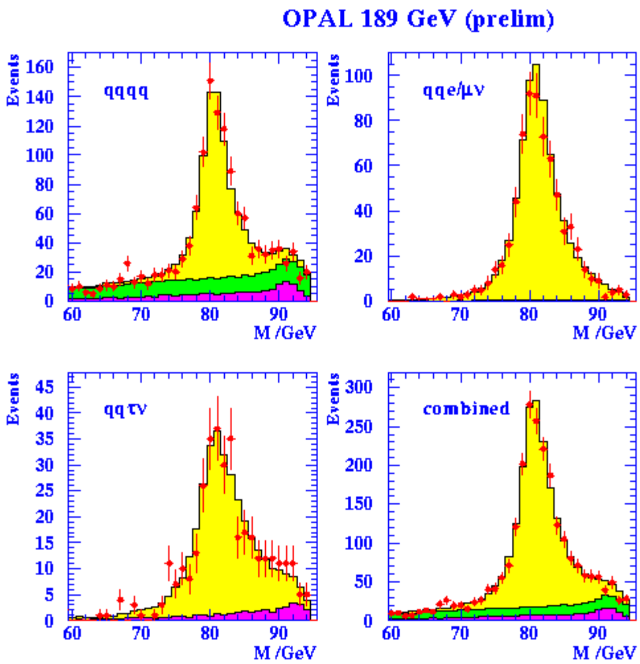


Measurement of m_W and Γ_W

Unlike $e^+e^- \to Z$, W boson production at LEP was not a resonant process.

m_W was measured by measuring the invariant mass in each event





4-momenta $p_{q1}, p_{q2}, p_e, p_\nu$

$m_W = \frac{1}{2}(m_{q\bar{q}} + m_{\ell\nu})$

$$m_W = 80.423 \pm 0.038 \text{ GeV}$$

$$\Gamma_W = 2.12 \pm 0.11 \text{ GeV}$$

W Boson Decay Width

In the Standard Model, the W boson decay width is given by

$$\Gamma(W^- \to e^-\bar{\nu}_e) = \frac{g_W^2 m_W}{48\pi} = \frac{G_F m_W^3}{6\sqrt{2}\pi}$$

μ-decay: $G_F = 1.166 \times 10^{-5}\text{ GeV}^{-2}$ LEP: $m_W = 80.423 \pm 0.038 \text{ GeV}$

$$\Rightarrow \Gamma(W^- \to e^-\bar{\nu}_e) = 227 \text{ MeV}$$

Total width is the sum over all partial widths:

$$W^- \to e^-\bar{\nu}_e,\ \mu^-\bar{\nu}_\mu,\ \tau^-\bar{\nu}_\tau,$$
$$W^- \to d'\bar{u},\ s'\bar{c}, \qquad \times 3 \text{ for colour}$$

If the W coupling to leptons and quarks is equal and there are 3 colours:

$$\Gamma = \sum_i \Gamma_i = (3 + 2 \times 3)\Gamma(W^- \to e^-\bar{\nu}_e) \sim 2.1 \text{ GeV}$$

Compare with measured value from LEP: $\Gamma_W = 2.12 \pm 0.11 \text{ GeV}$

- Universal coupling constant
- Yet more evidence for colour!

Summary of Electroweak Tests

Now have 5 precise measurements of fundamental parameters of the Standard Model

$$\alpha_{EM} = 1/(137.03599976 \pm 0.00000050) \qquad \text{(at } q^2 = 0 \text{)}$$

$$G_F = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$m_W = 80.385 \pm 0.015\ \mathrm{GeV}$$

$$m_Z = 91.1875 \pm 0.0021\ \mathrm{GeV}$$

$$\sin^2\theta_W = 0.23143 \pm 0.00015$$

In the Standard Model, only 3 are independent.

The measurements are consistent, which is an incredibly powerful test of the Standard Model of Electroweak Interactions.

Summary

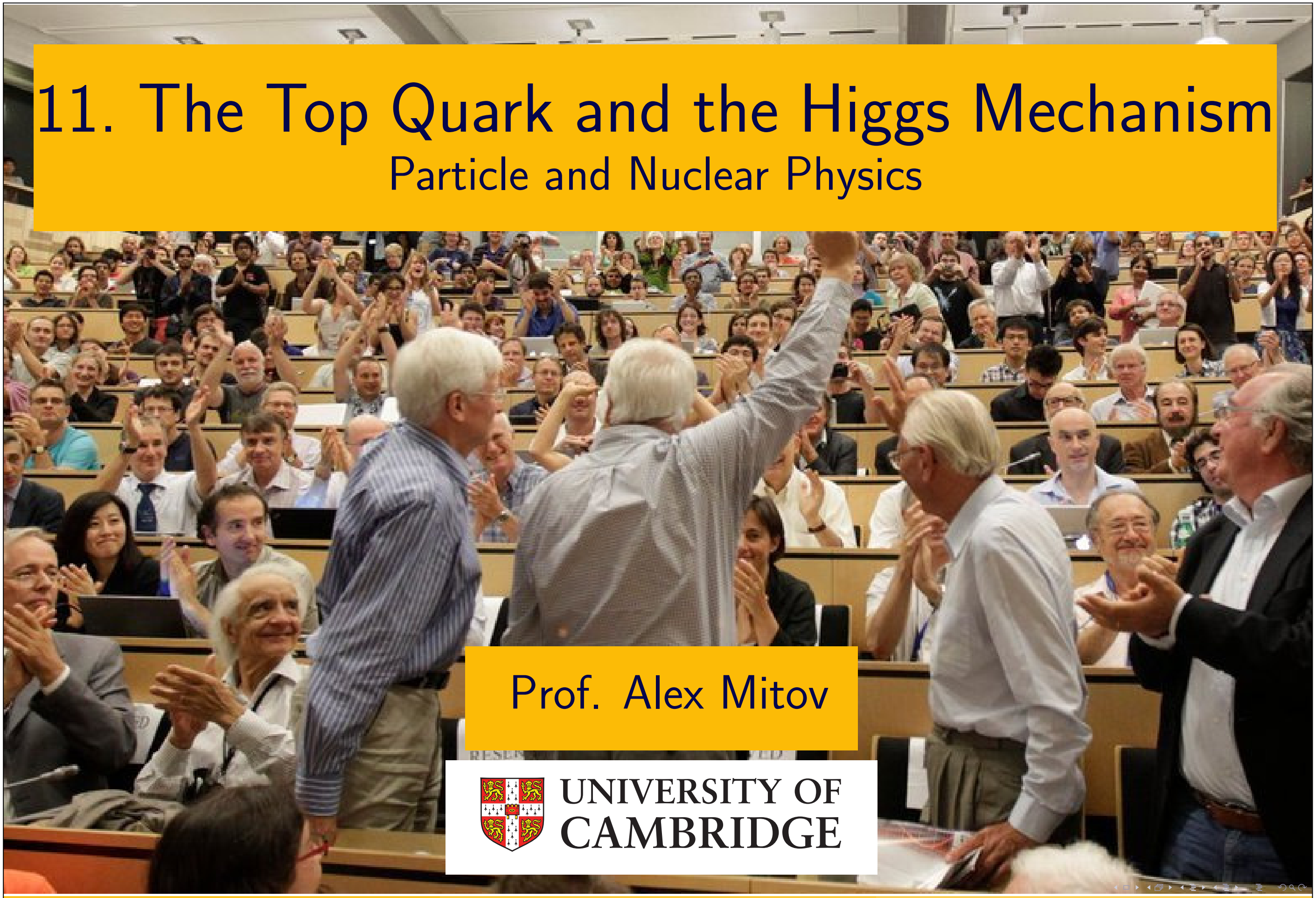
- Weak interaction with $W^{\pm}$ fails at high energy.
- Introduction of unified theory involving and relating Z and γ can resolve the divergences.
- One new parameter, θ_W, allows predictions of Z couplings and mass relations.
- Extensively and successfully tested at LEP.

Problem Sheet: q.26-27

Up next...
Section 11: The Top Quark and the Higgs Mechanism

11. The Top Quark and the Higgs Mechanism

Particle and Nuclear Physics



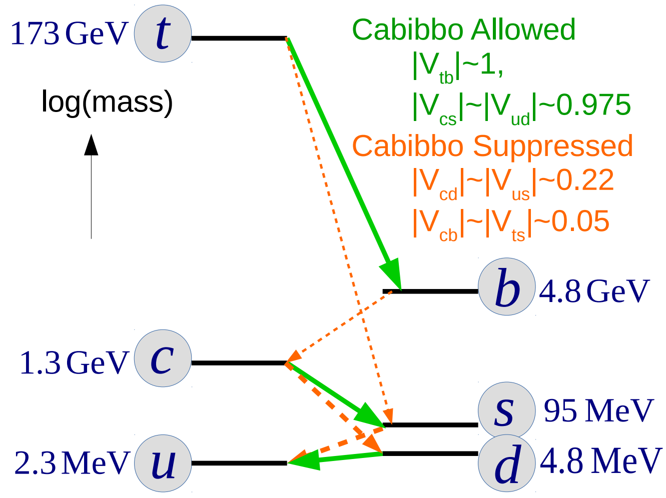
Prof. Alex Mitov

UNIVERSITY OF CAMBRIDGE

In this section...

- Focus on the most recent discoveries of fundamental particles
- The top quark – prediction & discovery
- The Higgs mechanism
- The Higgs discovery

Third Generation Quark Weak CC Decays

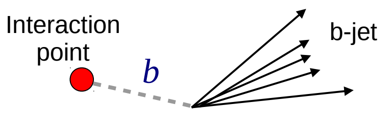


Top quarks are special.

- $m(t) \gg m(b)$ $(> m(W))$
- $\tau_t \sim 10^{-25}\,\mathrm{s} \Rightarrow$ decays before hadronisation
- $V_{tb} \sim 1 \Rightarrow$ BR$(t \to W + b)$=100%

Bottom quarks are also special.

- b quarks can only decay via the Cabbibo suppressed Wcb vertex. V_{cb} is very small – weak coupling!
  $\Rightarrow \tau(b) \gg \tau(u, c, d, s)$
- Jet initiated by b quarks look different to other jets. b quarks travel further from interaction point before decaying. b-jet traces back to a secondary vertex – "b-tagging".



The Top Quark *(non-examinable)*

The Standard Model predicted the existence of the top quark

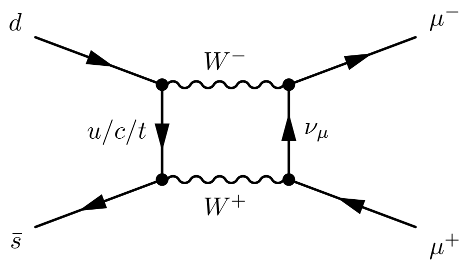
$$\begin{matrix} +\frac{2}{3}e \\ -\frac{1}{3}e \end{matrix} \quad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

which is required to explain a number of observations.

Example: Non-observation of the decay

$K^0 \to \mu^+\mu^-$ $\qquad B(K^0 \to \mu^+\mu^-) < 10^{-9}$

The top quark cancels the contributions from the u and c quarks.



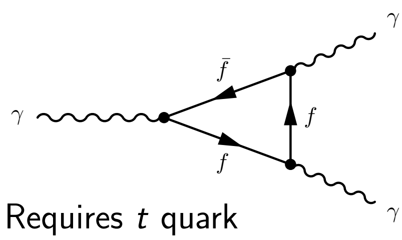
Example: Electromagnetic anomalies

This diagram leads to infinities in the theory unless

$\sum Q_f = 0$

where the sum is over all fermions (and colours)

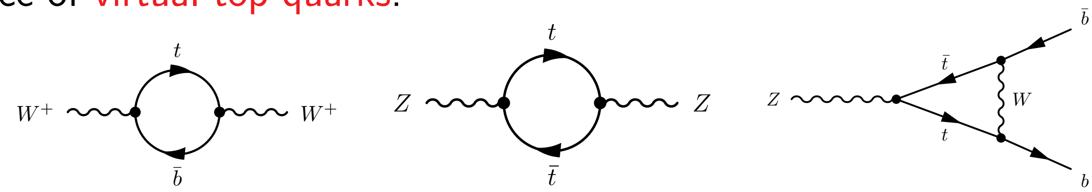
$$\sum_f Q_f = [3 \times (-1)] + \left[3 \times 3 \times \frac{2}{3}\right] + \left[3 \times 3 \times (-\frac{1}{3})\right] = 0$$



Requires t quark

The Top Quark

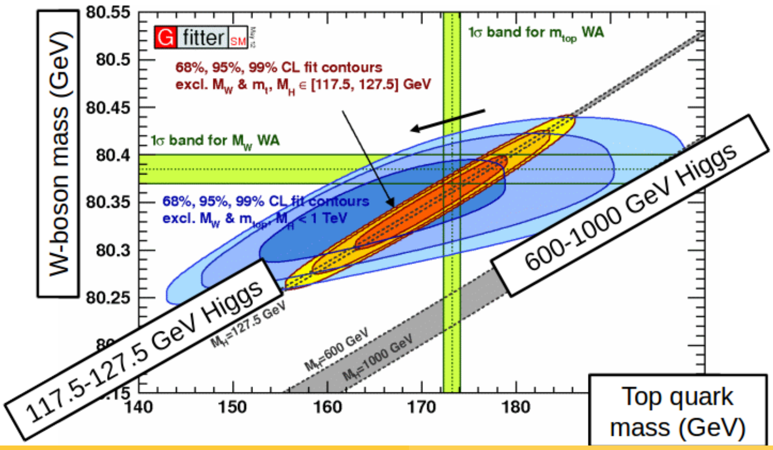
The top quark is too heavy for $Z \to t\bar{t}$ or $W^+ \to t\bar{b}$ so not directly produced at LEP.
However, precise measurements of m_Z, m_W, Γ_Z and Γ_W are sensitive to the existence of virtual top quarks:



Example

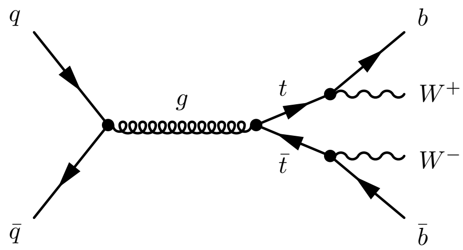
Standard Model prediction

Also depends on the Higgs mass



The Top Quark

- The top quark was discovered in 1994 by the CDF experiment at the worlds (then) highest energy pp collider ($\sqrt{s} = 1.8\ \mathrm{TeV}$), the Tevatron at Fermilab, US.



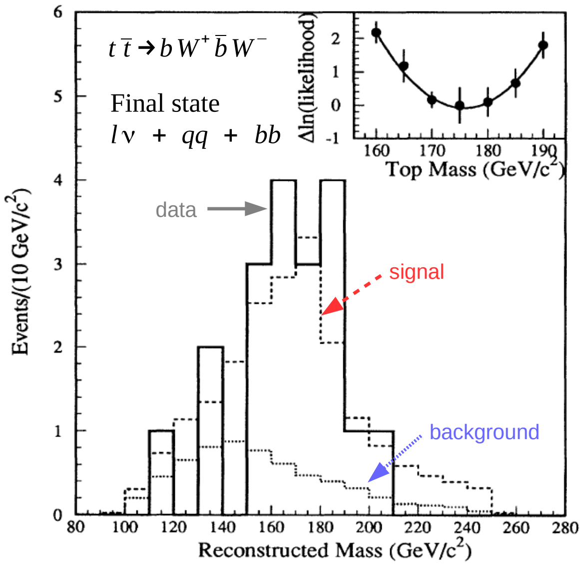
Final state $W^+W^-b\bar{b}$
Mass reconstructed in a similar manner to m_W at LEP, i.e. measure jet/lepton energies/momenta.

- $V_{tb} \sim 1$, so decay of top quark is ~100% $t \to bW^+$
- $m_t \gg m_W$, so the W^+ is real. The weak decay is just as fast as a strong decay ($\sim 10^{-25}$s), so the quark has no time to hadronise

  $\Rightarrow$ there are no t-hadrons
- Possible top quark decays are $t \to bq\bar{q}$ or $t \to b\ell\nu_\ell$
- In hadron collisions, multijet final states are the norm – for rare processes it's much easier to look for leptonic decays, accompanied by b-quark jets.

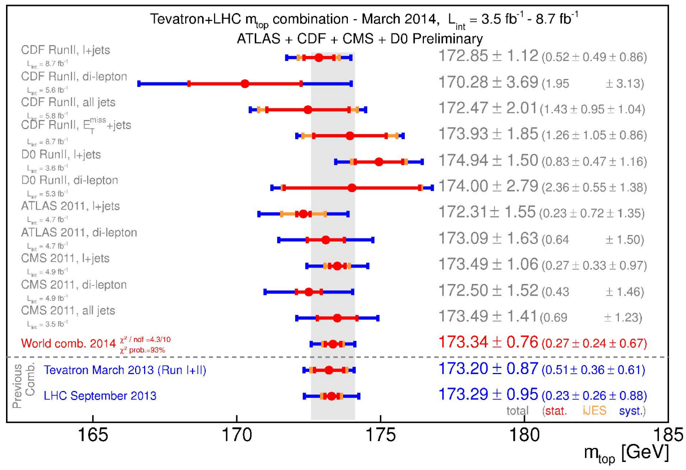
First observation of top (1995)

CDF collaboration published first PRL **74** 2626 (1995)



Current status

Results from LHC as well as Tevatron. All consistent, and in agreement with indirect expectation from LEP data.

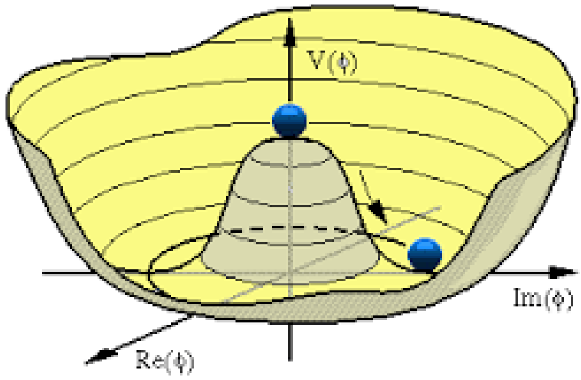


Higgs mechanism and the Higgs Boson

- Recall – the Klein-Gordon equation for massive bosons is:
$$\frac{\partial^2\psi}{\partial t^2} = \left(\nabla^2 - m^2\right)\psi$$
- However, the term $m^2\psi$ (or $\frac{1}{2}m^2\psi^2$ in the Lagrangian formulation), is not gauge invariant.
- So in gauge field theories, the gauge bosons should be massless. OK for QED and QCD, but plainly not for $W^\pm$ and Z.
- The Higgs mechanism tries to fix this. Imagine introducing a scalar Higgs field ϕ, which has interactions with the $W^\pm$ and Z fields, with coupling strength y, giving a term in Lagrangian $y\phi\psi\psi$.
- Looks like a mass term ($\propto \psi^2$). Mass of the bosons becomes effectively related to their coupling to the Higgs field.
- Requires the vacuum (lowest energy state of space) to have a non-zero expectation value for the Higgs field. How can this come about?

Higgs potential

- Suppose the Higgs field ϕ (actually a complex doublet) has self interactions yielding
$$V(\phi) = a\phi^4 - b\phi^2$$
- The equilibrium point, $\phi = 0$, respects the symmetry, but is unstable.



- The stable equilibrium point is at $|\phi^2_{\rm GS}| = b/2a$. The symmetry is "spontaneously broken".
- A weak boson propagating in the Higgs field will appear to have a mass $\sim y\phi_{\rm GS}$.
- Expanding about the ground state $V(\phi_{\rm GS} + x) = V_{\rm min} + 2bx^2$
- So can get excitations of the Higgs field about the minimum. These form the physical Higgs scalar boson, H – the observable physical manifestation of the operation of the Higgs mechanism.

Classical analogue of the Higgs mechanism

(non-examinable)

- Maxwell's equations lead to waves travelling at velocity c, hence to massless photons.
- Consider waves propagating in a charged plasma, with electron density n per unit volume.

Plasma: $\vec{J} = ne\vec{v}; \quad m_e\dfrac{\partial \vec{v}}{\partial t} = e\vec{E} \quad \Rightarrow \dfrac{\partial \vec{J}}{\partial t} = \dfrac{ne^2\vec{E}}{m_e}$

Maxwell:

$$\vec{\nabla}\wedge\vec{\nabla}\wedge\vec{E} = -\nabla^2\vec{E} = \vec{\nabla}\wedge\left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial \vec{\nabla}\wedge\vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left(\mu_0\vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}\right)$$

$$= -\frac{\mu_0 ne^2\vec{E}}{m_e} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} \qquad \Rightarrow \nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \frac{\mu_0 ne^2\vec{E}}{m_e}$$

- Compare with Klein-Gordon. Photon propagates with effective mass

$$m^2_{\rm eff} = \frac{\hbar\mu_0 ne^2}{m_e c^2}$$

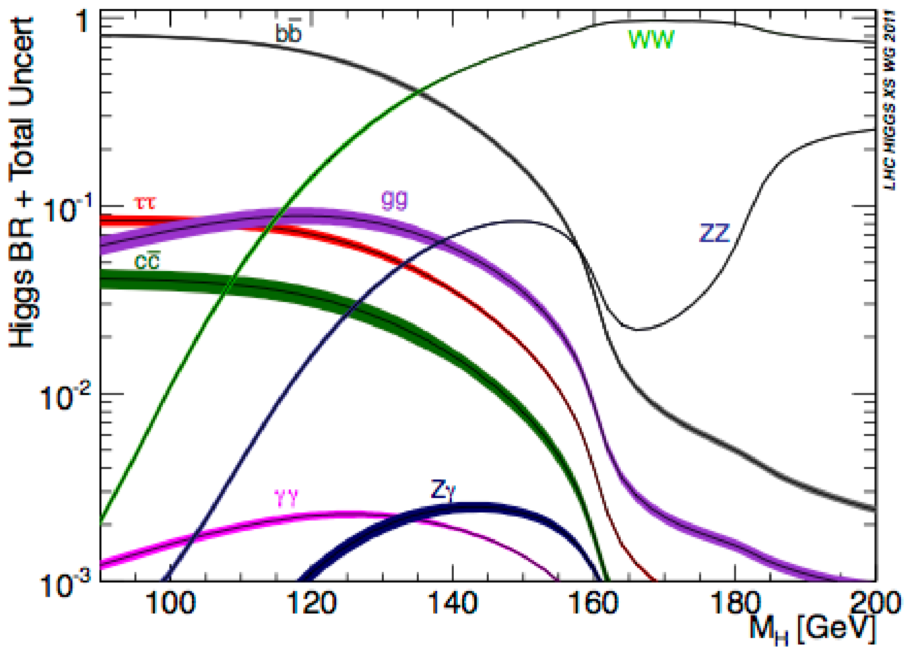
Note $m_{\rm eff} \propto e$, the coupling.

Higgs theory summary

- Gauge bosons (and also fermions) are intrinsically massless, and need to be so to satisfy Gauge Invariance.
- Nevertheless, interactions with the Higgs field make particles look like they have mass.
- Apparent masses are controlled by free parameters called Yukawa Couplings (the strength of the coupling to the Higgs field)
- A Higgs Boson arises as an excitation of the Higgs field. It must be a scalar particle to make everything work.
- The Higgs Boson has a mass, but the mass is not predicted by the theory – we have to find it experimentally.
- The Higgs Boson has couplings to all the particles to which it gives mass (and so has many ways it could decay), all fully calculable and determined by the theory as a function of its (a priori unknown) mass.

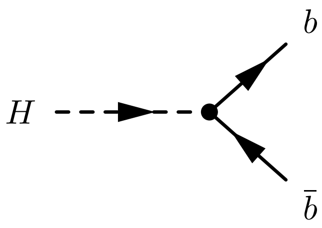
Higgs boson decays

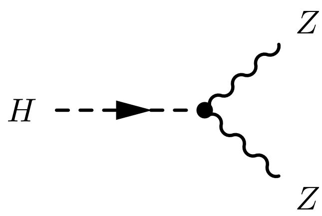
- Higgs Boson interacts via couplings which are proportional to masses.
- Higgs boson therefore decays preferentially to the heaviest particles that are kinematically accessible, depending on its mass.



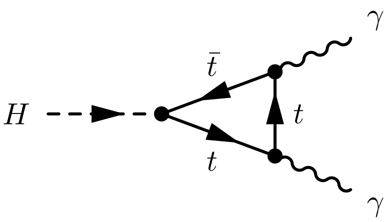
Higgs decay mechanisms

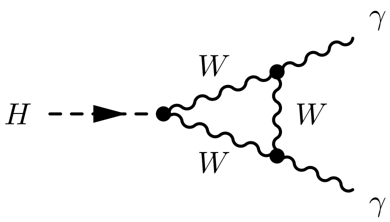
Directly to two fundamental fermions or bosons, coupling to mass, e.g.

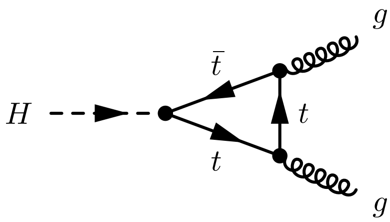




Indirectly to massless particles (photons or gluons) via massive loops



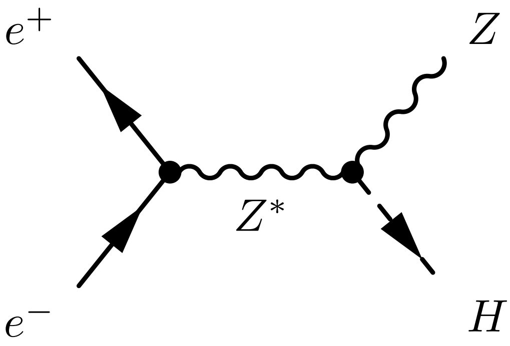




Higgs at LEP

Higgs Production at LEP (Large Electron Positron Collider – 1990s):

If $m_H < \sqrt{s} - m_Z$



"Higgsstrahlung" mechanism

In 2000, LEP operated with $\sqrt{s} \sim 207$ GeV, therefore had the potential to discover Higgs boson if $m_H < 116$ GeV.

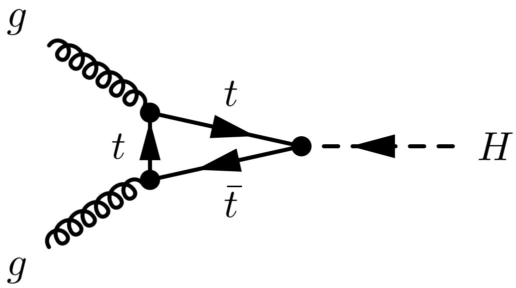
Searches were conducted in many possible final states (different decays for Z and H). All negative.

Ultimately, LEP excluded a Higgs Boson with a mass below 114 GeV.

Higgs at Large Hadron Collider

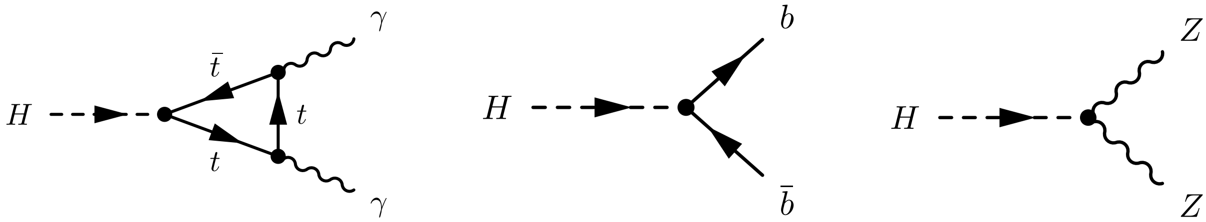
Higgs Production at the LHC

The dominant Higgs production mechanism at the LHC is



"Gluon fusion"

Higgs Decay at the LHC



Low mass

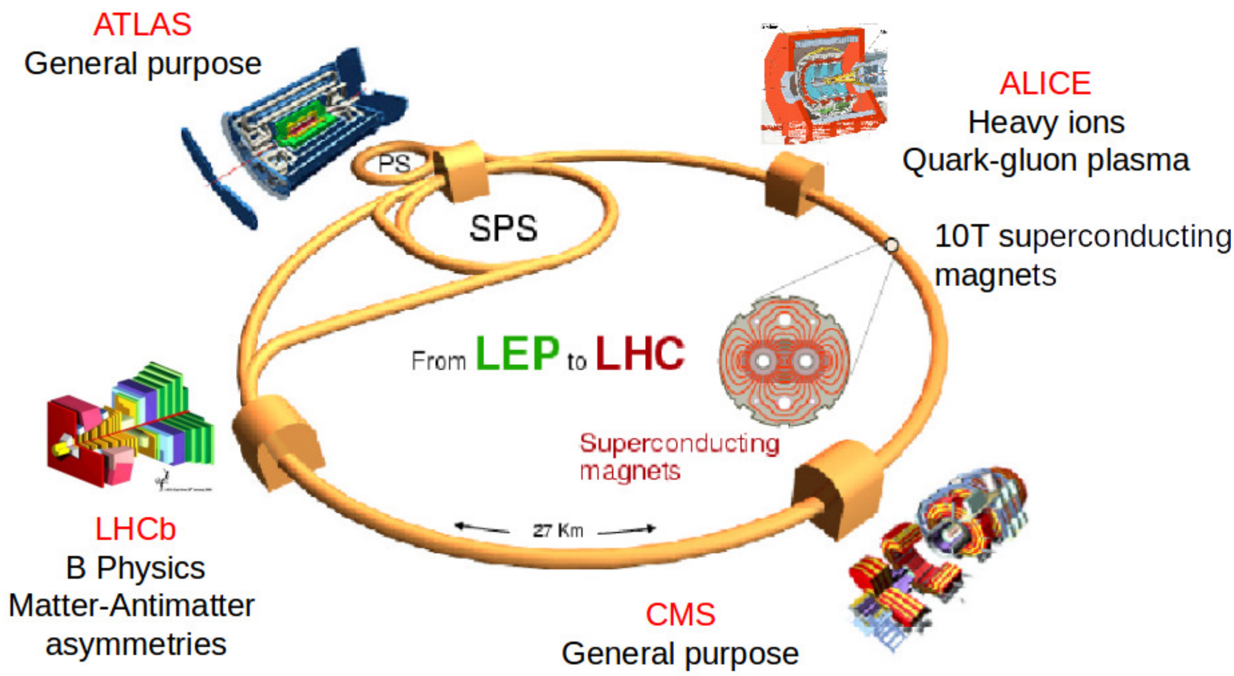
Medium mass

High mass

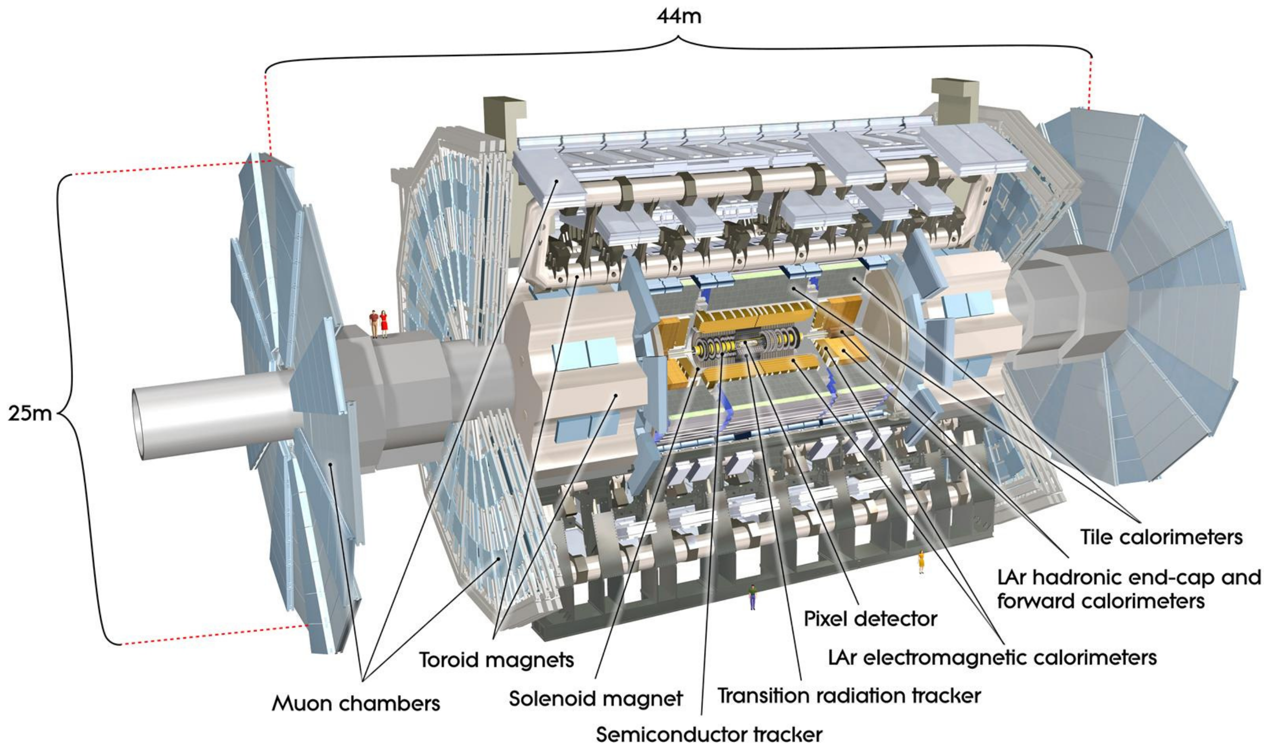
One Z may be virtual

The Large Hadron Collider

The LHC is a new proton-proton collider now running in the old LEP tunnel at CERN. In 2012 $4+4$ TeV; in 2015 $6.5+6.5$ TeV; ultimately $7+7$ TeV



ATLAS – a general purpose LHC detector



Higgs Observations (August 2014)

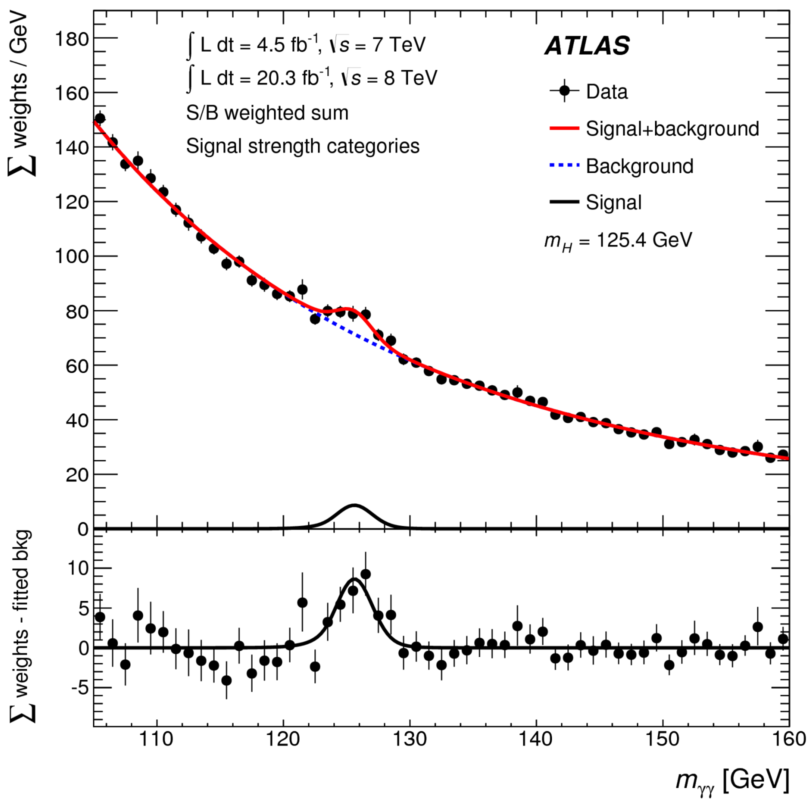
Indirect indications from LEP that Higgs mass should be not far above 115 GeV.

Dominant decay modes are all difficult:

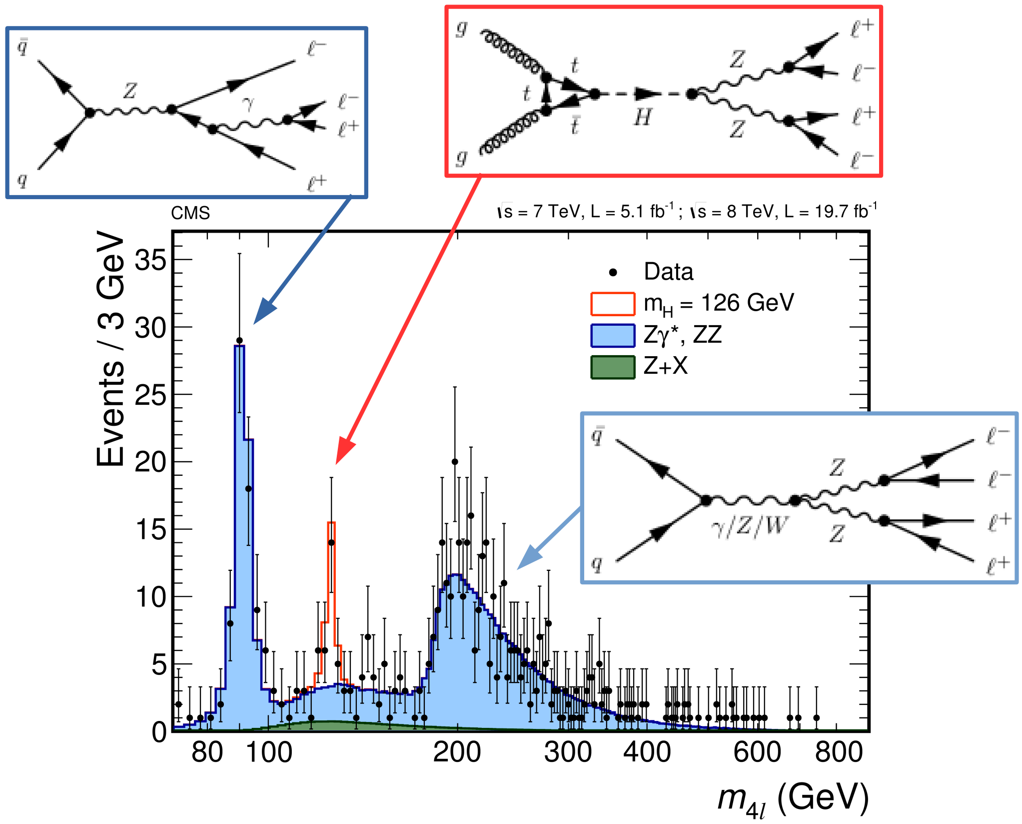
- $b\bar{b}$, $c\bar{c}$
  (swamped by QCD jets)
- W^+W^-, $\tau^+\tau^-$
  (missing neutrinos)

Best options are the rare decays:

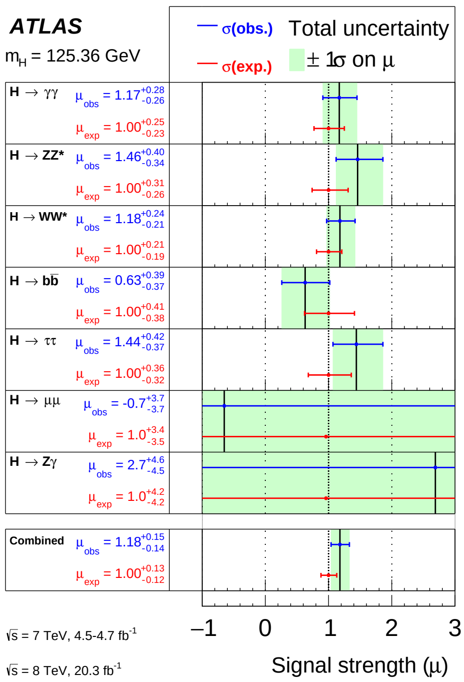
- $ZZ \to \ell^+\ell^-\ell^+\ell^-$
  $\gamma\gamma$



$H \to ZZ \to 4\ell$



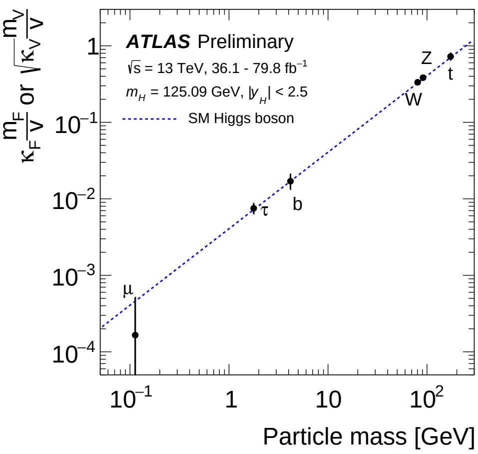
Higgs Boson Discovery



Convincing signal consistent with $m(H) = 125\ \mathrm{GeV}$ – seen in multiple decay modes & in two experiments.

Is it the Higgs boson of the SM?

Need to check its quantum numbers (should be $J^P = 0^+$).



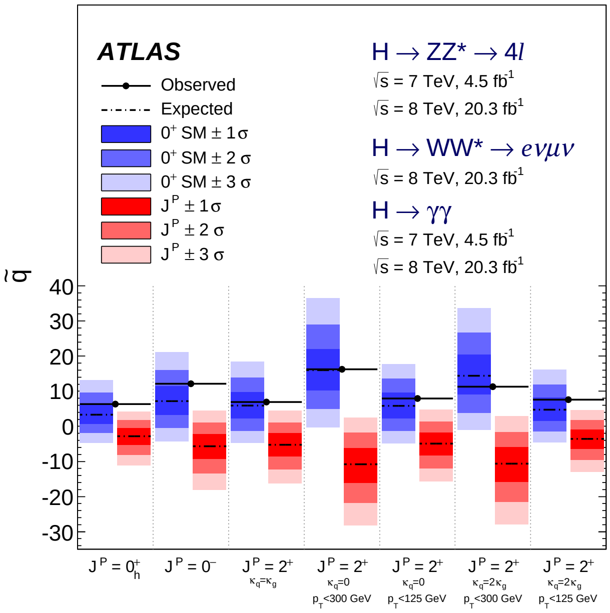
Check branching ratios and couplings. Look ok so far...

Higgs spin+parity?

Studied using angular distributions of decay products

So far it looks like the 0^+ SM Higgs.

Alternative spin-parity possibilities are disfavoured.

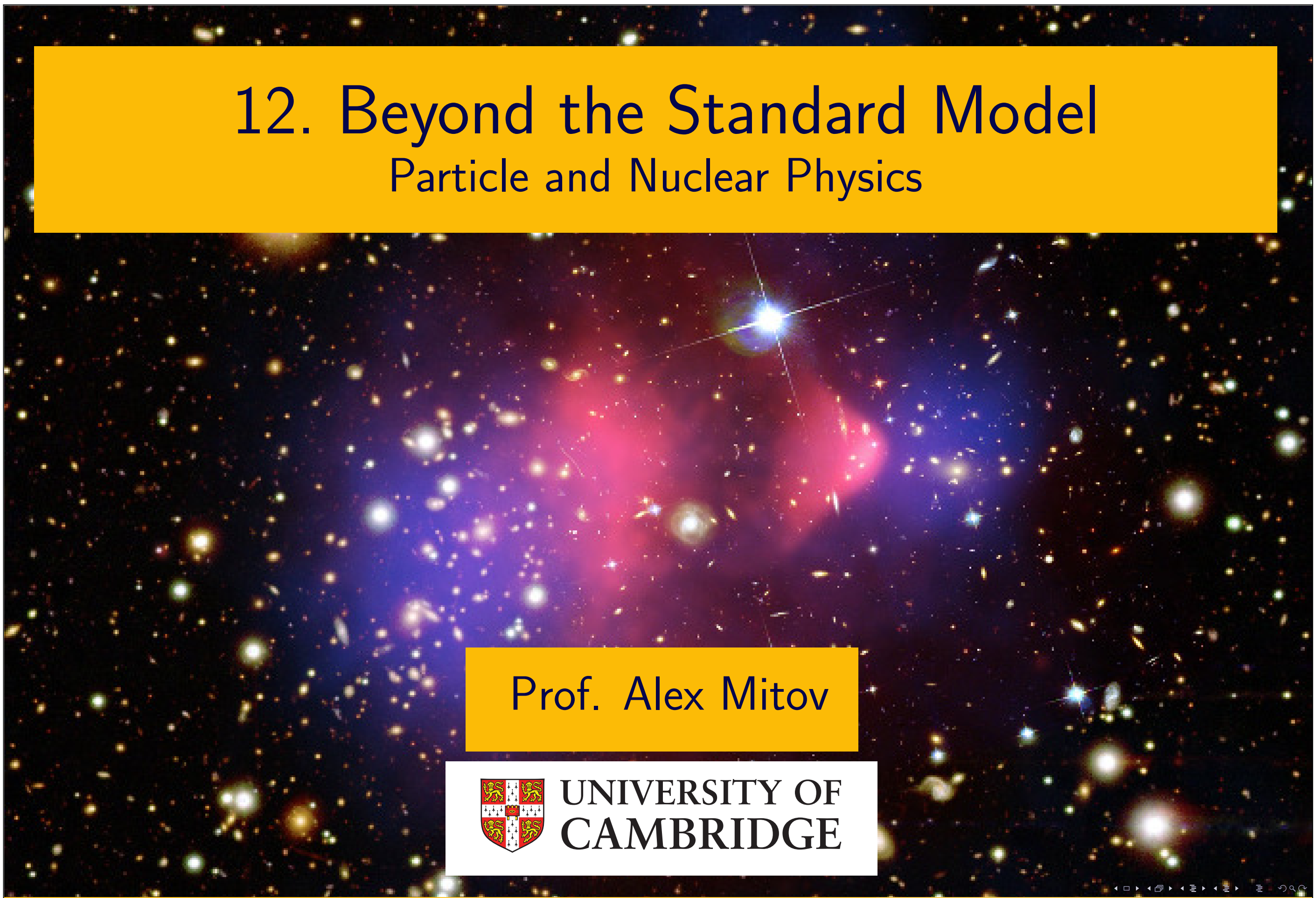


Summary

- Top quark – observed, and compatible with other precise electroweak measurements.
- Electroweak theory depends on the Higgs mechanism to endow particles with mass. This is a non-standard feature, which needs experimental verification.
- Higgs boson – detected in 2012 at 125 GeV. It is **the** Higgs boson of the Electroweak Standard Model.
  Work continues to determine all the Higgs properties precisely to see if any surprises are hiding...

Problem Sheet: q.28

Up next...
Section 12: Beyond the Standard Model



12. Beyond the Standard Model
Particle and Nuclear Physics

Prof. Alex Mitov

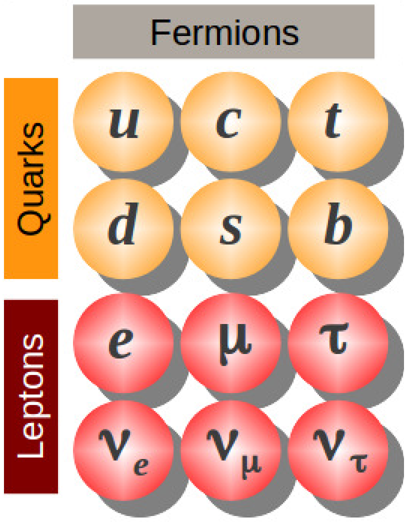
UNIVERSITY OF CAMBRIDGE

In this section...

- Summary of the Standard Model
- Problems with the Standard Model
- Neutrino oscillations
- Supersymmetry

The Standard Model (2012)

Matter: point-like spin $\frac{1}{2}$ Dirac fermions

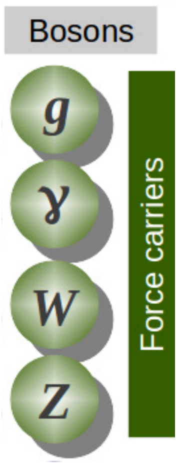


+ antiparticles

	Fermion		Charge [e]	Mass
1st gen.	Electron	e^-	-1	0.511 MeV
	Electron neutrino	ν_e	0	~ 0
	Down quark	d	$-1/3$	4.8 MeV
	Up quark	u	$+2/3$	2.3 MeV
2nd gen.	Muon	μ^-	-1	106 MeV
	Muon neutrino	ν_μ	0	~ 0
	Strange quark	s	$-1/3$	95 MeV
	Charm quark	c	$+2/3$	1.3 GeV
3rd gen.	Tau	τ^-	-1	1.78 GeV
	Tau neutrino	ν_τ	0	~ 0
	Bottom quark	b	$-1/3$	4.7 GeV
	Top quark	t	$+2/3$	173 GeV

The Standard Model (2012)

Forces: mediated by spin 1 bosons



Force	Particle	Mass
Electromagnetic	Photon γ	0
Strong	8 gluons g	0
Weak (CC)	$W^\pm$	80.4 GeV
Weak (NC)	Z	91.2 GeV

- The Standard Model also predicts the existence of a spin-0 Higgs boson which gives all particles their masses via its interactions. Evidence from LHC confirms this, with $m_H \sim 125$ GeV.
- The Standard Model successfully describes all existing particle physics data, with the exception of one

  $\Rightarrow$ Neutrino Oscillations $\Rightarrow$ Neutrinos have mass

  In the SM, neutrinos are treated as massless; right-handed states do not exist $\Rightarrow$ indication of physics Beyond the Standard Model

Problems with the Standard Model

The Standard Model successfully describes all existing particle physics data (though question marks over the neutrino sector).

But: many (too many?) input parameters:

- Quark and lepton masses
- Quark charge
- Couplings $\alpha_{\rm EM}$, $\sin^2\theta_W$, α_s
- Quark (+ neutrino) generation mixing – $V_{\rm CKM}$

23 free parameters in SM
- 9 fermion masses (e, μ, τ, u, d, s, c, b, t)
- 4 CKM: 3 mixing angles + CPV phase
- 4 PMNS: 3 mixing angles + CPV phase
- 3 gauge couplings: U(1), SU(2), SU(3)
- 3 other: QCD vacuum angle (strong CPV), Higgs VEV, Higgs mass

and: many unanswered questions:

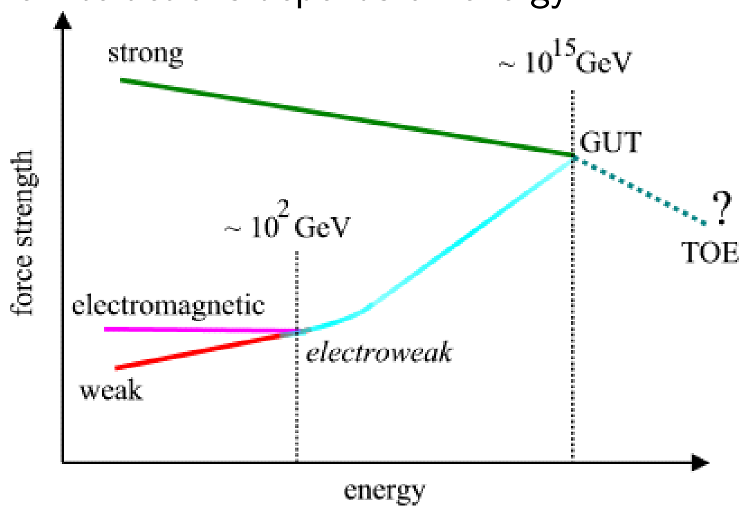
- Why so many free parameters?
- Why only three generations of quarks and leptons?
- Where does mass come from? (Higgs boson probably OK)
- Why is the neutrino mass so small and the top quark mass so large?
- Why are the charges of the p and e identical?
- What is responsible for the observed matter-antimatter asymmetry?
- How can we include gravity?

etc

Beyond the Standard Model – further unification??

Grand Unification Theories (GUTs) aim to unite the strong interaction with the electroweak interaction. Underpins many ideas about physics beyond the Standard Model.

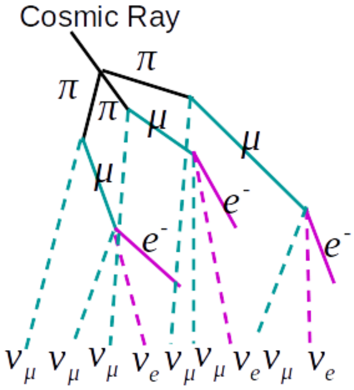
The strength of the interactions depends on energy:



- Suggests unification of all forces at $\sim 10^{15}$ GeV?
- Strength of Gravity only significant at the Planck Mass $\sim 10^{19}$ GeV

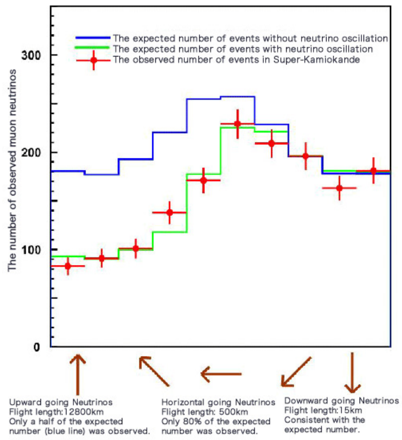
Neutrino Oscillations

In 1998 the Super-Kamiokande experiment announced convincing evidence for neutrino oscillations implying that neutrinos have mass.



$$\pi \to \mu\nu_\mu$$
$$\hookrightarrow e\nu_\mu\bar{\nu}_e$$

Expect
$$\frac{N(\nu_\mu)}{N(\nu_e)} \sim 2$$



Super-Kamiokande results indicate a deficit of ν_μ from the upwards direction. Upward neutrinos created further away from the detector.

- Interpreted as $\nu_\mu \to \nu_\tau$ oscillations
- Implies neutrino mixing and neutrinos have mass

Detecting Neutrinos

Neutrinos are detected by observing the lepton produced in charged current interactions with nuclei. e.g. $\nu_e + N \to e^- + X$ $\bar{\nu}_\mu + N \to \mu^+ + X$

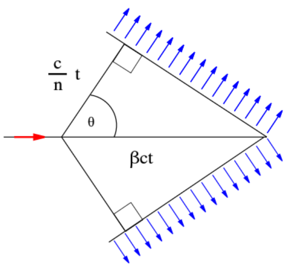
Size Matters:

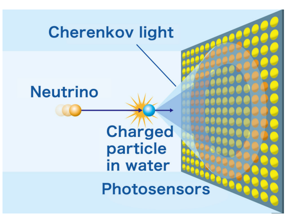
- Neutrino cross-sections on nucleons are tiny; $\sim 10^{-42}(E_\nu/\ \mathrm{GeV})\mathrm{m}^2$
- Neutrino mean free path in water $\sim$ light-years.
- Require very large mass, cheap and simple detectors.
- Water Čerenkov detection

Čerenkov radiation

- Light is emitted when a charged particle traverses a dielectric medium
- A coherent wavefront forms when the velocity of a charged particle exceeds c/n ($n =$ refractive index)
- Čerenkov radiation is emitted in a cone i.e. at fixed angle with respect to the particle.

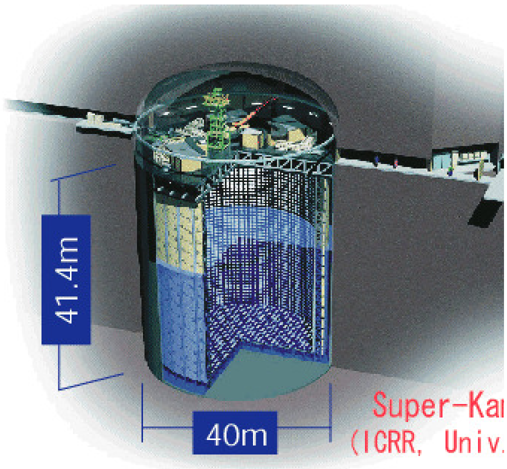
$$\cos\theta_C = \frac{c}{nv} = \frac{1}{n\beta}$$

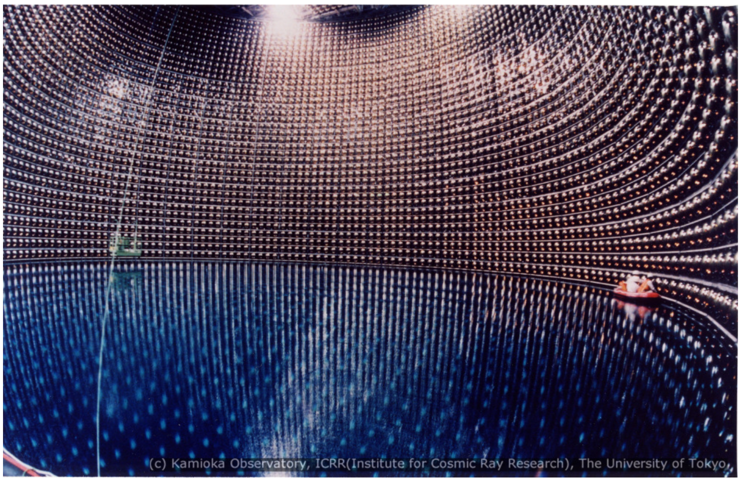




Super-Kamiokande

Super-Kamiokande is a Water Čerenkov detector sited in Kamioka, Japan



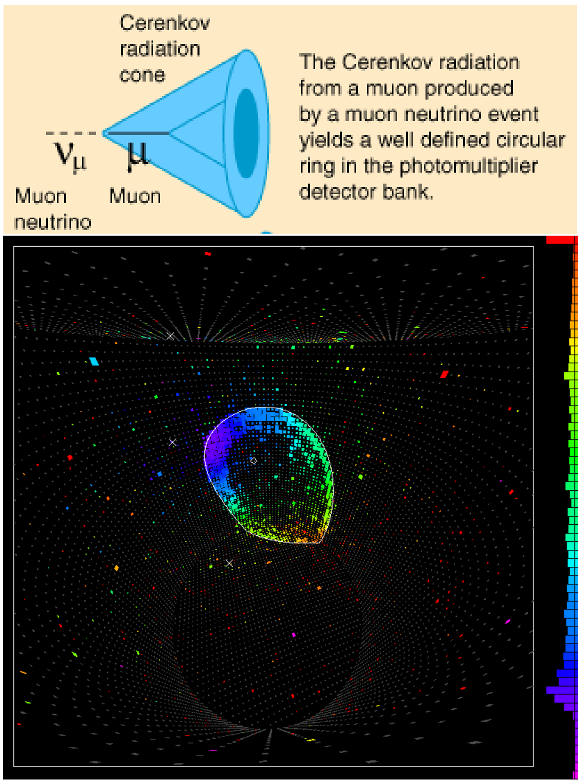


$50,000$ tons of water

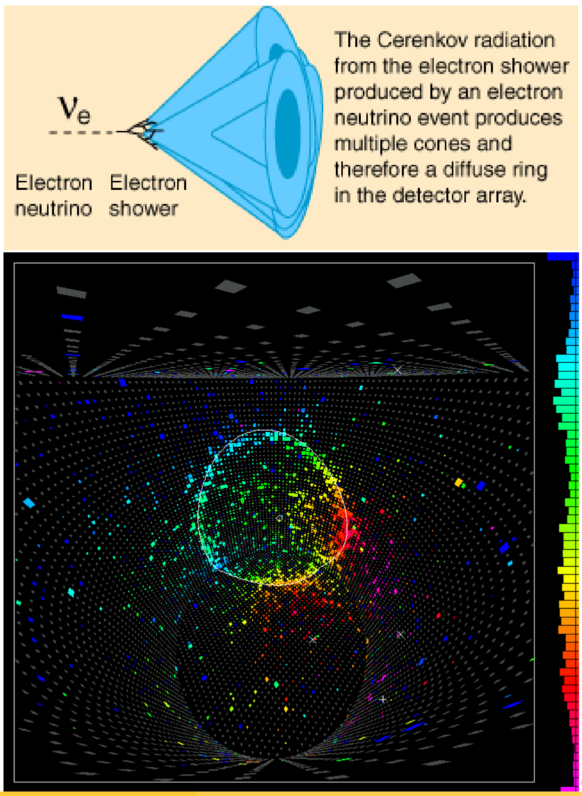
Surrounded by $11,146 \times 50\,\text{cm}$ diameter, photo-multiplier tubes

Super-Kamiokande *Examples of events*

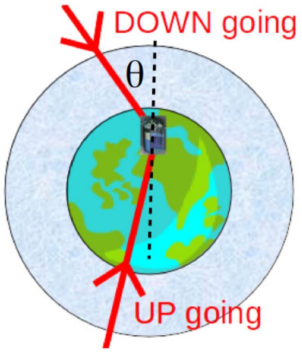
$\nu_\mu + N \to \mu^- + X$



$\nu_e + N \to e^- + X$

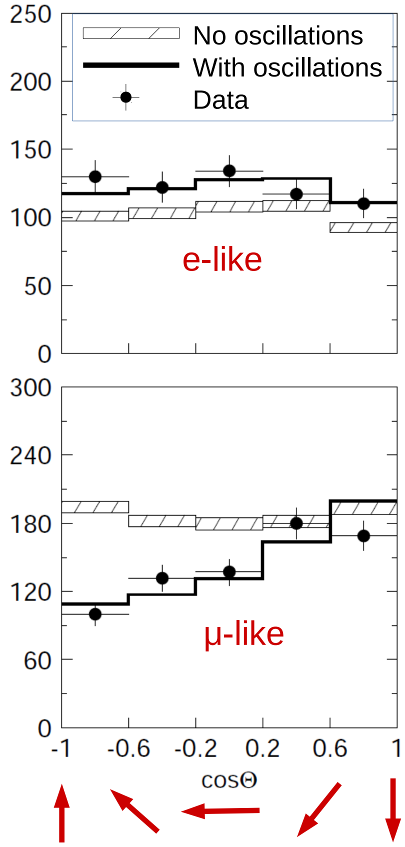


Super-Kamiokande *ν deficit*



Expect

- Isotropic (flat) distributions in $\cos\theta$
- $N(\nu_\mu) \sim 2N(\nu_e)$



Observe

- Deficit of ν_μ from **below**
- Whereas ν_e look as expected

Interpretation

- $\nu_\mu \to \nu_\tau$ oscillations
- $\Rightarrow$ neutrinos have mass

Neutrino Mixing

The quark states which take part in the weak interaction (d', s') are related to the flavour (mass) states (d, s)

Weak Eigenstates
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$
Mass Eigenstates

Cabibbo angle $\theta_C \sim 13^\circ$

Suppose the same thing happens for neutrinos. Consider only the first two generations for simplicity.

Weak Eigenstates

= flavour eigenstates
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$
Mass Eigenstates

Mixing angle θ

e.g. in π^+ decay produce μ^+ and ν_μ i.e. the neutrino state that couples to the weak interaction.

The ν_μ corresponds to a linear combination of the states with definite mass, ν_1 and ν_2
$$\nu_e = +\nu_1\cos\theta + \nu_2\sin\theta$$
$$\nu_\mu = -\nu_1\sin\theta + \nu_2\cos\theta$$

or expressing the mass eigenstates in terms of the weak eigenstates
$$\nu_1 = +\nu_e\cos\theta - \nu_\mu\sin\theta$$
$$\nu_2 = +\nu_e\sin\theta + \nu_\mu\cos\theta$$

Neutrino Mixing

Neutrino Mixing

Neutrino Mixing

Neutrino Mixing

Suppose a muon neutrino with momentum $\vec{p}$ is produced in a weak decay, e.g. $\pi^+ \to \mu^+ \nu_\mu$

At $t = 0$, the wavefunction

$$\psi(\vec{p}, t = 0) = \nu_\mu(\vec{p}) = \nu_2(\vec{p}) \cos\theta - \nu_1(\vec{p}) \sin\theta$$

The time evolution of ν_1 and ν_2 will be different if they have different masses

$$\nu_1(\vec{p}, t) = \nu_1(\vec{p}) e^{-iE_1 t} \; ; \qquad \nu_2(\vec{p}, t) = \nu_2(\vec{p}) e^{-iE_2 t}$$

After time t, state will in general be a mixture of ν_e and ν_μ

$$\begin{aligned}
\psi(\vec{p}, t) &= \nu_2(\vec{p}) e^{-iE_2 t} \cos\theta - \nu_1(\vec{p}) e^{-iE_1 t} \sin\theta \\
&= [\nu_e(\vec{p}) \sin\theta + \nu_\mu(\vec{p}) \cos\theta] \, e^{-iE_2 t} \cos\theta - [\nu_e(\vec{p}) \cos\theta - \nu_\mu(\vec{p}) \sin\theta] \, e^{-iE_1 t} \sin\theta \\
&= \nu_\mu(\vec{p}) \left[\cos^2\theta e^{-iE_2 t} + \sin^2\theta e^{-iE_1 t}\right] + \nu_e(\vec{p}) \left[\sin\theta \cos\theta \left(e^{-iE_2 t} - e^{-iE_1 t}\right)\right] \\
&= c_\mu \nu_\mu(\vec{p}) + c_e \nu_e(\vec{p})
\end{aligned}$$

Neutrino Mixing

Probability of oscillating into ν_e

$$
\begin{aligned}
P(\nu_e) = |c_e|^2 &= \left|\sin\theta\cos\theta\left(\mathrm{e}^{-iE_2t} - \mathrm{e}^{-iE_1t}\right)\right|^2 \\
&= \frac{1}{4}\sin^2 2\theta\left(\mathrm{e}^{-iE_2t} - \mathrm{e}^{-iE_1t}\right)\left(\mathrm{e}^{iE_2t} - \mathrm{e}^{iE_1t}\right) \\
&= \frac{1}{4}\sin^2 2\theta\left(2 - \mathrm{e}^{i(E_2-E_1)t} - \mathrm{e}^{-i(E_2-E_1)t}\right) \\
&= \sin^2 2\theta \sin^2\left[\frac{(E_2-E_1)t}{2}\right]
\end{aligned}
$$

But $\quad E = \sqrt{\vec{p}^2 + m^2} = \vec{p}\sqrt{1 + \frac{m^2}{\vec{p}^2}} \sim \vec{p} + \frac{m^2}{2\vec{p}} \quad$ for $m \ll E$

$1 + x \sim (1 + x/2)^2$

when x is small, can ignore x^2 term

$$\Rightarrow E_2(\vec{p}) - E_1(\vec{p}) \sim \frac{m_2^2 - m_1^2}{2\vec{p}} \sim \frac{m_2^2 - m_1^2}{2E}$$

$$\Rightarrow P(\nu_\mu \to \nu_e) = \sin^2 2\theta \sin^2\left[\frac{(m_2^2 - m_1^2)t}{4E}\right]$$

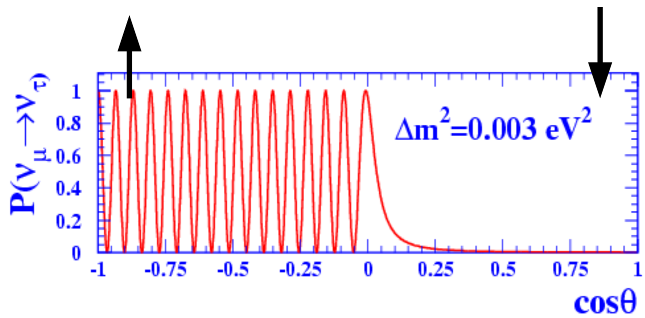
Neutrino Mixing

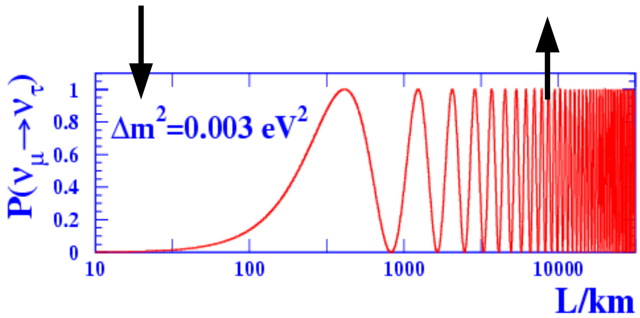
For $\nu_\mu \to \nu_\tau$ $\quad P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\left[\frac{(m_3^2 - m_2^2)t}{4E}\right] = \sin^2 2\theta \sin^2\left[\frac{1.27\Delta m^2 L}{E_\nu}\right]$

where L is the distance travelled in km,
$\Delta m^2 = m_3^2 - m_2^2$ is the mass difference in $(\,\mathrm{eV})^2$
and E_ν is the neutrino energy in GeV.

Interpretation of Super-Kamiokande Results

For $E(\nu_\mu) = 1\ \mathrm{GeV}$ (typical of atmospheric neutrinos)





Results are consistent with $\nu_\mu \to \nu_\tau$ oscillations:

$$|m_3^2 - m_2^2| \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2; \qquad \sin^2 2\theta \sim 1$$

Neutrino Mixing – Comments

- Neutrinos almost certainly have mass
- Neutrino oscillation only sensitive to mass differences
- More evidence for neutrino oscillations

  Solar neutrinos (SNO experiment)

  Reactor neutrinos (KamLand)

  suggest $|m_2^2 - m_1^2| \sim 8 \times 10^{-5}\ \mathrm{eV}^2$.
- More recent experiments use neutrino beams from accelerators or reactors; observe energy spectrum of neutrinos at a distant detector.
- At fixed L, observation of the values of E_ν at which minima/maxima are seen determines Δm^2, while depth of minima determine $\sin^2 2\theta$.
- Note all these experiments only tell us about mass **differences**.
- Best constraint on absolute mass comes from the end point in Tritium β-decay, $m(\nu_e) < 2\ \mathrm{eV}$.

Three-flavour oscillations

This whole framework can be generalised...
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

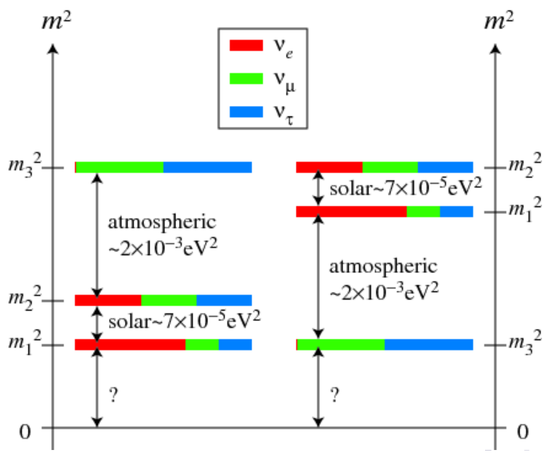
where
$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{23}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

defining $\cos\theta_{12} = c_{12}$ etc.

This is an active field!

Current status...

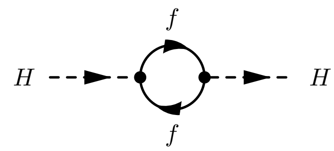
- $\sin^2\theta_{12} = 0.304 \pm 0.014$
- $\sin^2\theta_{23} = 0.51 \pm 0.06$
- $\sin^2\theta_{13} = 0.0219 \pm 0.0012$



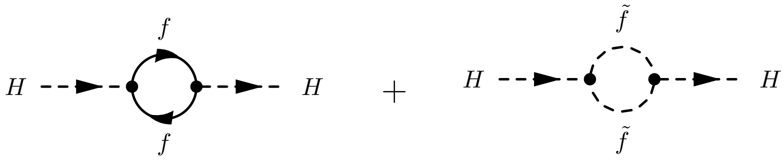
Supersymmetry (SUSY)

A significant problem is to explain why the Higgs boson is so light.

- The effect of loop corrections on the Higgs mass should be to drag it up to the highest energy scale in the problem (i.e. unification, or Planck mass).

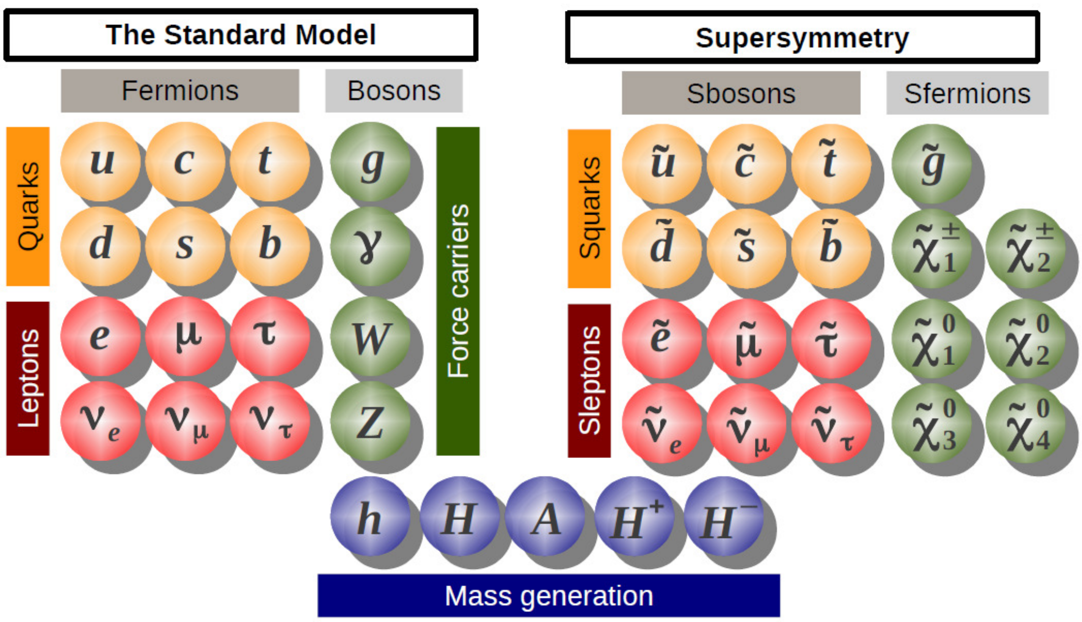


- One attractive solution is to introduce a new space-time symmetry, "supersymmetry" which links fermions and bosons (the only way to extend the Poincaré symmetry of special relativity and respect quantum field theory.)
- Each fermion has a boson partner, and vice versa, with the same couplings. Boson and fermion loops contribute with opposite sign, giving a natural cancellation in their effect on the Higgs mass.



- Must be a broken symmetry, because we clearly don't see bosons and fermions of the same mass.
- However, this doubles the particle content of the model, without any direct evidence (yet), and introduces lots of new unknown parameters.

The Supersymmetric Standard Model



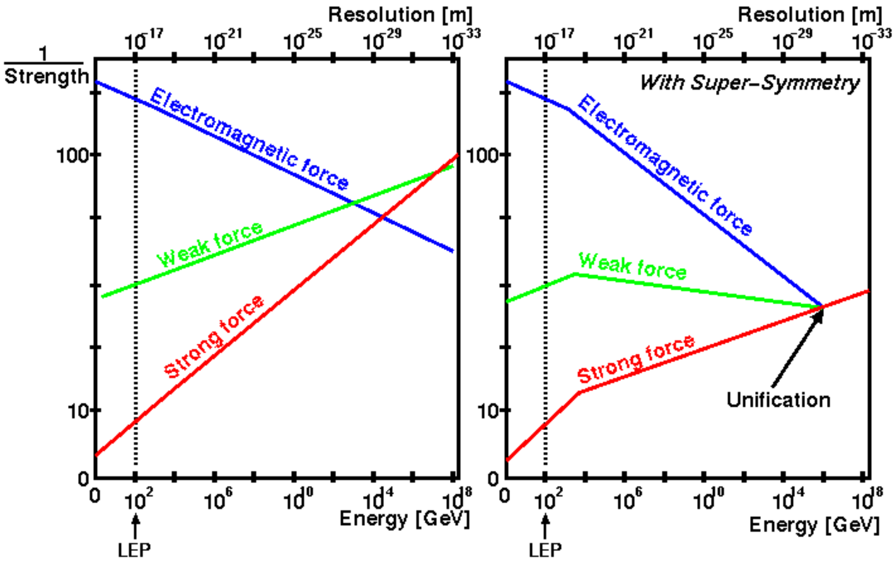
$\text{SM}: \ W^{\pm},\ W^0,\ B \xrightarrow{\text{mixing}} W^{\pm},\ Z,\ \gamma$

$\text{SUSY}: \ \tilde{H}_u^0,\ \tilde{H}_d^0,\ \tilde{W}^0,\ \tilde{B}^0 \xrightarrow{\text{mixing}} \tilde{\chi}_1^0,\ \tilde{\chi}_2^0,\ \tilde{\chi}_3^0,\ \tilde{\chi}_4^0$

$\tilde{H}_u^+,\ \tilde{H}_d^-,\ \tilde{W}^+,\ \tilde{W}^- \xrightarrow{\text{mixing}} \tilde{\chi}_1^{\pm},\ \tilde{\chi}_2^{\pm}$

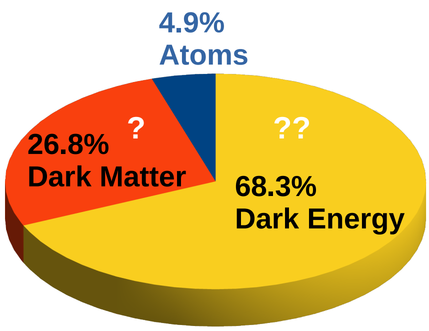
SUSY and Unification

- In the Standard Model, the interaction strengths are not quite unified at very high energy.
- Add SUSY, the running of the couplings is modified, because sparticle loops contribute as well as particle loops.
- Details depend on the version of SUSY, but in general unification much improved.



SUSY and cosmology

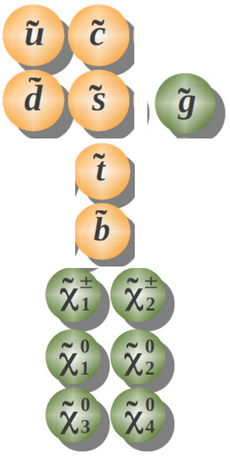
- SUSY, or any unified theory, tends to have potential problems with explaining the non-observation of proton decay.
- For this reason, many versions of SUSY introduce a conserved quantity "R-parity", which means that sparticles have to be produced in pairs.
- A consequence is that the lightest sparticle would have to be stable. In many scenarios this would be a "neutralino" $\tilde{\chi}_1^0$ (a mixture of neutral "gauginos" and "Higgsinos").
- Cosmologists tell us that $\sim 25\%$ of the mass in the universe is in the form of "dark matter", which interacts gravitationally, but otherwise only weakly.
- The lightest sparticle could be a candidate for the "WIMPs" (Weakly Interacting Massive Particles) which could comprise dark matter.

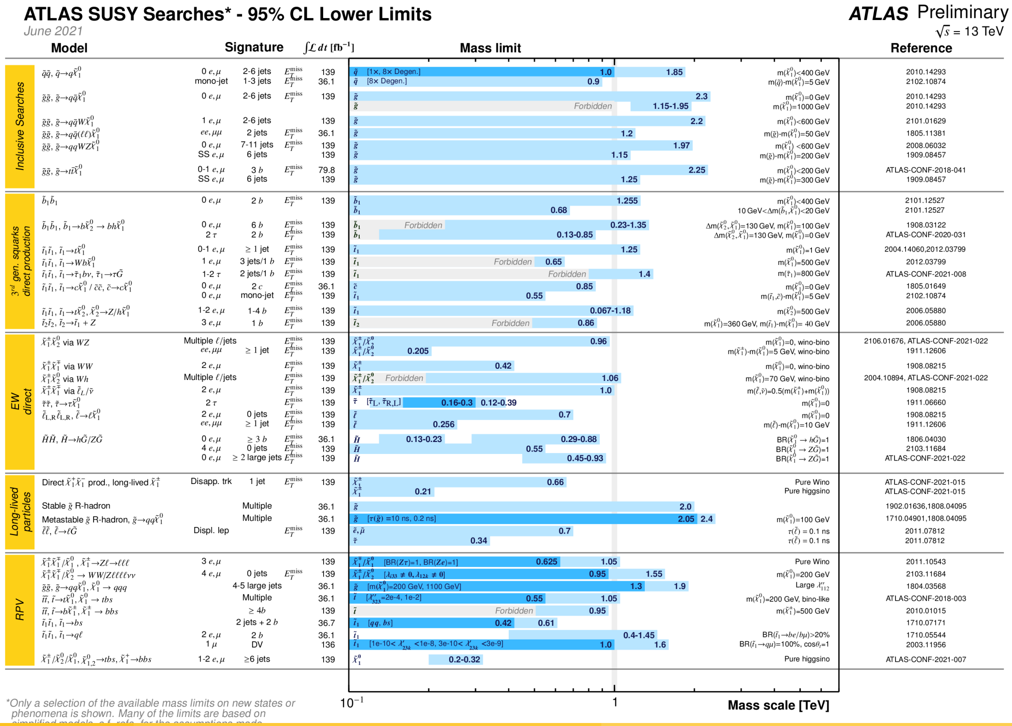


- So there are several different reasons why SUSY is attractive.

However, no sign of supersymmetry yet...

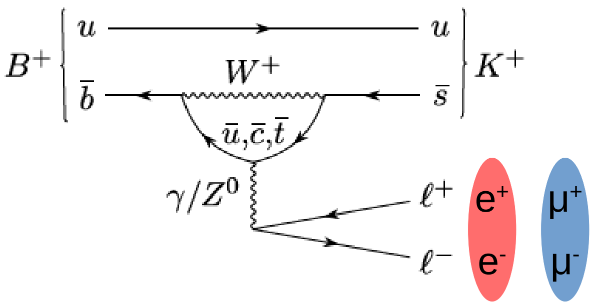
On general grounds, some sparticles ought to be seen at energies around 1 TeV or lower. So LHC ought to be able to see them, especially squarks+gluinos (high σ @LHC).

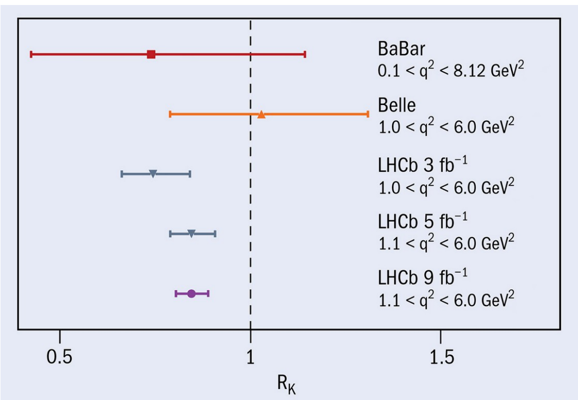




Signs of anything else? *(non-examinable)*

LHCb Flavour Anomalies





Lepton universality in SM predicts $R = \frac{\mu\mu}{ee} = 1$

Test using rare decays of B mesons

- easy to see deviations from small values
- precise theory predictions

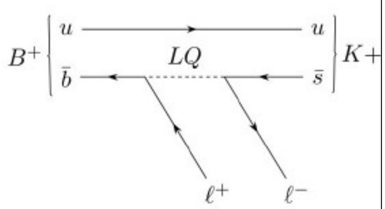
$$R_K = 0.85 \pm 0.04(stat.) \pm 0.01(syst.)$$

3 standard deviations from prediction.
Evidence of something new!

5 std.dev is gold standard for discovery.

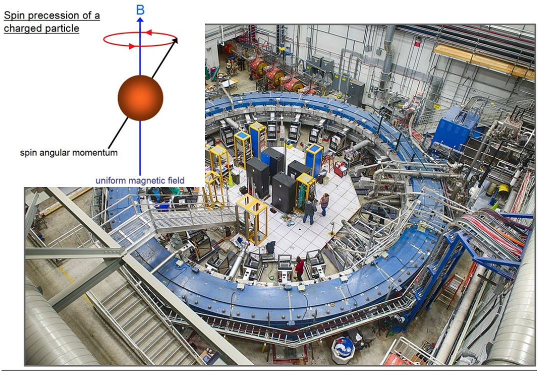
Similar effects seen in several rare decay modes.

This might be the first glimpse of new particles affecting decay rates, e.g. Leptoquarks



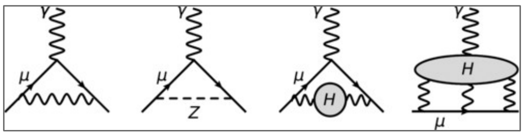
Signs of anything else? *(non-examinable)*

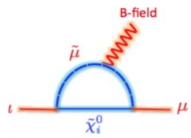
Muon g-2 Anomaly

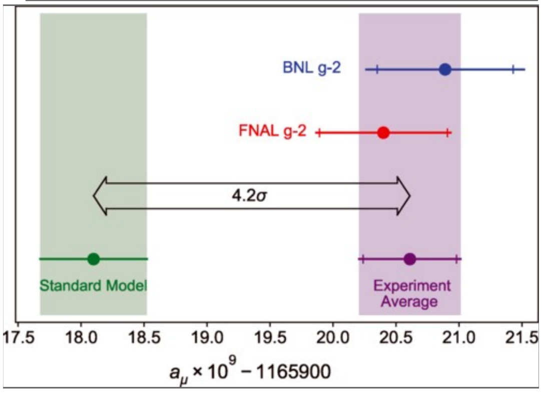


Measure muon spin precession in magnetic field. Precision test of QED – precession frequency depends on how much it interacts with the magnetic field.

All known particles contribute to the muon's magnetic moment. Measure this very precisely and look for deviations.







20 year anomaly has been confirmed with a new measurement at Fermilab – measured muon magnetic moment to 0.46 ppm.
4.2 standard deviations from prediction.
Evidence of something new! Perhaps smuons?

Follow the results from LHC yourself!

To date (2024) LHC has taken only ∼5% of its planned total dataset.
Stay tuned!!

http://atlas.ch
http://cms.web.cern.ch
http://lhcb-public.web.cern.ch/lhcb-public/

Summary

- Over the past 50 years our understanding of the fundamental particles and forces of nature has changed beyond recognition.
- The Standard Model of particle physics is an enormous success. It has been tested to very high precision and can model almost all experimental observations so far.
- The Higgs "hole" is now becoming closed, though some other aspects of the SM are not quite yet under as much experimental "control" as one might wish for (the neutrino sector, the CKM matrix, etc).
- Good reasons to expect that the next few years will bring many more (un)expected surprises (more Higgs or gauge bosons, SUSY?).

Problem Sheet: q.29-30

Up next...
Section 13: Nuclear Physics, Basic Nuclear Properties