Non-examinable

More on spin, Dirac equation and more.

1) Origin of spin = ?

It was an unexpected property of atomic systems discovered in the

Stern – Gerlach experiment (1922)

Apparently atoms (and particles) have intrinsic angular-momentum - like property of discrete values!

No idea then how to explain this.

2) First theoretical basis: the Pauli equation (1927)

It is a Schrödinger equation like, but describes a particle interacting with an E-M potential.

$$\left\{ \frac{(\vec{p} - q\vec{A})^2}{2m} - \frac{q\,\vec{\sigma}\cdot\vec{B}}{2m} + q\phi \right\} u_A = E_{kin}\, u_A$$

non-standard term with a 2×2 matrix $\vec{\sigma}$ (3 of them)

$\Rightarrow u_A$ is a $\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$ 2-component spinor.

3) Connection with the Dirac equation:

One can show that in the non-relativistic limit

$$u_B \sim \frac{v}{c} u_A$$

$$\Rightarrow \psi = \begin{pmatrix} u_A \\ u_B \end{pmatrix} \approx \begin{pmatrix} u_A \\ 0 \end{pmatrix} \approx u_A$$

i.e. the 4-component Dirac spinor becomes effectively a 2-spinor ~~that~~ satisfies the Pauli equation.

This gives us a solid basis to interpet

Dirac is relativistic needs 4 components for $e^{\pm}$ $s = \pm\frac{1}{2}$

Pauli → non-relativistic for e^- with $s = \pm\frac{1}{2}$

in particular: the operator

$$\boxed{\frac{\vec{\sigma}}{2} \equiv \vec{S}} \Leftarrow \text{spin operator}$$

can now easily be interpreted. $\vec{\mu} = \frac{q}{m}\vec{S}$ ← intrinsic magnetic moment.

"Dirac spinor" or just "spinor"?

Nominally, a spinor is a 2-component object.

A Dirac spinor is also known as a

bi-spinor.

i.e. "two spinors put together".

Spinors and bi-spinors transform under different representations of the Lorentz group

$$\underbrace{\left(0, \tfrac{1}{2}\right)}_{2} \text{ or } \underbrace{\left(\tfrac{1}{2}, 0\right)}_{2} \quad \text{v.s.} \quad \underbrace{\left(0, \tfrac{1}{2}\right) \oplus \left(\tfrac{1}{2}, 0\right)}_{4}$$

Now, let's review Appendix II

Dimension of Dirac matrices:

why 4×4?

1) Recall: we defined them by the algebra:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \Big| \quad \gamma^\mu \ 4 \text{ of them}$$

Besides we can also introduce:

$$[\gamma^\mu, \gamma^\nu] \quad \Big| \quad 6 \text{ of them}$$

$$\gamma^5 = \frac{i}{4!}\varepsilon^{\mu\nu\sigma\rho}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho = i\gamma^0\gamma^1\gamma^2\gamma^3 \quad \Big| \quad 1$$

$$\gamma^\mu\gamma^5 \quad \Big| \quad 4 \text{ of them}$$

In total we have $\mathbb{1} + 4 + 6 + 1 + 4 = 16$

Any 4×4 matrix has 16 components.

Since the above 16 matrices are independent (no proof here) $\Rightarrow$ they form a basis for the 4×4 matrices $\Rightarrow$ they are 4×4.

End non-examinable.