1 Question 1

(a) *[Bookwork]*

There are two diagrams – one s channel annihilation, and one t channel scattering.

(b) *[Bookwork]*

They could write down

$$-iM = \bar{u}_{c\uparrow}(ie\gamma^{\mu})v_{d\downarrow}\left(\frac{-ig_{\mu\nu}}{(a+b)^2}\right)\bar{v}_{b\downarrow}(ie\gamma^{\nu})u_{a\uparrow}$$
$$+\bar{u}_{c\uparrow}(ie\gamma^{\mu})u_{a\uparrow}\left(\frac{-ig_{\mu\nu}}{(a-c)^2}\right)\bar{v}_{b\downarrow}(ie\gamma^{\nu})v_{d\downarrow}$$

provided that they also indicate that:

$$u_{c\uparrow} = \sqrt{E}\begin{pmatrix}\hat{c}\\ \hat{s}\\ \hat{c}\\ \hat{s}\end{pmatrix}, v_{d\downarrow} = \sqrt{E}\begin{pmatrix}\hat{s}\\ -\hat{c}\\ \hat{s}\\ -\hat{c}\end{pmatrix}, \qquad \text{and that} \qquad u_{a\uparrow} = \sqrt{E}\begin{pmatrix}1\\ 0\\ 1\\ 0\end{pmatrix}, v_{b\downarrow} = \sqrt{E}\begin{pmatrix}0\\ -1\\ 0\\ -1\end{pmatrix},$$

wherein the last two may be deduced by specialising the first two.

(c) *[Partly bookwork; similar in problem sheets]*

(i)

By simply substituting in the supplied gamma matrices and spinors (which is facilitated most efficiently by reducing them to two x two matrices of pauli matrices and acting on spinors with upper and lower parts grouped together) one finds

$$j^{\mu}_{ab} = 2\begin{pmatrix}\bar{a}_1 b_1 + \bar{a}_2 b_2\\ \bar{a}_1 b_2 + \bar{a}_2 b_1\\ -i\bar{a}_1 b_2 + i\bar{a}_2 b_1\\ \bar{a}_1 b_1 - \bar{a}_2 b_2\end{pmatrix}.$$

(ii)

Simply doing the dot product and cancelling a few terms leads to:

$$P_{abcd} = 8(\bar{a}_1 b_1 \bar{c}_2 d_2 + \bar{a}_2 b_2 \bar{c}_1 d_1) - 8(\bar{a}_1 b_2 \bar{c}_2 d_1 + \bar{a}_2 b_1 \bar{c}_1 d_2).$$

(iii) Hopefully they will now realise that their matrix element can be re-written in the form:

$$M = -e^2\left(\frac{P_{cdba}}{s} + \frac{P_{cabd}}{t}\right).$$

To evaluate P_{cdba}, using the answer to (b), they should see that they need to use

$$\left\{\begin{pmatrix}a_1\\ a_2\end{pmatrix}, \begin{pmatrix}b_1\\ b_2\end{pmatrix}, \begin{pmatrix}c_1\\ c_2\end{pmatrix}, \begin{pmatrix}d_1\\ d_2\end{pmatrix}\right\} = \left\{\begin{pmatrix}\hat{c}\\ \hat{s}\end{pmatrix}, \begin{pmatrix}\hat{s}\\ -\hat{c}\end{pmatrix}, \begin{pmatrix}0\\ -1\end{pmatrix}, \begin{pmatrix}1\\ 0\end{pmatrix}\right\}$$

leading to $P_{cdba} = 8E^2(-\hat{c}^2) = 4E^2(-2\hat{c}^2) = -4E^2(1+\cos\theta)$.

Similarly, to evaluate P_{cabd}, using the answer to (b), they should see that they need to use

$$\left\{ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \right\} = \left\{ \begin{pmatrix} \hat{c} \\ \hat{s} \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} \hat{s} \\ -\hat{c} \end{pmatrix} \right\}$$

leading to $P_{cabd} = 8E^2(\hat{c}^2) = 4E^2(2\hat{c}^2) = 4E^2(1+\cos\theta)$.

This results in

$$M = e^2 4E^2(1+\cos\theta)\left(\frac{1}{s} - \frac{1}{t}\right)$$

or

$$|M|^2 = e^4 16E^4(1+\cos\theta)^2\left(\frac{1}{s} - \frac{1}{t}\right)^2.$$

[Extension of lecture ideas] At this point we may wish to comment things like "It is good that we have a $(1+\cos\theta)^2$ term on the top, as this is what we expect from the spin 1 initial state going to the spin 1 final state, from consideration of overall angular momentum. We also see that we have s and t propagator terms, corresponding to our s and t channel diagrams. s takes tha value $4E^2$ and so is always positive. t, on the other hand, is found to take the value $-4E^2\sin^2(\theta/2)$ and so is always negative. This means that M itself is real and never negative (even before we take its modulus), and is only able to reach zero when evaluated at $\theta = -\pi$, i.e. when conservation of angular momentum forbids the scattering.

(iv)*[Extension of lecture ideas]*

To write $|M|^2$ entirely in terms of Mandelstam s and t it is only necessary to rewrite the $4E^2(1+\cos\theta)$ part. We have

$$4E^2(1+\cos\theta) = 2(b-a)^\mu(c-b)_\mu = 2(b.c - a.c + a.b) = (-u+t+s).$$

But noting that $s+t+u=0$ we have

$$4E^2(1+\cos\theta) = 2(s+t)$$

and so

$$(4E^2(1+\cos\theta))^2 = 4(s+t)^2$$

and so

$$|M|^2 = 4e^4(s+t)^2\left(\frac{1}{s} - \frac{1}{t}\right)^2$$

implying that $A=4$, $B=2$, $C=-1$ and $D=2$.

2 Question 2

[Bookwork]

The K^0 and $\bar{K}^0$ belong to a $J^{PC} = 0^{-+}$ multiplet so

$$CP|K^0> = -|\bar{K}^0>, \qquad CP|\bar{K}^0> = -|K^0>.$$

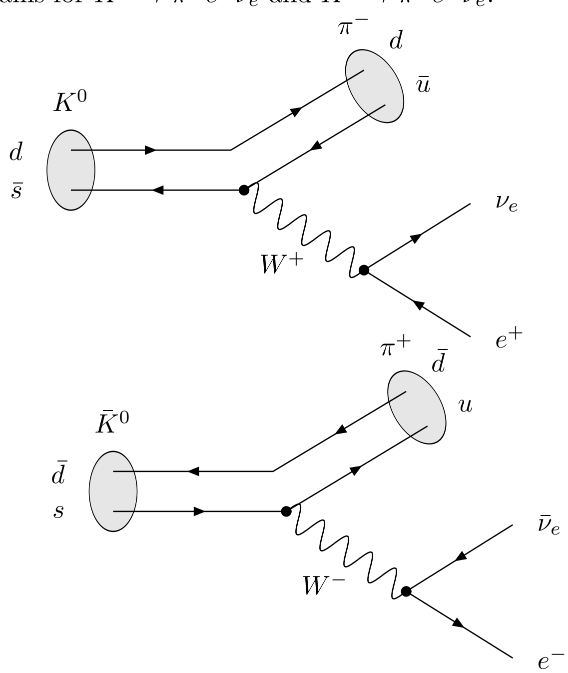
The CP eigenstates K_1 and K_2 can then be constructed as

$$|K_1> = \frac{1}{\sqrt{2}}\left(|K^0> - |\bar{K}^0>\right) \qquad CP|K_1> = +|K_1>$$
$$|K_2> = \frac{1}{\sqrt{2}}\left(|K^0> + |\bar{K}^0>\right) \qquad CP|K_2> = -|K_2>$$

If CP violation is neglected, the states K_S and K_L decay only via $K_S \to \pi\pi$ and $K_L \to \pi\pi\pi$. The $\pi\pi$ system has $CP = +1$ and the $\pi\pi\pi$ system has $CP = -1$, and we can therefore identify

$$|K_S> = |K_1> = \frac{1}{\sqrt{2}}\left(|K^0> - |\bar{K}^0>\right)$$
$$|K_L> = |K_2> = \frac{1}{\sqrt{2}}\left(|K^0> + |\bar{K}^0>\right)$$

Feynman diagrams for $K^0 \to \pi^- e^+ \nu_e$ and $\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$:



Thus the decays $K^0 \to \pi^- e^+ \nu_e$ and $\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$ are allowed, while the decays $\bar{K}^0 \to \pi^- e^+ \nu_e$ and $K^0 \to \pi^+ e^- \bar{\nu}_e$ are forbidden, *i.e.* the final state $\pi^- e^+ \nu_e$ determines the K^0 component in the beam while $\pi^+ e^- \bar{\nu}_e$ determines the $\bar{K}^0$ component.

For a pure $|K^0>$ beam at $t = 0$, the initial wavefunction is

$$|\psi(0)> = |K^0> = \frac{1}{\sqrt{2}}\left(|K_L> + |K_S>\right)$$

The wavefunction ψ evolves with time as

$$\begin{aligned}|\psi(t)> &= \frac{1}{\sqrt{2}}\left(|K_L(t)> + |K_S(t)>\right) \\ &= \frac{1}{\sqrt{2}}\left(|K_L>e^{-im_Lt-\Gamma_Lt/2} + |K_S>e^{-im_St-\Gamma_St/2}\right) \ .\end{aligned}$$

The decay rate into $\pi^-e^+\nu_e$ is determined by the K^0 component of the beam:

$$\begin{aligned}\Gamma({K^0}_{t=0}\to\pi^-e^+\nu_e) &= \left|\left\langle K^0|\psi(t)\right\rangle\right|^2 \\ &= \left|\left\langle \frac{1}{\sqrt{2}}\left(K_L+K_S\right)|\frac{1}{\sqrt{2}}\left(K_Le^{-im_Lt-\Gamma_Lt/2} + K_Se^{-im_St-\Gamma_St/2}\right)\right\rangle\right|^2 \\ &= \frac{1}{4}\left|e^{-im_Lt-\Gamma_Lt/2} + e^{-im_St-\Gamma_St/2}\right|^2 \\ &= \frac{1}{4}\left(e^{-\Gamma_St} + e^{-\Gamma_Lt} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta mt\right)\end{aligned}$$

where $\Delta m \equiv m_L - m_S$. Similarly,

$$\Gamma({K^0}_{t=0}\to\pi^+e^-\bar{\nu}_e) = \left|\left\langle \bar{K}^0|\psi(t)\right\rangle\right|^2 = \frac{1}{4}\left(e^{-\Gamma_St} + e^{-\Gamma_Lt} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta mt\right) \ .$$

[Similar on problem sheet] The two decay rates become equal when $\cos\Delta mt = 0$, *i.e.* when $\Delta mt = \pi/2$. Since $L = vt_{lab}$, $t_{lab} = \gamma t$, $\gamma = E/m$ and $v = p/E$, we have

$$\begin{aligned}\Delta m &= \frac{\pi}{2}\frac{1}{t} = \frac{\pi}{2}\frac{\gamma}{t_{lab}} = \frac{\pi}{2}\frac{E/m}{L/v} = \frac{\pi}{2L}\frac{p}{m} \\ &= \frac{\pi}{2\times(17.8\,\mathrm{m})}\times\frac{100GeV}{0.498GeV}\times(0.197GeV.\mathrm{fm}) = 3.5\times10^{-15}GeV \ .\end{aligned}$$

The K_L lifetime is about 500 times greater than the K_S lifetime, so at large times, only the $e^{-\Gamma_Lt}$ term survives. The two decay rates are then approximately equal:

$$\Gamma({K^0}_{t=0}\to\pi^-e^+\nu_e) \approx \Gamma({K^0}_{t=0}\to\pi^+e^-\bar{\nu}_e) \approx \frac{1}{4}e^{-\Gamma_Lt} \ .$$

Since the beam is almost pure K_L at large times, this gives (in the absence of CP violation)

$$\Gamma(K_L\to\pi^-e^+\nu_e) = \Gamma(K_L\to\pi^+e^-\bar{\nu}_e) \ .$$

[Mainly bookwork, extension at the end] With CP violation:

$$\begin{aligned}|K_L> &= \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2> + \epsilon|K_1>\right) \\ &= \frac{1}{\sqrt{1+|\epsilon|^2}}\left[\frac{1}{\sqrt{2}}\left(|K^0> + |\bar{K}^0>\right) + \frac{\epsilon}{\sqrt{2}}\left(|K^0> - |\bar{K}^0>\right)\right] \\ &= \frac{1}{\sqrt{1+|\epsilon|^2}}\cdot\frac{1}{\sqrt{2}}\left[(1+\epsilon)|K^0> + (1-\epsilon)|\bar{K}^0>\right] \ .\end{aligned}$$

Hence the decay rates to $\pi^- e^+ \nu_e$ and $\pi^+ e^- \bar{\nu}_e$ are

$$I(\pi^- e^+ \nu_e) \propto \left|\left\langle K^0 | K_L \right\rangle\right|^2 \propto |1+\epsilon|^2$$
$$I(\pi^+ e^- \bar{\nu}_e) \propto \left|\left\langle \bar{K}^0 | K_L \right\rangle\right|^2 \propto |1-\epsilon|^2$$

The decay rate asymmetry is

$$\begin{aligned}\delta &\equiv \frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)} \\ &= \frac{|1+\epsilon|^2 - |1-\epsilon|^2}{|1+\epsilon|^2 + |1-\epsilon|^2} \\ &= \frac{(1+\epsilon)(1+\epsilon^*) - (1-\epsilon)(1-\epsilon^*)}{(1+\epsilon)(1+\epsilon^*) + (1-\epsilon)(1-\epsilon^*)} \\ &= \frac{\epsilon + \epsilon^*}{1+|\epsilon|^2} \\ &\approx 2\mathbb{R}e(()\epsilon)\end{aligned}$$

2.1 (b) SU(3) multiplets and q qbar q qbar exotic states

[Hint is bookwork, rest is unseen] This is not book work as this question requires the students to extend something they saw in the lectures and in the notes to a slightly more complicated situation. In the lectures and notes the students were shown (in some detail) how to multiply together SU(3) multiplets and re-express the answer as a direct sum of other SU(3) multiplets. They were also given considerable instruction on the nature of singlet states, and their relation to the colour confinement hypothesis. However, they were only *shown* a symmetric colour singlet emerging from 3x3bar = 8+1 (for qqbar mesons), and an antisymmetric colour singlet emerging from 3x3x3 = 10+8+8+1 (for qqq hadrons). The ABSENCE of a colour singlet in 3x3=6+3bar was used to explain (together with the colour confinement hypothesis) the lack of qq hadrons. The existence of singlets in qqbarqqbar and qqqqbar hadrons was mentioned but not proved. This question asks the students to have a go at extending the proofs they have already used to the qqbarqqbar case.

The students should already know that 3x3bar is 8+1, so they only have to square 8+1 in order to perform the desired decomposition. Because of the existence of the singlet in 8+1 that squaring process itself is almost trivial:

$$(8 \oplus 1) \otimes (8 \oplus 1) = 8 \otimes 8 \oplus 8 \oplus 8 \oplus 1$$

meaning that the only tricky thing to expand is the $8 \otimes 8$ itself. This, I assume, they will do graphically using the method explained in lectures. This involves drawing 64 dots as follows

		1		2		1		
	2		6		6		2	
1		6		10		6		1
	2		6		6		2	
		1		2		1		

and noting that this decomposes to

$$\begin{array}{ccccccc} 1 & & 1 & & 1 & & 1 \\ & 1 & & 1 & & 1 & \\ & & 1 & & 1 & & \\ & & & 1 & & & \end{array}$$

plus

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 1 & & 1 & \\ 1 & & 1 & & 1 & & 1 \end{array}$$

plus

$$\begin{array}{ccccc} & 1 & & 1 & \\ 1 & & 2 & & 1 \\ & 1 & & 1 & \end{array}$$

plus

$$\begin{array}{ccccc} & 1 & & 1 & \\ 1 & & 2 & & 1 \\ & 1 & & 1 & \end{array}$$

plus

$$1$$

or in group-speak

$$8 \otimes 8 = 27 \oplus 10 \oplus \bar{1}0 \oplus 8 \oplus 8 \oplus 1.$$

Putting this together with the "trival" first part of the overall product we find:

$$3 \otimes \bar{3} \otimes 3 \otimes \bar{3} = 27 \oplus 10 \oplus \bar{1}0 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 1 \oplus 1.$$

The interesting comment here is that there are *two* singlets in this decomposition, not one as in the previous cases the students had seen in lectures. One of these singlets emerges from squaring the ordinary meson singlet, whereas the other one emerged from the 8 times 8. As they are different singlets, they will have different wavefunctions (i.e. with different symmetries) and so represent two different ways that qqbarqqbar states could be 'colourless'. Any remarks along roughly those lines, together with an evident understanding of what the colour confinement hypothesis is in relation to singlet states of colour SU(3) will get the two comment marks.

3 Question 3

This is an extended notes question (i.e. not much more than bullet points is needed for answers to be acceptable) and is entirely bookwork.

Last year I had huge trouble diferentiating between students on the brief notes question as many appeared to use the extra time that this exam now has to write (in most cases) about 30 independent points on each topic, where the mark shceme only envisaged rewarding 15. In a few cases, three times as many points as the mark-scheme envisaged were recorded. This made it very hard to hit the target mark for the question without penalising people who simply put down 15 succinct clear points for each answer and then moved on. A simple rescaling would give these students close to a fail mark, for no good reason.

To prevent this problem happening again this year, I have removed the choice of topic, forcing people to write on ONE topic, for all 30 marks, rather than on TWO topics for 15 marks each. It is my hope that this will make it much harder for students to simply saturate the markscheme, thereby allowing me to mark more freely – giving credit where it appears to be due, etc, rather than having to split students on the smallest of differences and/or attempt huge re-scalings.

The removal of choice should also favour those who tried to revise a bit of everything, rather than those who chose a small number of topics in the hope that at least one would come up.

Last year, in the solutions/mark-scheme, I produced a list of N suitable bullet points. I then found it was worthless for actually marking things, as real answers are so variable in style and construction. In the end, the mark scheme was implemented as a single mark each time the student appeared to make an independent point that seemed relevant, insightful, and sufficiently different from the points he/she had previously made. All students seemed to understand this process of "regurgitating and summarising their notes", and I think any interested external or internal examiner can do so too. Rather than fritter my time away fruitlessly creating a list that will never be useful for any purpose, I will instead refer the reader to handouts 5, 6 and 10 in the course, which are the ones that the students will be summarising when they answer this question.