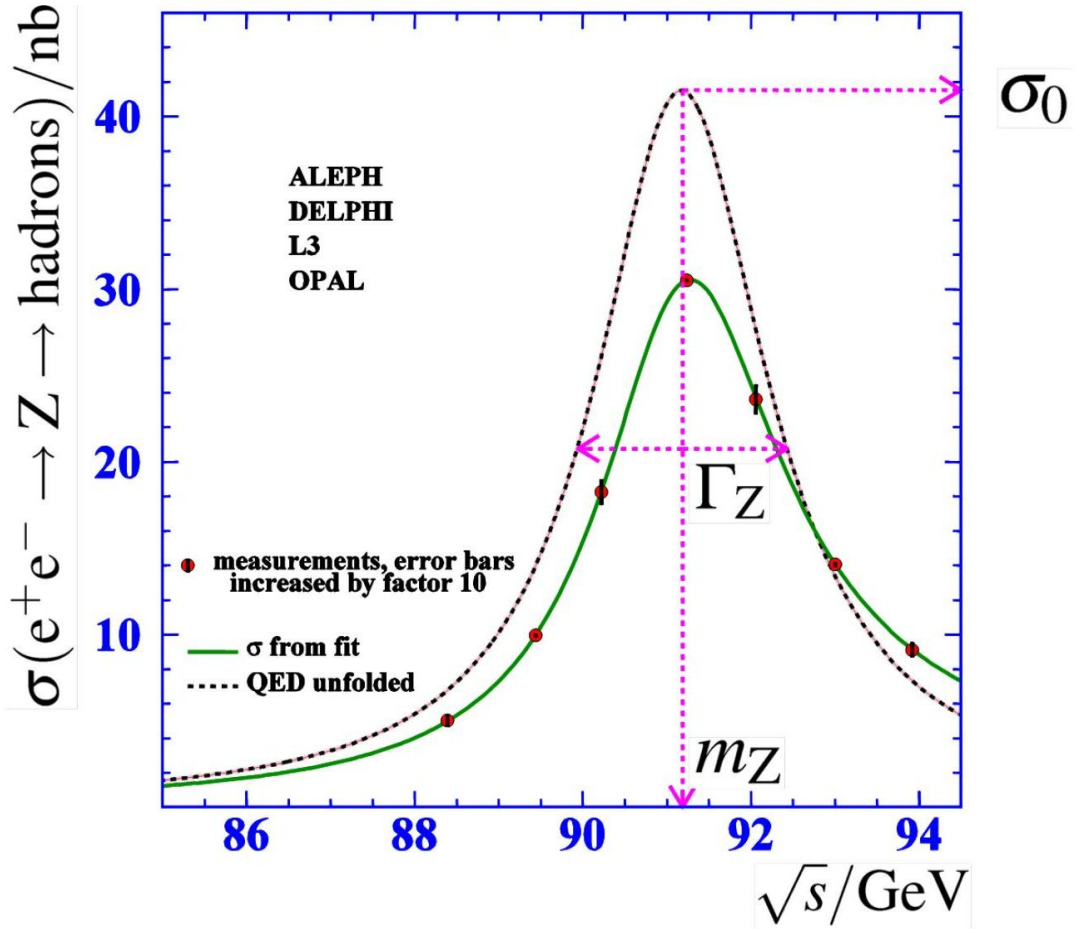
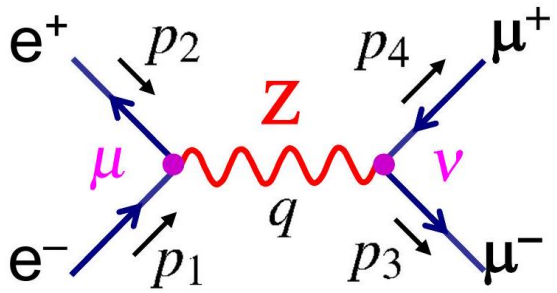


H14: Precision Tests of the Standard Model

The Z Resonance ($e^+e^- \to Z \to \mu^+\mu^-$)

- The Feynman rules from the last handout have told us that the matrix element for the process shown on the right is:



$$M_{fi} = \frac{-g_Z^2 g_{\mu\nu}}{q^2 - m_Z^2} \left[\bar{v}(p_2)\frac{\gamma^\mu}{2}\left(c_V(e) - c_A(e)\gamma^5\right)u(p_1)\right]\cdot\left[\bar{u}(p_3)\frac{\gamma^\nu}{2}\left(c_V(\mu) - c_A(\mu)\gamma^5\right)v(p_4)\right]$$

- As we have found previously, it is convenient to evaluate M_{fi} with external particles in helicity eigenstates in the ultra-relativistic limit so that helicity=chirality.
We will therefore prefer to use c_L and c_R in M_{fi} rather than c_V and c_A.
We can switch them over using (172) and (173):

$$\tfrac{1}{2}\left(c_V - c_A\gamma^5\right) = \underbrace{c_L\tfrac{1}{2}\left(1-\gamma^5\right) + c_R\tfrac{1}{2}\left(1+\gamma^5\right)}_{\text{LH and RH projections operators}}$$

- Thus:

$$M_{fi} = -\frac{g_Z^2 g_{\mu\nu}}{q^2 - m_Z^2}\left[c_L(e)\bar{v}(p_2)\gamma^\mu \tfrac{1}{2}\left(1-\gamma^5\right)u(p_1) + c_R(e)\bar{v}(p_2)\gamma^\mu \tfrac{1}{2}\left(1+\gamma^5\right)u(p_1)\right]$$
$$\times\left[c_L(\mu)\bar{u}(p_3)\gamma^\nu \tfrac{1}{2}\left(1-\gamma^5\right)v(p_4) + c_R(\mu)\bar{u}(p_3)\gamma^\nu \tfrac{1}{2}\left(1+\gamma^5\right)v(p_4)\right].$$

- Apply projection operators remembering that in the ultra-relativistic limit

$$\tfrac{1}{2}\left(1-\gamma^5\right)u = u_\downarrow; \quad \tfrac{1}{2}\left(1+\gamma^5\right)u = u_\uparrow, \quad \tfrac{1}{2}\left(1-\gamma^5\right)v = v_\uparrow, \quad \tfrac{1}{2}\left(1+\gamma^5\right)v = v_\downarrow$$

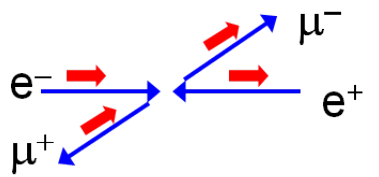
to get
$$M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}\left[c_L(e)\bar{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R(e)\bar{v}(p_2)\gamma^\mu u_\uparrow(p_1)\right]$$
$$\times\left[c_L(\mu)\bar{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R(\mu)\bar{u}(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$
$$= -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}\left[c_L(e)\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R(e)\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)\right]$$
$$\times\left[c_L(\mu)\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R(\mu)\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)\right]$$

since (in ultra-relativistic limit) $\bar{u}_\uparrow\gamma^\nu u_\uparrow = 0$ (see page 122).

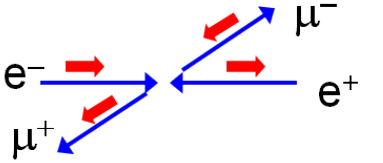
- M_{fi} is therefore a sum of the four terms M_{RR}, M_{RL}, M_{LR}, M_{LL} detailed overleaf.

Matrix elements for allowed Z-boson annihilation processes (ultra rel)

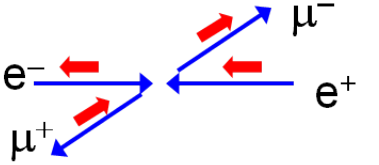
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R(e) c_R(\mu) g_{\mu\nu} \left[\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)\right] \left[\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)\right]$$



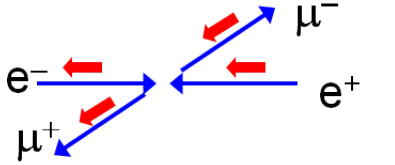
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R(e) c_L(\mu) g_{\mu\nu} \left[\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)\right] \left[\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)\right]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L(e) c_R(\mu) g_{\mu\nu} \left[\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)\right] \left[\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)\right]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L(e) c_L(\mu) g_{\mu\nu} \left[\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)\right] \left[\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)\right]$$



(L/R refer are helicities of the initial/final state particles (not anti-particles).)

Fortunately we have calculated terms like these on pages 113 and 114 (e.g. (57)) when considering $e^+e^- \to \gamma \to \mu^+\mu^-$.

- Re-using QED results for the Z-exchange matrix elements on page 114 gives:

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 \cdot c_R^2(e) \cdot c_R^2(\mu) \cdot (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 \cdot c_R^2(e) \cdot c_L^2(\mu) \cdot (1 - \cos\theta)^2$$

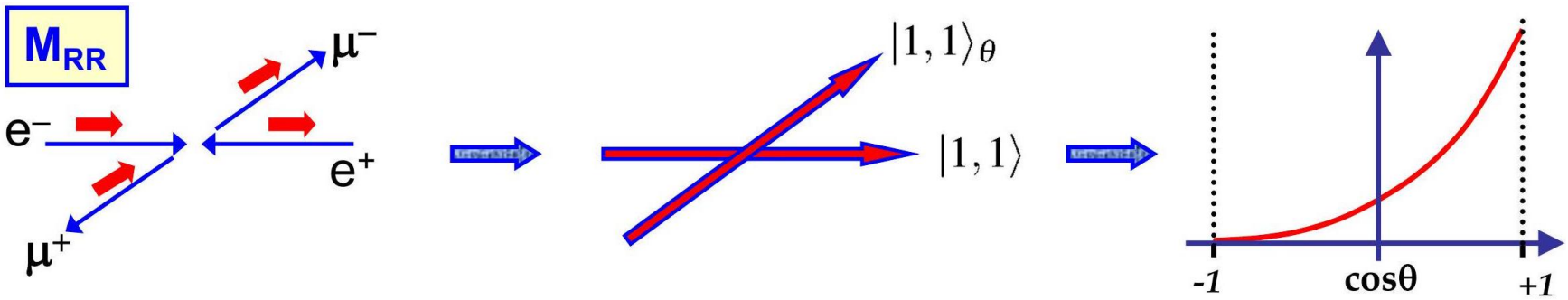
$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 \cdot c_L^2(e) \cdot c_R^2(\mu) \cdot (1 - \cos\theta)^2$$

$$|M_{LL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 \cdot c_L^2(e) \cdot c_L^2(\mu) \cdot (1 + \cos\theta)^2$$

$$\frac{e^2}{q^2} \to \frac{g_Z^2}{q^2 - m_Z^2} c(e)c(\mu)$$

where $q^2 = s = 4E_e^2$

- As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles. For example:



The Breit-Wigner Resonance I

- Note that the propagator term $1/\left(s - m_Z^2\right)$ diverges when the C.o.M. energy is equal to the rest mass of the Z boson. Is this physical? No. It is an artefact of using only the simplest diagram. Other diagrams account for the Z-boson's instability.
- A stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}.$$

- For an unstable particle the equivalent must amount to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \quad \text{with} \quad \tau = \frac{1}{\Gamma_Z}.$$

- This change could be effected by making the replacement $m \to m - i\Gamma/2$ which motivates making the substitution:

$$m_Z \to m_Z - i\Gamma_Z/2$$

in the Z boson propagator to model some of the effects of higher order diagrams:

$$\left(s - m_Z^2\right) \longrightarrow \left[s - \left(m_Z - i\Gamma_Z/2\right)^2\right] = s - m_Z^2 + i m_Z \Gamma_Z + \frac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + i m_Z \Gamma_Z$$

in which the approximation has assumed that $\Gamma_Z \ll m_Z$.

The Breit-Wigner Resonance II

- The aforementioned substitution results in

$$\left|\frac{1}{s-m_Z^2}\right|^2 \to \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}$$

and the Matrix elements change to things like this:

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}\cdot c_R^2(e)\cdot c_R^2(\mu)\cdot(1+\cos\theta)^2$$

which do not diverge as $s \to m_Z^2$ and are much more physical.

Physically reasonable c.o.m. differential cross sections for $e^+e^- \to Z \to \mu^+\mu^-$

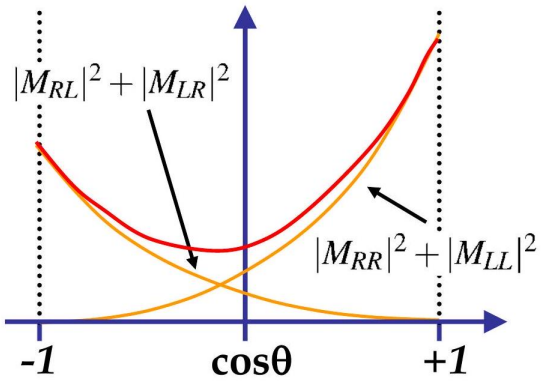
If we neglect m_e and m_μ, the result (10) on page 29 tells us that $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$ and so the differential cross sections for $e^+e^- \to Z \to \mu^+\mu^-$ broken down by particle helicty are:

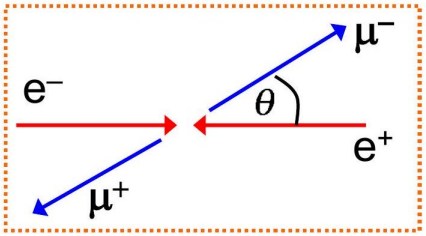
$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \cdot c_R^2(e) \cdot c_R^2(\mu) \cdot (1+\cos\theta)^2$$

$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \cdot c_L^2(e) \cdot c_L^2(\mu) \cdot (1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \cdot c_L^2(e) \cdot c_R^2(\mu) \cdot (1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \cdot c_R^2(e) \cdot c_L^2(\mu) \cdot (1-\cos\theta)^2$$





Aside regarding 'forward-backward asymmetry' (much more detail on this on slide 444)

Because $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).

Total cross section for Z-boson production (unpolarized beams) I

- To calculate the total cross section need to sum over all matrix elements and average over the initial spin states. Here, assuming unpolarized beams (i.e. both $\mathrm{e}^+$and both e^-spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\begin{aligned}\left\langle |M_{fi}|^2 \right\rangle =& \frac{1}{2}\cdot\frac{1}{2}\cdot\left(|M_{RR}|^2+|M_{LL}|^2+|M_{LR}|^2+|M_{RL}|^2\right)\\ =& \frac{1}{2}\cdot\frac{1}{2}\frac{g_Z^4 s^2}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}\times\left\{\left[c_R^2(e)c_R^2(\mu)+c_L^2(e)c_L^2(\mu)\right](1+\cos\theta)^2\right.\\ &\left.+\left[c_L^2(e)c_R^2(\mu)+c_R^2(e)c_L^2(\mu)\right](1-\cos\theta)^2\right\}.\end{aligned}$$

- The $\{\ldots\}$ part of the expression can be rearranged:

$$\begin{aligned}\{\ldots\} =& \left[c_R^2(e)+c_L^2(e)\right]\left[c_R^2(\mu)+c_L^2(\mu)\right]\left(1+\cos^2\theta\right)+\\ & 2\left[c_R^2(e)-c_L^2(e)\right]\left[c_R^2(\mu)-c_L^2(\mu)\right]\cos\theta\end{aligned}$$

and using $c_V^2+c_A^2=2\left(c_L^2+c_R^2\right)$ and $c_V c_A = c_L^2-c_R^2$

$$\{\ldots\}=\tfrac{1}{4}\left[c_V^2(e)+c_A^2(e)\right]\left[c_V^2(\mu)+c_A^2(\mu)\right]\left(1+\cos^2\theta\right)+2c_V^{(e)}c_A^{(e)}c_V^{(\mu)}c_A^{(\mu)}\cos\theta.$$

Total cross section for Z-boson production (unpolarized beams) II

- Hence the complete expression for the unpolarized differential cross section is:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{1}{64\pi^2 s}\left\langle |M_{fi}|^2 \right\rangle \\ &= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}\times \\ &\quad \left\{\tfrac{1}{4}\left[c_V^2(e)+c_A^2(e)\right]\left[c_V^2(\mu)+c_A^2(\mu)\right]\left(1+\cos^2\theta\right)+2c_V^{(e)}c_A^{(e)}c_V^{(\mu)}c_A^{(\mu)}\cos\theta\right\}.\end{aligned}$$

- Integrating over solid angle $\mathrm{d}\Omega = \mathrm{d}\phi\mathrm{d}(\cos\theta) = 2\pi\mathrm{d}(\cos\theta)$

$$\int_{-1}^{+1}\left(1+\cos^2\theta\right)\mathrm{d}(\cos\theta) = \int_{-1}^{+1}\left(1+x^2\right)dx = \frac{8}{3} \text{ and } \int_{-1}^{+1}\cos\theta\cdot\mathrm{d}(\cos\theta) = 0$$

and so the total cross section is:

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{\left(s-m_Z^2\right)^2+m_Z^2\Gamma_Z^2}\left[c_V^2(e)+c_A^2(e)\right]\left[c_V^2(\mu)+c_A^2(\mu)\right]. \quad (176)$$

Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions $c_V^2(f)+c_A^2(f)$.

Relativistic Breit-Wigner Form of Z cross section

- The Z-boson decay rates (partial widths) from (175) on page 423 were:

$$\Gamma_{ee} = \Gamma\left(Z \to e^+e^-\right) = \frac{g_Z^2 m_Z}{48\pi}\left[c_V^2(e) + c_A^2(e)\right]$$

and

$$\Gamma_{\mu\mu} = \Gamma\left(Z \to \mu^+\mu^-\right) = \frac{g_Z^2 m_Z}{48\pi}\left[c_V^2(\mu) + c_A^2(\mu)\right] ..$$

- The above formulae contain **sums of vector and axial couplings** they can be used to eliminate those same sums from our last formula for the total cross section (176):

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{\left(s - m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}\left[c_V^2(e) + c_A^2(e)\right]\left[c_V^2(\mu) + c_A^2(\mu)\right].$$

This leaves us with the much simpler formula for the total Z-boson cross section:

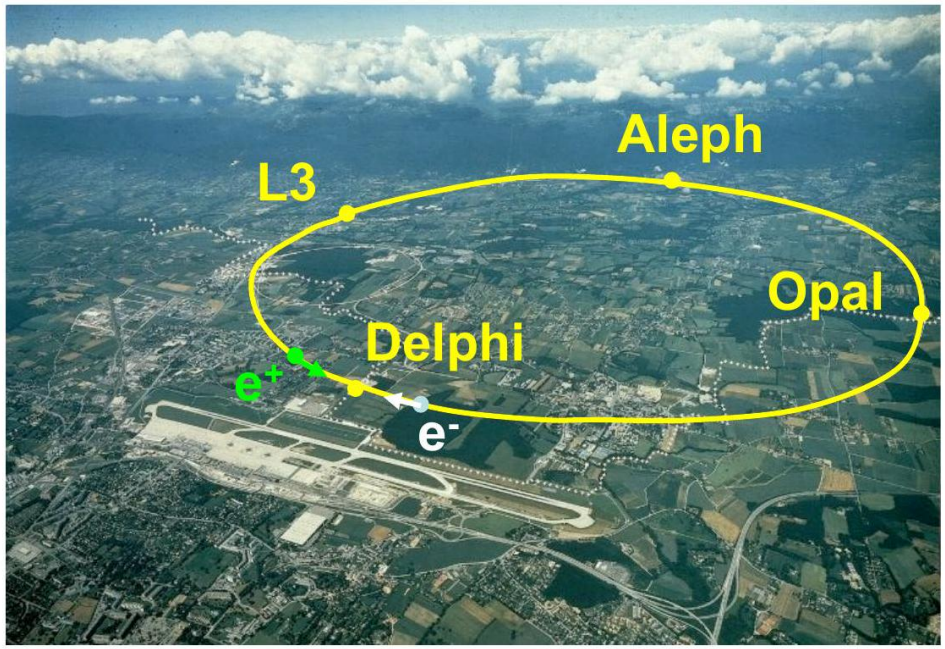
$$\sigma\left(e^+e^- \to Z \to f\bar{f}\right) = \frac{12\pi}{m_Z^2}\frac{s}{\left(s - m_Z^2\right)^2 + m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \tag{177}$$

where $\boldsymbol{f}$ is the final state fermion flavour.

(A comparison of (177) with the non-relativistic Breit-Wigner formula seen in the Part II Course is given in Appendix XXV.)

Electroweak Measurements at LEP

The Large Electron Positron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.

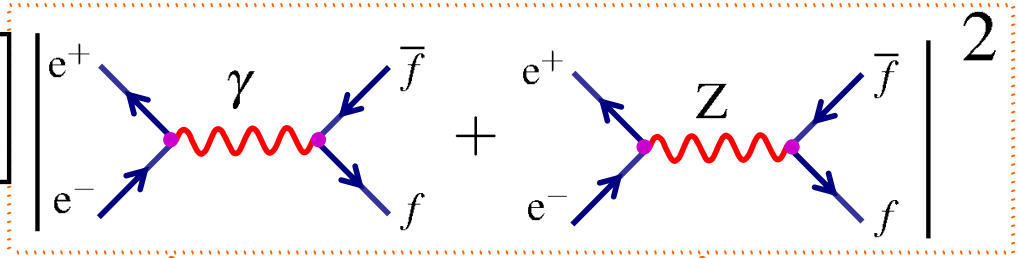


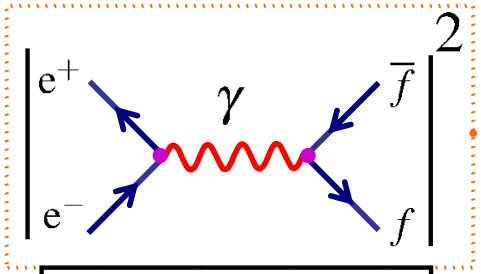
- 26 km circumference accelerator straddling French/Swiss border
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):ALEPH, DELPHI, L3, OPAL

- 1989-1995: collisions at $\sqrt{s} = 91.2\ \mathrm{GeV}$
  - 17 Million Z bosons detected.
- 1996-2000 : collisions at $\sqrt{s} = 161 - 208\ \mathrm{GeV}$
  - 30,000 W^+W^- events detected.
- LEP was therefore sometimes referred to as a Z and W 'factory'.

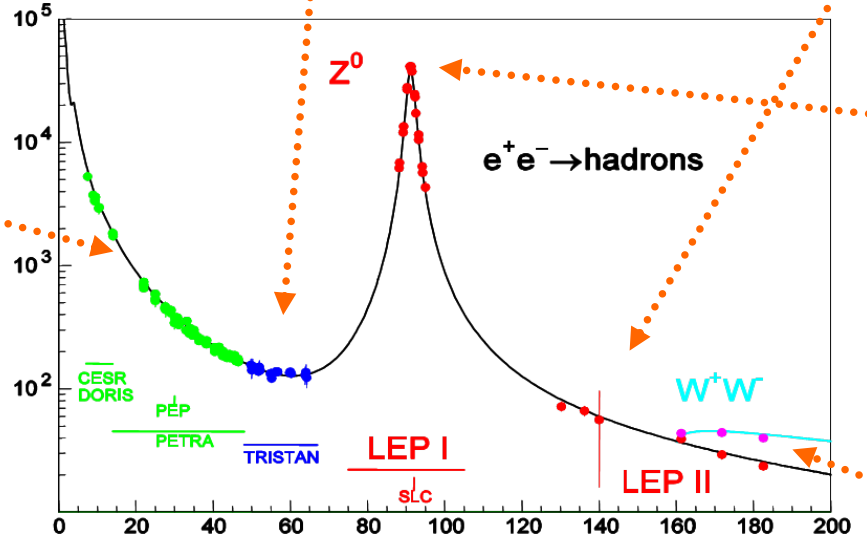
e^+e^- annihilation Feynman Diagrams – changes in importance with $\sqrt{s}$

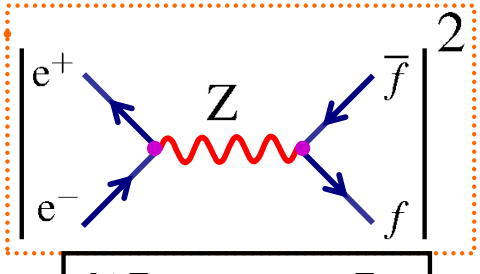
In general e⁺e⁻ annihilation involves both photon and Z exchange : + interference





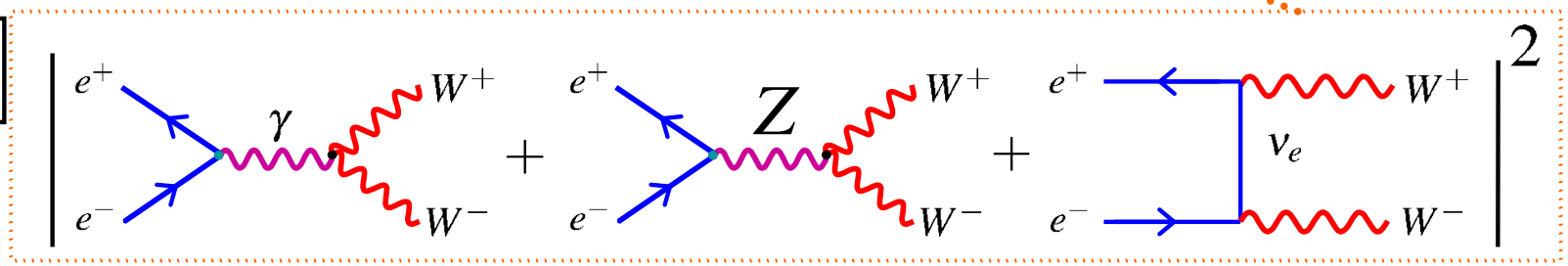
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

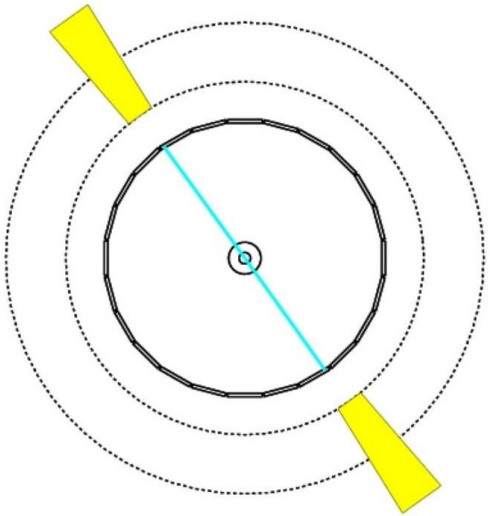
High energies: WW production



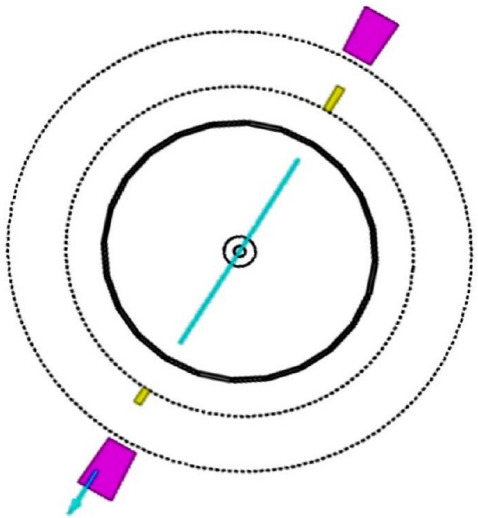
Z-boson cross section measurements and integrated luminosity

- On resonance there are a small number of event types including, e.g.:

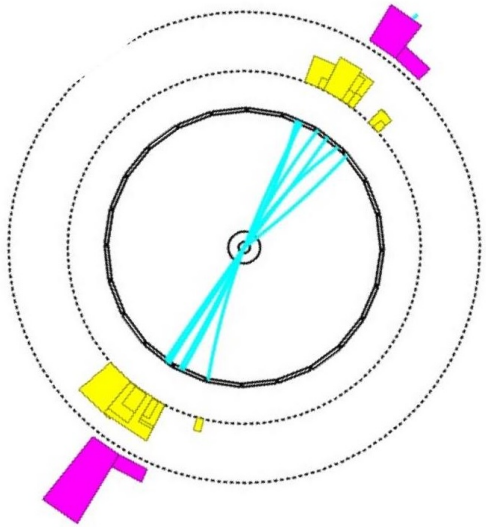
$e^+e^- \to Z \to e^+e^-$



$e^+e^- \to Z \to \mu^+\mu^-$



$e^+e^- \to Z \to \text{jet}^+\text{jet}^-$



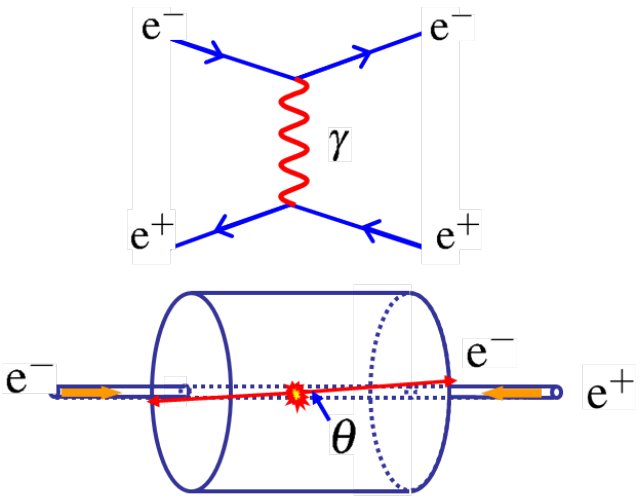
- To experimentally measure the cross section, σ, we can use $N_{\text{events}} = L\sigma$ where
  - N_{events} is the number of observed events of each type, and
  - L is the 'integrated luminosity'.
- Note that our original defintion of cross section in (6) (see page 25) did not use L. It defined σ instead in terms of incident flux and interactions per unit time.

No exciting new physics hides behind the introduction of the 'integrated luminosity', L.

View L just as the name for the correct constant of proportionality that links N_{events} to σ.

'Stealing' the integrated luminosity from a well understood process

- Calculating/predicting the collider's integrated luminosity from beam properties alone is very difficult. Uncertainty of $\sim 10\%$ as beam profiles are hard to measure.
- But since **'integrated luminosity' is a collider property** (not a property of any event) the value of L can be extracted from a (different) process that is well understood.



- Use the QED **Bhabha Scattering** process.
- σ_{Bhabha} can be calclated very precisely from QED.
- Reaction is very forward peaked — i.e. the electron tends not to get deflected much
  $\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{sin^4(\theta/2)} \rightarrow \frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}$
  (Photon propagator), (see (82) on page 134)
- V. large forward cross section (small statistical errors).
- Measure N_{Bhabha}, number of events where the electron is scattered in the very forward direction.
- $N_{\text{Bhabha}} = L\sigma_{\text{Bhabha}}$ is then used to find L.

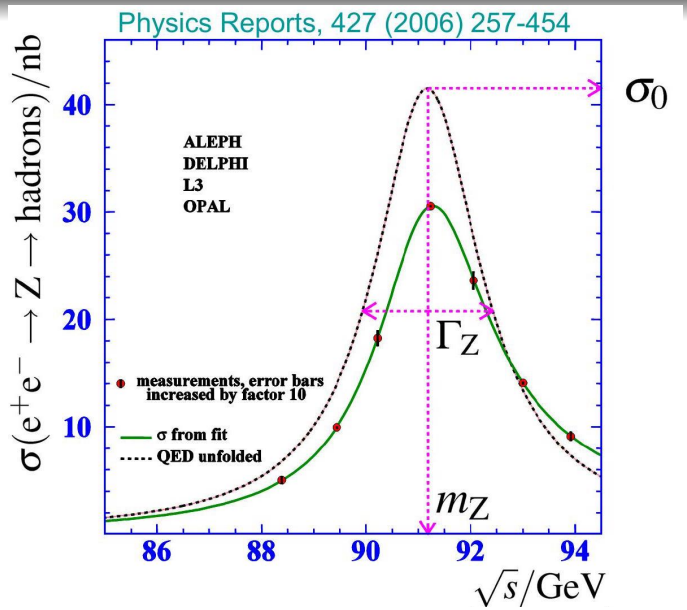
- Hence Z-boson (or other) cross sections can be expressed as multiple of Bhabha:

$$\sigma_{Z\to ff} = \frac{N_{Z\to ff}}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}. \tag{178}$$

Nice! Z-boson cross section measuring is now 'mostly' counting!

Measurements of the Z line-shape

- The way the Z-boson's cross section varies as a function of $\sqrt{s}$ is referred to as its 'line-shape'.
- Measurements of the Z-boson line-shape determine:
  - m_Z : peak of the resonance,
  - Γ_Z : FWHM of resonance,
  - Γ_f : Partial decay widths,
  - N_V : Number of light neutrino generations.
- We have recently proved in (177) that



$$\sigma\left(e^+e^- \to Z \to f\bar{f}\right) = \frac{12\pi}{m_Z^2} \frac{s}{\left(s - m_Z^2\right)^2 + m_Z^2\Gamma_Z^2} \Gamma_{ee}\Gamma_{ff} \tag{179}$$

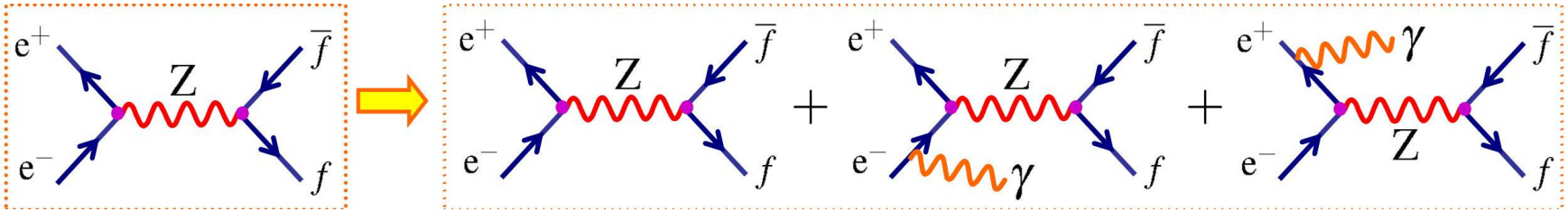
so maximum cross section occurs at $\sqrt{s} = m_Z$ and peak cross section is equal to

$$\boxed{\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}}. \tag{180}$$

That the cross section falls to half peak value at $\sqrt{s} \approx m_z \pm \frac{\Gamma_Z}{2}$ can be seen in (179).

Take home message: $\Gamma_Z = \frac{\hbar}{\tau_Z}$ can be 'seen' in the FWHM of the resonance lineshape.

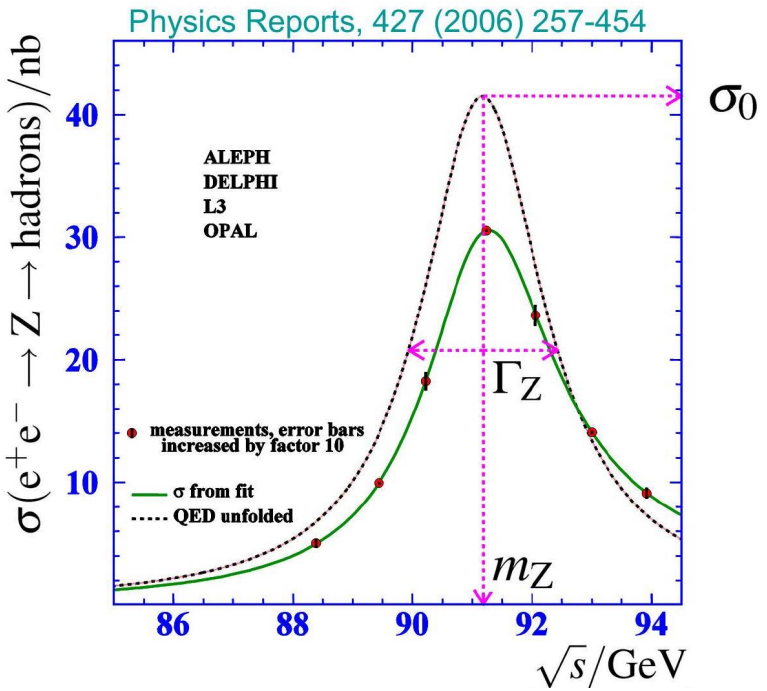
- In practice, it is not that simple, QED corrections distort the measured line-shape.
- One particularly important correction: initial state radiation (ISR):



- Initial state radiation reduces the centre-of-mass energy of the 'inner' e^+e^- collision.
- Measured cross section, σ_{meas}, can be written:

$$\sigma_{\text{meas}}\left(\sqrt{s}\right) = \int_0^{\sqrt{s}} \sigma(\sqrt{s'})p(\sqrt{s'} \mid \sqrt{s})\mathrm{d}\sqrt{s'}$$

where $p(\sqrt{s'} \mid \sqrt{s})$ is the probability density for the post-ISR $e^+e^- \to Z$ part of the collision to occur at $\sqrt{s'}$ given that the initial state energy available pre-ISR was $\sqrt{s}$, and where σ is the ISR-free cross section we calculated earlier (see (177) or (179)).



- Fortunately QED can be used to calculate $p(\cdot \mid \cdot)$.
- The ISR-distorted σ_{meas} measured lineshape can therefore be compared with calculations/predictions based on the convolution of the ISR-free σ and $p(\cdot \mid \cdot)$.

Precision measurements of Z-boson mass and width

World average at 2024 (largely dominated by LEP measurements) is:

$$\boxed{m_Z = 91.1875 \pm 0.0021 \text{ GeV}} \qquad \boxed{\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}}$$

Not examinable:

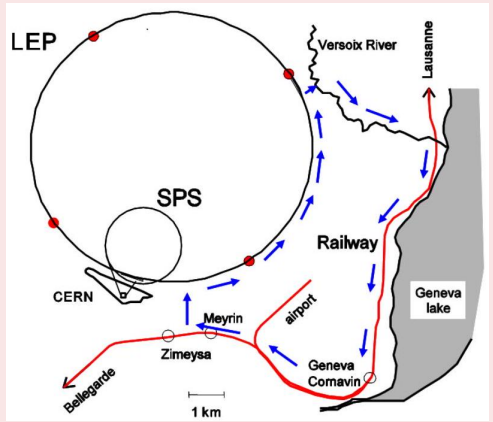
Achieving such high precision required other surprising factors to be taken into account beyond ISR. Examples include:

Position of the Moon:

- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly and the nominal radius of the accelerator of 4.3 km varies by ± 0.15 mm.
- This changes beam energy by ~ 10 MeV, so need to correct for tidal effects!

Train tables:

- Leakage currents from the TGV railway line returned to Earth following the path of least resistance, a small part of which was via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator changing the LEP beam energy by ~ 10 MeV.



Count the number of generations via the Z's 'invisible width' ?

Observed Z-width, Γ_Z, is the sum of all partial widths including to invisible particles!

- 3-generation Standard Model predicts:

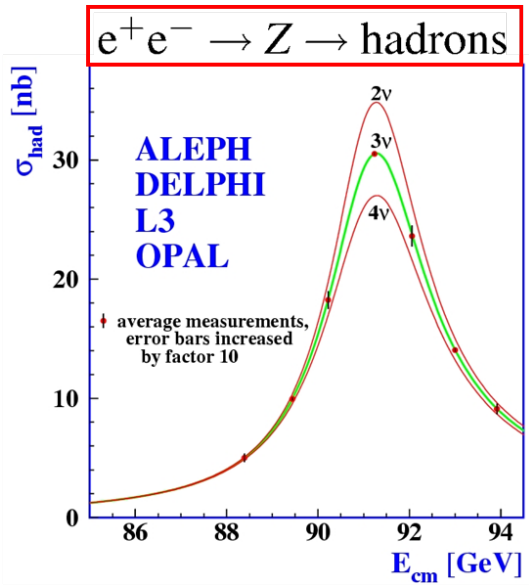
$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} = \boxed{\Gamma_{\text{visible}} + 3 \cdot \Gamma_{\nu_1\nu_1}}.$$

- If there were an additional 4th generation would expect $Z \to \nu_4\bar{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. $> m_z/2$) so
- $\geq$ 4-generation Standard Model predicts:

$$\Gamma_Z \geq \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + \Gamma_{\nu_4\nu_4} = \boxed{\Gamma_{\text{visible}} + 4 \cdot \Gamma_{\nu_1\nu_1}}.$$

Recall from (180) that

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}.$$



Given:

- Γ_Z measured from Z lineshape,
- $\Gamma_{\nu_1\nu_1}$ calculated (e.g. as in qu. 26), and
- $\Gamma_{ee}, \Gamma_{\mu\mu}, \ldots$ measured from peak cross sections, $\sigma^0_{ee}, \sigma^0_{\mu\mu}, \ldots$,

a fit of N_ν to $\boxed{\Gamma_Z = \Gamma_{\text{visible}} + N_\nu \cdot \Gamma_{\nu_1\nu_1}}$ finds:

$$\boxed{N_\nu = 2.9840 \pm 0.0082}.$$

Forward-Backward Asymmetry for $e^+e^- \to Z \to \mu^+\mu^-$

On pages 433 and 434 we obtained the expression for the differential cross section:

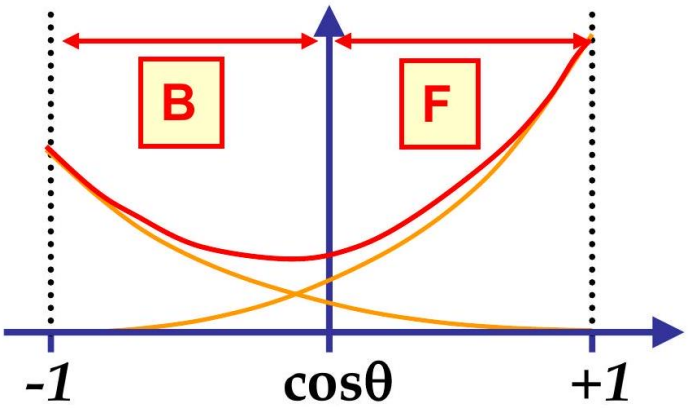
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \kappa\left[A\left(1+\cos^2\theta\right)+2B\cos\theta\right] \tag{181}$$

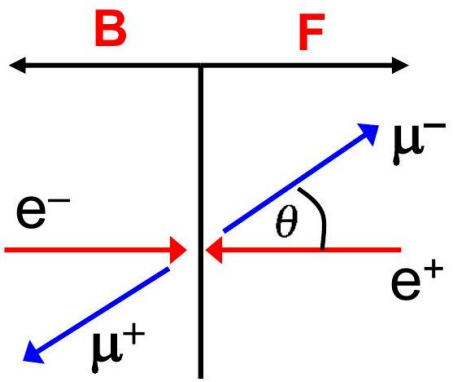
where κ was independent of the muon scattering angle, θ, and where A and B were

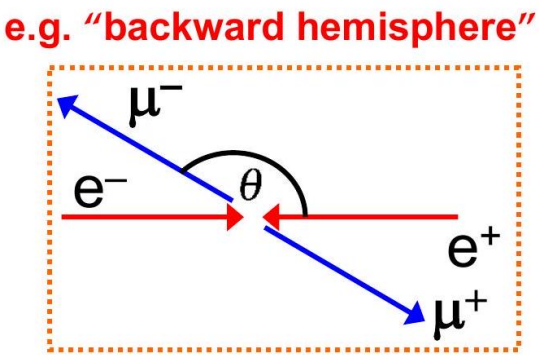
$$A = \left[c_L^2(e)+c_R^2(e)\right]\left[c_L^2(\mu)+c_R^2(\mu)\right] \quad \text{and} \quad B = \left[c_L^2(e)-c_R^2(e)\right]\left[c_L^2(\mu)-c_R^2(\mu)\right].$$

We define the FORWARD and BACKWARD cross sections as follows:

$$\sigma_F \equiv \sigma_{\cos\theta>0} = \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta \quad \text{and} \quad \sigma_B \equiv \sigma_{\cos\theta<0} = \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta.$$

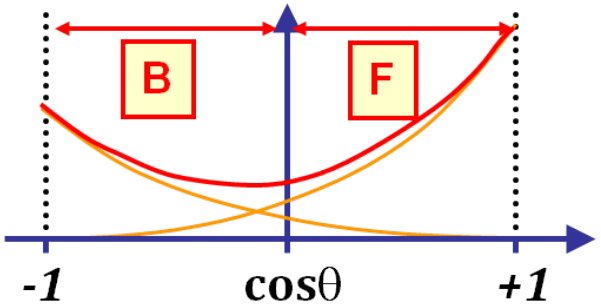






- The Forward-Backward Asymmetry, A_{FB}, is **defined** by:

$$\boxed{A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}}.$$



- Integrating equation (181) (while replacing $\cos\theta$ with x) gives:

$$\sigma_F \propto \int_0^1 \left[A\left(1+x^2\right)+2Bx\right]\mathrm{d}x = \frac{4}{3}A + B \propto \boxed{4A+3B}$$

$$\sigma_B \propto \int_{-1}^0 \left[A\left(1+x^2\right)+2Bx\right]\mathrm{d}x = \frac{4}{3}A - B \propto \boxed{4A-3B}, \qquad \text{so}$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{6B}{8A} = \frac{3}{4}\left[\frac{c_L^2(e)-c_R^2(e)}{c_L^2(e)+c_R^2(e)}\right]\cdot\left[\frac{c_L^2(\mu)-c_R^2(\mu)}{c_L^2(\mu)+c_R^2(\mu)}\right] = \frac{3}{4}A_eA_\mu$$

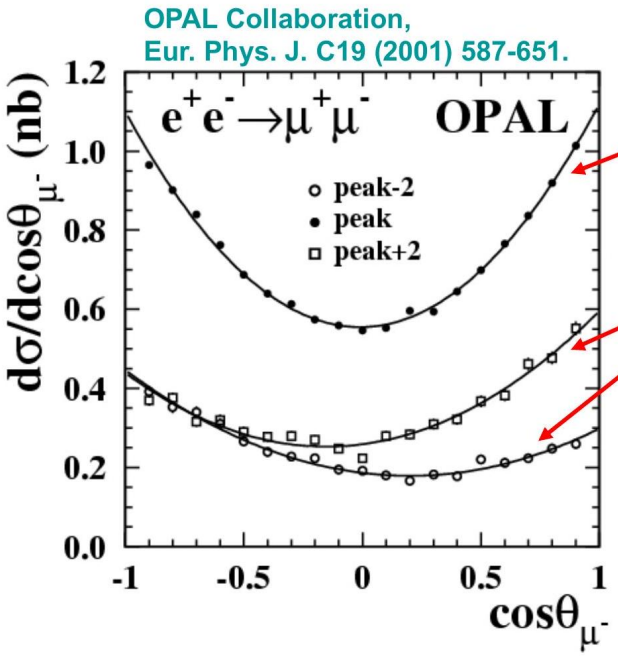
where

$$\boxed{A_x \equiv \frac{c_L^2(x)-c_R^2(x)}{c_L^2(x)+c_R^2(x)} = \frac{2c_V(x)c_A(x)}{c_V^2(x)+c_A^2(x)} = \frac{2\cdot\left(c_V(x)/c_A(x)\right)}{1+\left(c_V(x)/c_A(x)\right)^2} = f\left(\frac{c_V(x)}{c_A(x)}\right)}. \quad (182)$$

- We observe a non-zero asymmetry A_{FB} because the couplings of the Z to L and R particles differ so $A_e \neq 0 \neq A_\mu$. Contrast with QED where the L and R couplings are the same (parity is conserved) and the interaction is FB symmetric.

Measured Forward-Backward Asymmetries

These are the observed muon $\cos\theta$ distributions from the LEP experiment 'OPAL':



Because $\sin^2\theta_w \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

- A_e, A_μ and A_τ values can be extracted from the above A_{FB} values via $A_{FB}^{\mu} = \frac{3}{4} A_e A_\mu$.

- A mixture of the above measurements and other related measurments (e.g. A_{LR} of Appendix XXVI) finds that

$$\begin{aligned} A_e &= 0.1514 \pm 0.0019, \\ A_\mu &= 0.1456 \pm 0.0091, \\ A_\tau &= 0.1449 \pm 0.0040. \end{aligned} \tag{183}$$

Measuring the Weak Mixing Angle, θ_W, from F-B asymmetries

- We saw already in (182) that A_x is a function of just $c_V(x)/c_A(x)$.

- Proof below shows that $c_V(x)/c_A(x)$ is only a function of the weak mixing angle θ_W.

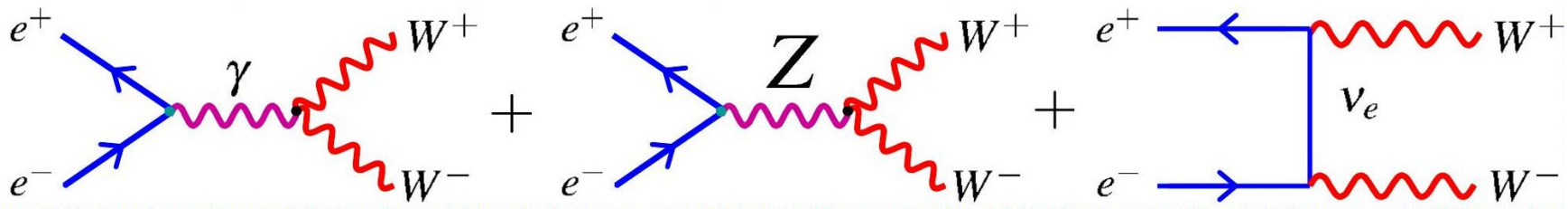
$$\begin{aligned}
\frac{c_V(x)}{c_A(x)} &= \frac{I_W^3(x_L) - 2Q_x \sin^2\theta_W}{I_W^3(x_L)} && \text{(by (170) and (171))} \\
&= 1 - \frac{2Q_x}{I_3(x_L)} \sin^2\theta_W \\
&= 1 - 4|Q_x| \sin^2\theta_W && \text{(true for every row of the table on page 421)} \\
&= 1 - 4\sin^2\theta_W && \text{(only true for } x \in \{e, \mu, \tau\}\text{).}
\end{aligned}$$

Therefore a clean measurement of the weak mixing angle θ_W can be obtained from the measurements of A_e, A_μ and A_τ listed in (183). It is found that:

$$\sin^2\theta_W = 0.23154 \pm 0.00016.$$

W^+W^- Production

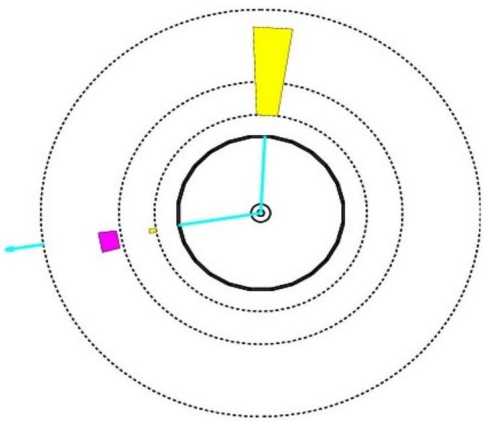
- From 1995-2000 LEP operated above the threshold for W-pair production.
- The three tree level diagrams for W-pair production are shown below.



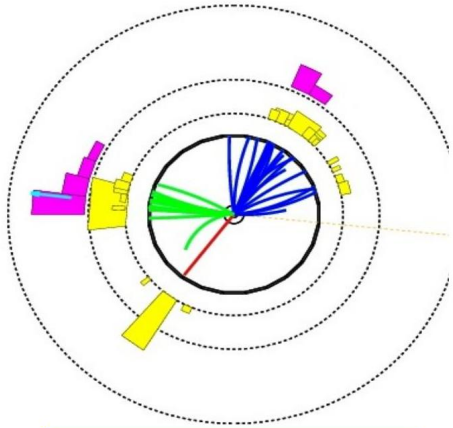
- [Page 409]: W-bosons decay to either to leptons or hadrons with branching fractions:

$$\text{Br}\left(W^- \to \text{ hadrons }\right) \approx 0.67 \quad \text{Br}\left(W^- \to e^- \bar{\nu}_e\right) \approx 0.11$$
$$\text{Br}\left(W^- \to \mu^- \bar{\nu}_\mu\right) \approx 0.11 \quad \text{Br}\left(W^- \to \tau^- \bar{\nu}_\tau\right) \approx 0.11$$

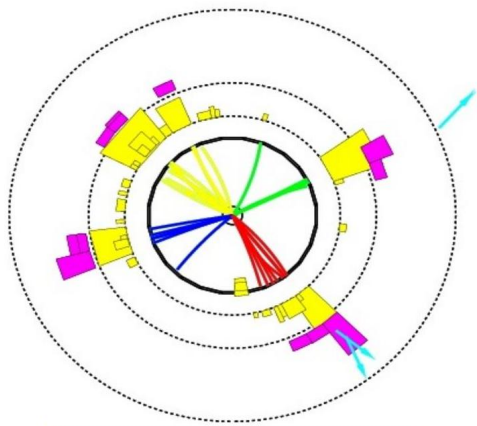
giving rise to three distinct signatures:



$W^+W^- \to \ell^+ \nu \ell^- \bar{\nu}$



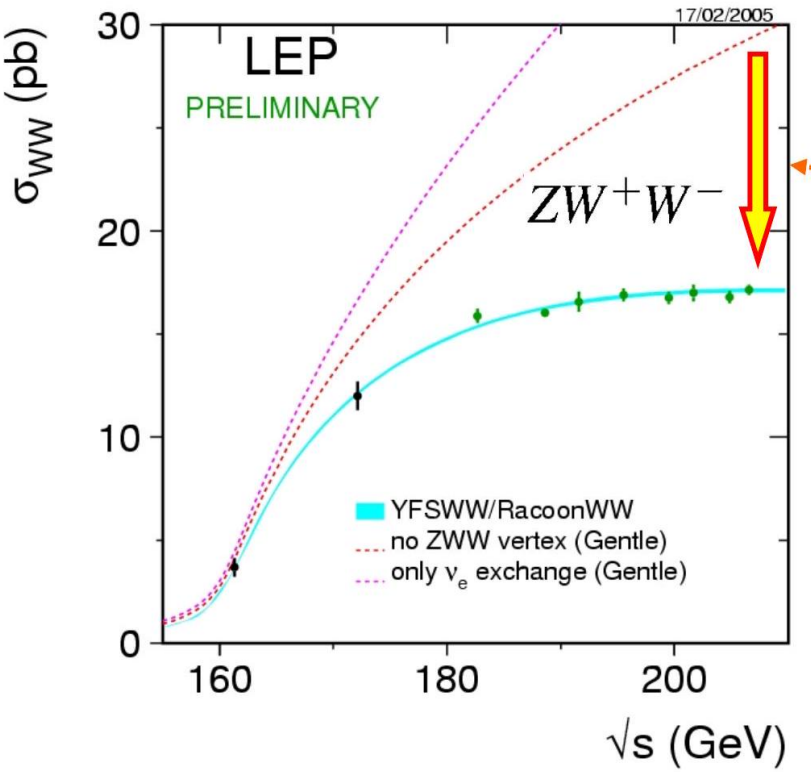
$W^+W^- \to q\bar{q}\ell\nu$



$W^+W^- \to q\bar{q}q\bar{q}$

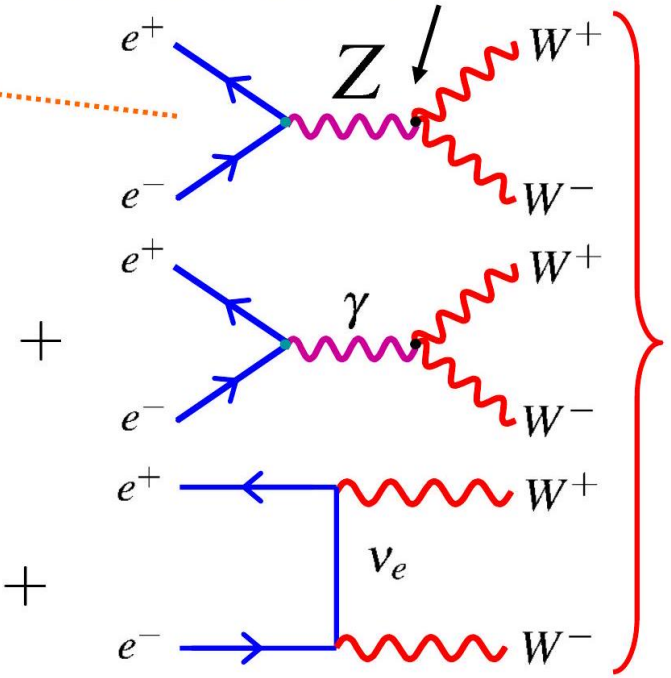
$e^+e^- \to W^+W^-$ line shape provides confirmation of ZWW vertex

Cross section is again normalised against Bhabha scattering (see (178) on page 439).



★ **Data consistent with SM expectation**

★ **Provides a direct test of ZW^+W^- vertex**



- Without the Z diagram the WW cross section would violate unitarity.
- The presence of the Z fixes this problem.

W-mass and W-width

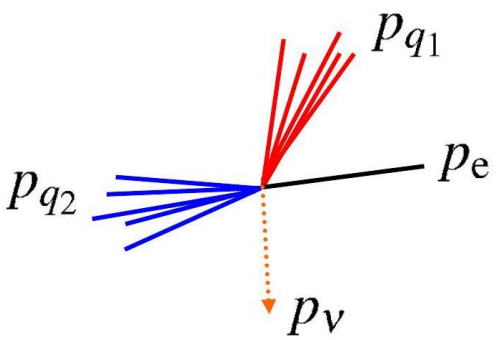
- The W line-shape is not used to find the W mass as production is not resonant. Instead, W masses are directly reconstructed in each event (often twice).

- E.g. in $W^+W^- \to q\bar{q}e^-\bar{\nu}_e$ the two mass estimates would be defined by

$$m^2_{W1} = (p_e + p_{\nu_e})^\mu (p_e + p_{\nu_e})_\mu$$
$$m^2_{W2} = (p_{q_1} + p_{q_2})^\mu (p_{q_1} + p_{q_2})_\mu$$

  in which the momentum $p^\mu_{\nu_e}$ of the unobserved neutrino would be deduced by solving

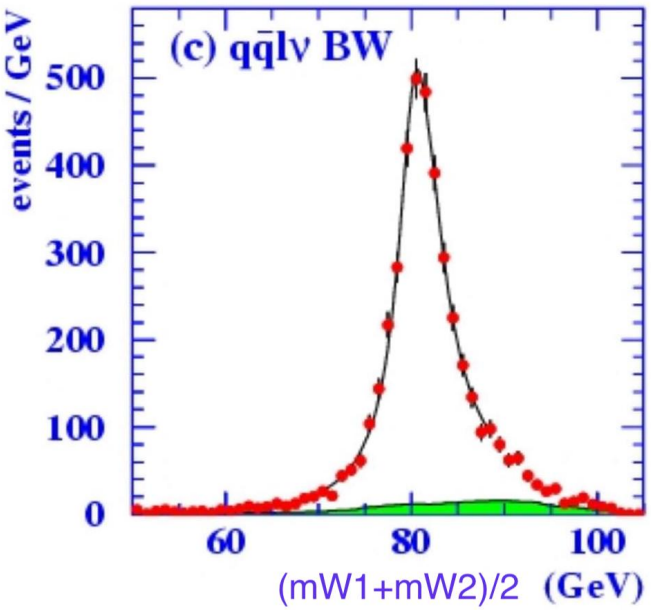
$$(p_e + p_{\nu_e} + p_{q_1} + p_{q_2})^\mu = (\sqrt{s}, 0)^\mu.$$



- Shape of reconstructed mass distribution gives:

$$m_W = 80.376 \pm 0.033 \text{ GeV}$$ and

$$\Gamma_W = 2.196 \pm 0.083 \text{ GeV}.$$



Hadron Colliders after LEP (Tevatron, LHC) have improved upon these results.

The Higgs Mechanism

- Higgs Fields can be used to give masses to both fermions and gauge bosons - but mechanism is different in the two cases.

- Explaining how the Higgs mechanism gives the W and Z gauge bosons masses, while leaving the photon massless, is (unfortunately) beyond this course. [See Gauge Field Theory minor option).]

- We do, howwever, attempt to describe the way interactions with the Higgs field gives masses to fermions.

Higgs Learning Objectives Overview

Higgs field is more important than Higgs Boson

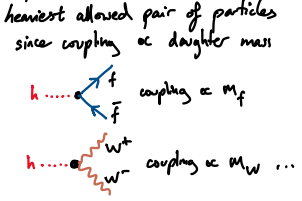
Higgs field gives mass to bosons and fermions in different ways.

Mass given to $W^\pm$ & Z bosons by:
- Spontaneous Symmetry Breaking
- Goldstone Bosons
- Gauge Field Theory Course!

Higgs Boson

Higgs Boson must be a scalar (spin-0) because Yukawa interaction (see later) has no spare indices to contract with a spinor or vector field.

prefers to decay to heaviest allowed pair of particles since coupling $\propto$ daughter mass



Mass for Fermions is supplied by Yukawa interactions:

[Complex Scalar Higgs Field $SU(2)_L$ Doublet]

$$\mathcal{L} = \left(\ldots + y_e \overline{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L} \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} e_R + \ldots\right) + \text{complex conjugate}$$

not Dirac mass term:

$$\mathcal{L} = \ldots + m_e \bar{e} e + \ldots$$

because:
- Yukawa mass $\mathcal{L}$ is $SU(2)_L$ Gauge Invariant
- Dirac mass $\mathcal{L}$ is not $SU(2)_L$ Gauge Invariant

} Why?

and because
- Yukawa interactions "act like" Dirac mass terms if $\langle\phi^0\rangle \neq 0$.

} Why?

$\langle\phi^0\rangle$ is called the "Higgs V.E.V." (vacuum expectation value)

[Measured value: $\langle\phi^0\rangle = 246\,\text{GeV}$]

Under $SU(2)_L$ transformation:

$$\left\{ \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \to e^{i\frac{\theta\cdot\sigma}{2}} \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L = \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \right\}$$
$$\left\{ \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \to e^{i\frac{\theta\cdot\sigma}{2}} \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \right\}$$
$$e_R \to e_R$$

hence:

$$y_e \overline{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L} \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} e_R \to y_e \overline{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L} e^{-i\frac{\theta\cdot\sigma}{2}} e^{i\frac{\theta\cdot\sigma}{2}} \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} e_R = y_e \overline{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L} \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} e_R$$ ✓ invariant

but:

$$m_e \bar{e} e = m_e \bar{e}(P_L + P_R) e \qquad (P_L + P_R = 1)$$
$$= m_e \bar{e}(P_L^2 + P_R^2) e \qquad (P_L^2 = P_L \;\&\; P_R^2 = P_R)$$
$$= m_e \bar{e} P_L^2 e + m_e \bar{e} P_R^2 e$$
$$= m_e \bar{e}_R e_L + m_e \bar{e}_L e_R \qquad (\bar{\psi} P_{R/L} = \bar{\psi}_{L/R} \;\&\; P_{R/L}\psi = \psi_{R/L})$$

$$\to m_e \bar{e}_R (c\,\nu_{eL} + d\,e_L) + m_e (\bar{c}\,\bar{\nu}_{eL} + \bar{d}\,\bar{e}_L) e_R .$$

Not $m_e \bar{e} e$! × Not invariant!

Yukawa Mass Term $= y_e \overline{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L} \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} e_R$ + c.c. ("complex conjugate")

$$= \left(y_e \bar{\nu}_L \phi^+ e_R + y_e \bar{e}_L \phi^0 e_R\right) + \text{c.c.}$$
$$= \left(y_e \bar{e}_L \phi^0 e_R + y_e \bar{e}_R \phi^0 e_L\right) + \left(y_e \bar{\nu}_{eL} \phi^+ e_R + \text{c.c.}\right)$$
$$= (y_e \phi^0)\left(\bar{e}_L e_R + \bar{e}_R e_L\right) + \left(y_e \bar{\nu}_{eL} \phi^+ e_R + \text{c.c.}\right)$$
$$= (y_e \phi^0)\,\bar{e}\,e + \left(y_e \bar{\nu}_{eL} \phi^+ e_R + y_e \bar{e}_R \phi^- \nu_{eL}\right)$$

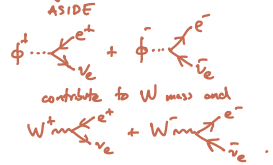
Behaves like $m_e \bar{e} e$ (Dirac mass term)

provided ϕ^0 does not wobble too much

and provided $\langle y_e \phi^0 \rangle = m_e$

i.e. provided $\underset{\text{Higgs VEV}}{\langle\phi^0\rangle} = \frac{m_e}{y_e} \neq 0$.

ASIDE

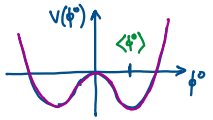


But why should $\langle\phi^0\rangle$ bother to take a non-zero value?

Give Lagrangian a Higgs Potential $V(\phi^0)$ which is minimised at $\phi^0 = \langle\phi^0\rangle_{VEV} \neq 0$.

e.g.:

$$V(\phi^0) = a(\phi^0)^4 - b(\phi^0)^2$$



ASIDE (Not Examinable!)

Strictly one must use something like:

$$a\left[\begin{pmatrix}\phi^{0\dagger} & \phi^{+\dagger}\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\right]^2 - b\begin{pmatrix}\phi^{+\dagger} & \phi^{0\dagger}\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$$

to maintain $SU(2)_L$ Gauge Invariance

Higgs field is more important than Higgs Boson

Higgs field gives mass to bosons and fermions in different ways.

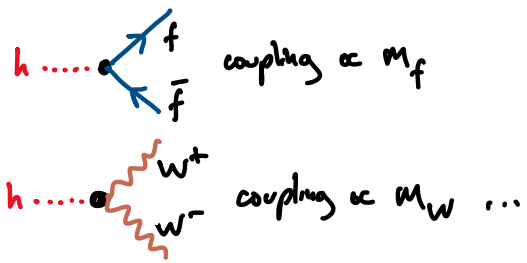
Mass given to $W^{\pm}$ & Z bosons by:

- Spontaneous Symmetry Breaking
- Goldstone Bosons
- Gauge Field Theory Course!

Higgs Boson

Higgs Boson must be a scalar (spin-0) because Yukawa interaction has no spare indices to contract with a spinor or vector field.

prefers to decay to heaviest allowed pair of particles since coupling $\propto$ daughter mass



Mass for Fermions is supplied by

Yukawa interactions:

[Complex Scalar Higgs Field $SU(2)_L$ Doublet]

$$\mathcal{L} = \left(\ldots + y_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \ldots \right) + \text{complex conjugate}$$

not **Dirac mass term:**

$$\mathcal{L} = \ldots + m_e \bar{e} e + \ldots$$

because:

- Yukawa mass $\mathcal{L}$ is $SU(2)_L$ Gauge Invariant
- Dirac mass $\mathcal{L}$ is not $SU(2)_L$ Gauge Invariant

} Why?

and because

- Yukawa interactions "act like" Dirac mass terms if $\langle \phi^0 \rangle \neq 0$.

} Why?

lue)

$\langle \phi^0 \rangle$ is called the "Higgs V.E.V." (vacuum expectation value)

[Measured value: $\langle \phi^0 \rangle = 246\,\text{GeV}$]

• Yukawa interactions act like mass terms if $\langle \phi^0 \rangle \neq 0$.

Under $SU(2)_L$ transformation:

$$\left\{ \begin{array}{l} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \to e^{i\underline{\theta}\cdot\underline{\sigma}} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \\ \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to e^{i\underline{\theta}\cdot\underline{\sigma}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\ e_R \to e_R \end{array} \right\}$$

hence:

$$y_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R \to y_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} e^{-i\underline{\theta}\cdot\underline{\sigma}} e^{i\underline{\theta}\cdot\underline{\sigma}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R = y_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R$$ ✓ invariant

but:

$$\begin{aligned} m_e \bar{e} e &= m_e \bar{e} (P_L + P_R) e && (P_L + P_R = 1) \\ &= m_e \bar{e} (P_L^2 + P_R^2) e && (P_L^2 = P_L \ \& \ P_R^2 = P_R) \\ &= m_e \bar{e} P_L^2 e + m_e \bar{e} P_R^2 e \\ &= m_e \bar{e}_R e_L + m_e \bar{e}_L e_R && (\bar{\psi} P_{R/L} = \bar{\psi}_{L/R} \ \& \ P_{R/L} \psi = \psi_{R/L}) \end{aligned}$$

$$\to m_e \bar{e}_R (c\, \nu_{e_L} + d\, e_L) + m_e (\bar{c}\, \bar{\nu}_{e_L} + \bar{d}\, \bar{e}_L) e_R .$$

Not $m_e \bar{e} e$!
× Not invariant!

Yukawa Mass Term $= y_e \overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \text{c.c.}$ ("complex conjugate")

$= \left(y_e \bar{\nu}_{eL} \phi^+ e_R + y_e \bar{e}_L \phi^0 e_R \right) + \text{c.c.}$

$= \left(y_e \bar{e}_L \phi^0 e_R + y_e \bar{e}_R \phi^0 e_L \right) + \left(y_e \bar{\nu}_{eL} \phi^+ e_R + \text{c.c.} \right)$

$= (y_e \phi^0)(\bar{e}_L e_R + \bar{e}_R e_L) + \left(y_e \bar{\nu}_{eL} \phi^+ e_R + \text{c.c.} \right)$

$= \underbrace{(y_e \phi^0) \bar{e} e}_{\downarrow} + \underbrace{\left(y_e \bar{\nu}_{eL} \phi^+ e_R + y_e \bar{e}_R \phi^- \nu_{eL} \right)}_{\downarrow}$

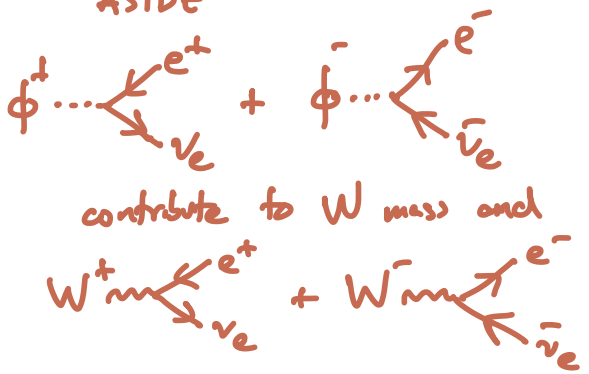
Behaves like $m_e \bar{e} e$ (DIRAC MASS TERM)

provided ϕ^0 does not wobble too much

and provided $\langle y_e \phi^0 \rangle = m_e$

i.e. provided $\underbrace{\langle \phi^0 \rangle}_{\text{HIGGS VEV}} = \frac{m_e}{y_e} \neq 0.$

ASIDE

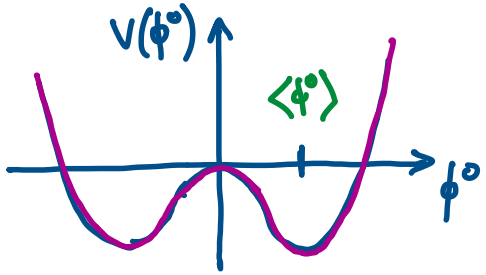


contribute to W mass and

Give Lagrangian a Higgs Potential $V(\phi^0)$ which is minimised at $\phi^0 = \langle \phi^0 \rangle_{\text{VEV}} \neq 0$.

e.g. :

$$V(\phi^0) = a(\phi^0)^4 - b(\phi^0)^2$$



But why should $\langle \phi^0 \rangle$ bother to take a non-zero value?

Behaves like $m_e \bar{e} e$ (DIRAC MASS TERM)

provided ϕ^0 does not wobble too much

and provided $\langle y_e \phi^0 \rangle = m_e$

i.e. provided $\langle \phi^0 \rangle_{\text{HIGGS VEV}} = \frac{m_e}{y_e} \neq 0$.

Not examinable

ASIDE (Not Examinable!)

Strictly one must use something like:

$$V = a\left[\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}^\dagger\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\right]^2 - b\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}^\dagger\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$$

to maintain $SU(2)_L$ Gauge Invariance

Not examinable

Higgs theory summary for fermions:

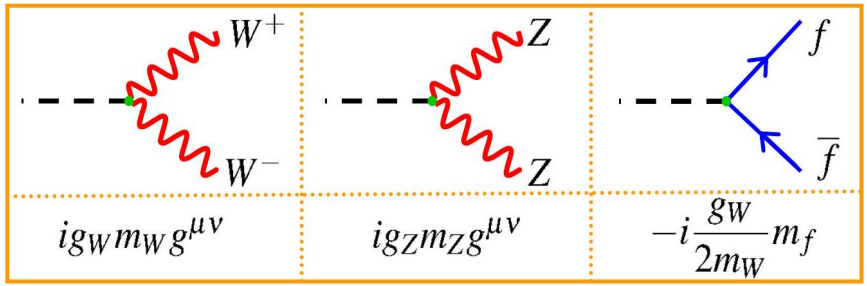
- Fermions are intrinsically massless, and need to be so to be Gauge Invariant.
- Nevertheless, interactions with the Higgs field make fermions look like they have mass at "low temperature" (i.e. when the Higgs field is near its ground state)
- Apparent fermion masses are controlled by free parameters called Yukawa Couplings (the strength of the coupling to the Higgs field).
- A Higgs Boson is an excitation of the Higgs Field.
- The Higgs Boson must be a scalar particle to make everything work.
- The Higgs Boson has a mass, but the mass is not predicted by the theory - we have to find it experimentally.
- The Higgs Boson has couplings to all the particles it gives mass to (and indeed to gauge bosons too!) and so has many ways it could decay, all fully calculable and determined by the theory as a function of its (as yet unknown) mass.
- For proper discussion of the Higgs mechanism see the Gauge Field Theory minor option.

Higgs mechanism for gauge bosons:

Not examinable

- The Higgs mechanism results in absolute predictions for masses of gauge bosons.
- In the SM, fermion masses are also ascribed to interactions with the Higgs field - however, here no prediction of the masses - just put in by hand.

- The Higgs is electrically neutral but carries weak hypercharge of $\frac{1}{2}$.
- The photon does not couple to the Higgs field and remains massless.
- The W and Z bosons couple to weak hypercharge and become massive.



Feynman Vertex factors

Within the SM, electroweak unification and the Higgs mechanism implies the following relations between SM parameters:

$$m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W} \quad \text{and} \quad m_Z = \frac{m_W}{\cos\theta_W}.$$

Hence, if you know any three of : $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ predict the other two.

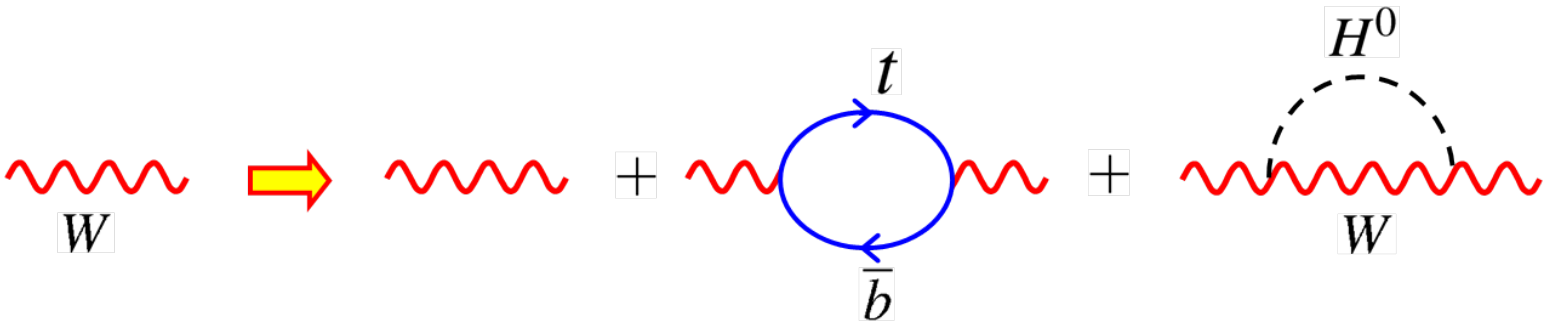
Not examinable

So called 'Precision Electroweak' tests at LEP

- At leading order, our theory predicts $m_W = m_Z \cos\theta_W$.
- Since we measure:

$$\left\{\begin{array}{ll} m_Z & = 91.1875 \pm 0.0021 \text{ GeV} \\ \sin^2\theta_W & = 0.23154 \pm 0.00016 \end{array}\right\}$$

  we expect: $m_W = 79.946 \pm 0.008$ GeV but measure $m_W = 80.376 \pm 0.033$ GeV.
- Close, but not quite right! We have only considered lowest order diagrams!
- Higher order calculation shows that preduction for W boson mass also includes terms from virtual loops:



$$m_W = m_W^0 + a \cdot m_t^2 + b \cdot \ln\left(\frac{m_H}{m_W}\right)$$

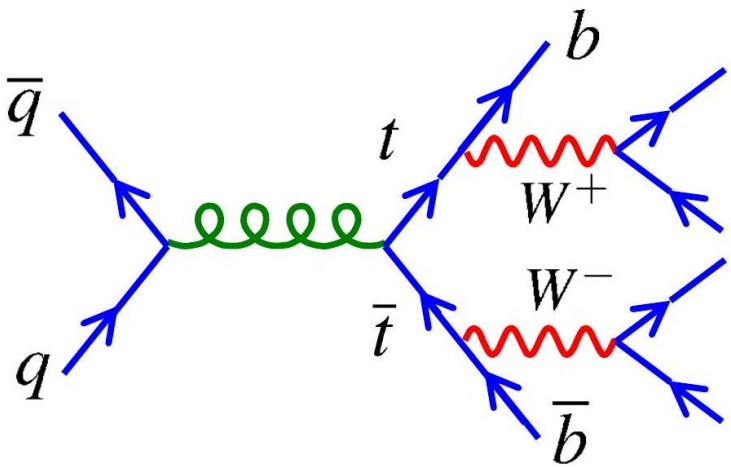
- So very high precision measurements provide sensitivity to the masses of particles inside the virtual loops!

The Top Quark

- Virtual loop corrections and precise LEP data predicted the top quark mass would be:

$$m_t^{\text{loop}} = 173 \pm 11 \text{ GeV}.$$

- This was subsequently confirmed by the discovery in 1994 of the top quark at Fermilab's Tevatron (proton anti-proton collider) at the predicted mass!



- The top quark almost exclusively decays to a bottom quark since

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2 .$$

leading to complicated final states like:

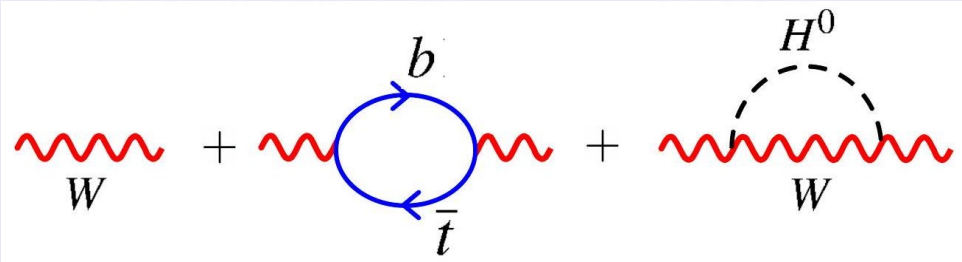
$$t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to \text{6jets}$$
$$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to \text{4jets} + \ell + \nu$$
$$t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to \text{2jets} + 2\ell + 2\nu.$$

- The top-quark mass is usually determined by direct reconstruction (methods similar to those used for W-boson mass, page 450). Including later LHC results, the 2024 world estimate is:

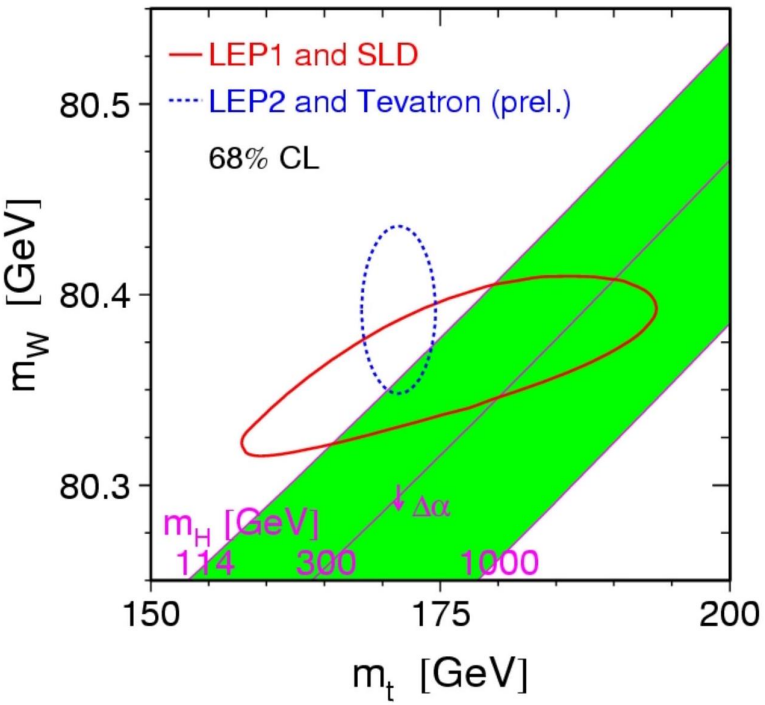
$$m_t^{\text{meas}} = 172.76 \pm 0.3 \text{ GeV}.$$

Logarithmic corrections to W-mass placed LEP limits on Higgs Mass

The W mass depends on the Higgs mass (albeit only logarithmically):

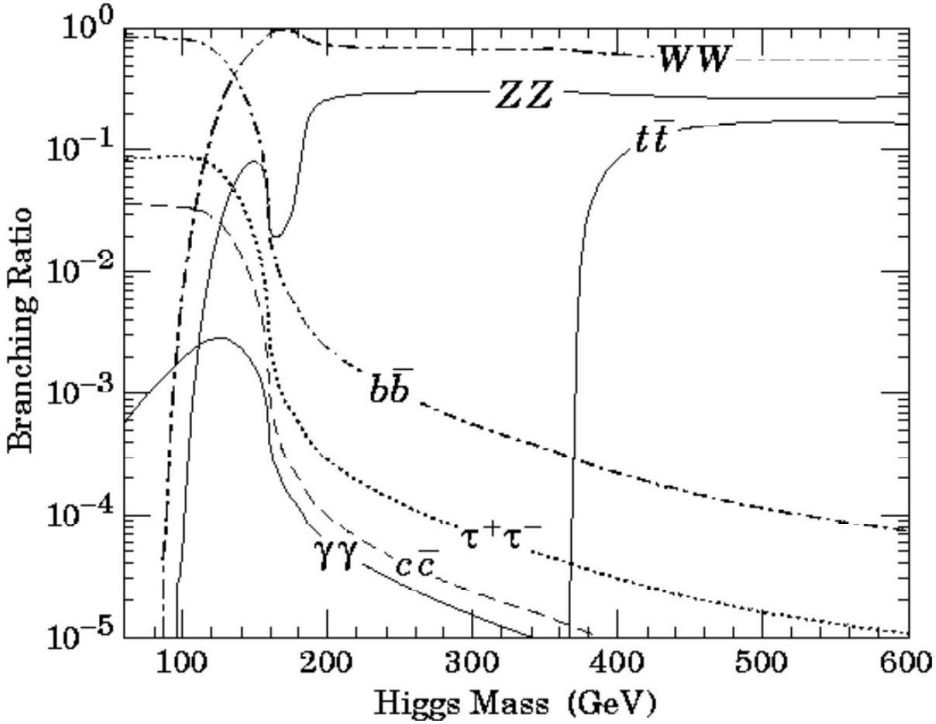


$$m_W = m_W^0 + a \cdot m_t^2 + b \cdot \ln\left(\frac{m_H}{m_W}\right)$$

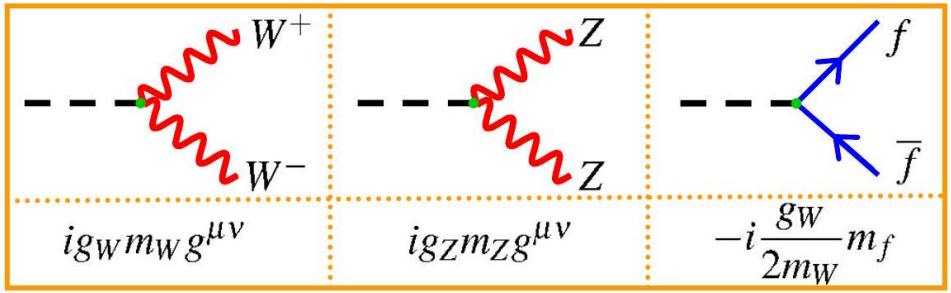


- LEP measurements were sufficiently precise to have some sensitivity to the Higgs mass.
- Direct and indirect values of the top-quark and W mass could be compared to prediction for different Higgs masses.
  - Direct: W and top masses from direct reconstruction
  - Indirect: from SM interpretation of Z mass, θ_W etc.
- Data favoured a light Higgs: $\boxed{m_H < 200\,GeV}$.

Possible Higgs Boson Decays at LEP



- The LEP searches needed to pay attention to how the Higgs decays.



- Since Higgs boson couplings are proportional to mass, most Higgs decays are to the heaviest kinematically allowed particle:

Summary of typical Higgs decay modes at LEP

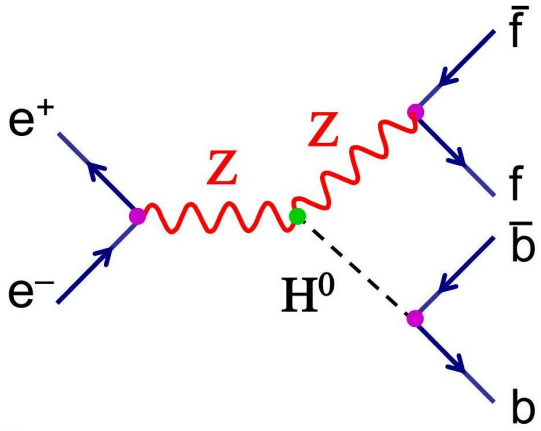
$m_H < 2m_W$: mainly $H^0 \to b\bar{b}$ but also around 10% of $H^0 \to \tau^+\tau^-$.

$2m_W < m_H < 2m_t$: almost entirely $H^0 \to W^+W^-$ and $H^0 \to ZZ$.

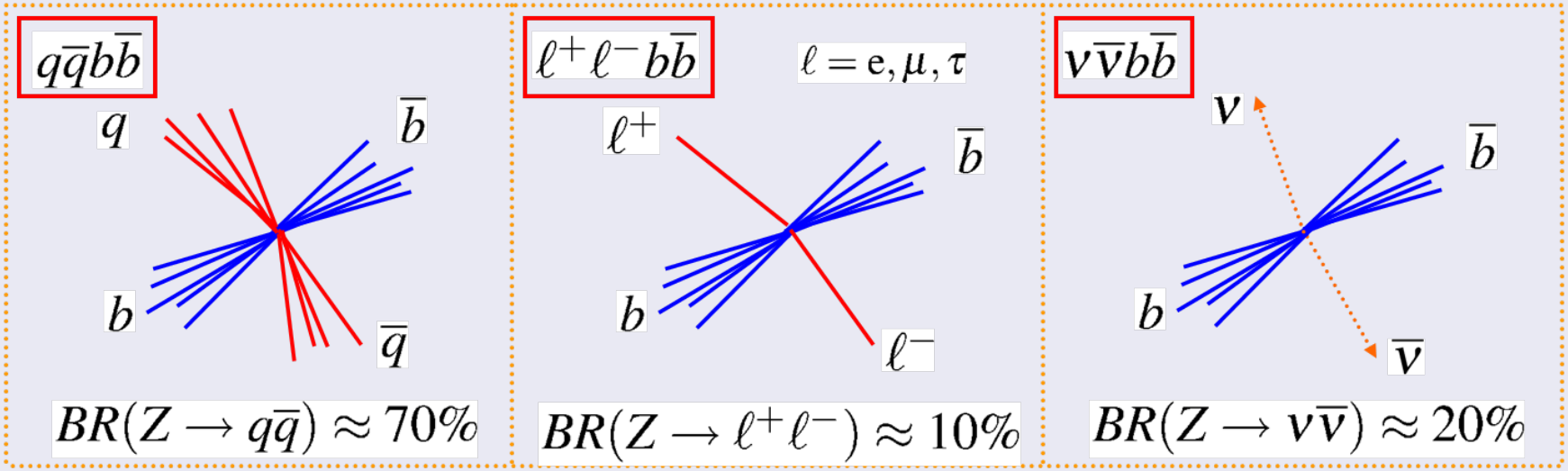
$m_H > 2m_t$: either $H^0 \to W^+W^-$, $H^0 \to ZZ$, or $H^0 \to t\bar{t}$.

Could LEP observe Higgs Bosons ?

- LEP operated with a C.o.M. energies up to 207 GeV.
- At these energies the main production mechanism would be the 'Higgsstrahlung' process (shown right).
- The Z-boson in the final state means that process is viable only for Higgs bosons with masses below ~ 207 GeV $- m_Z \sim 116$ GeV.

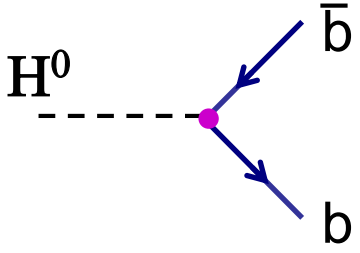


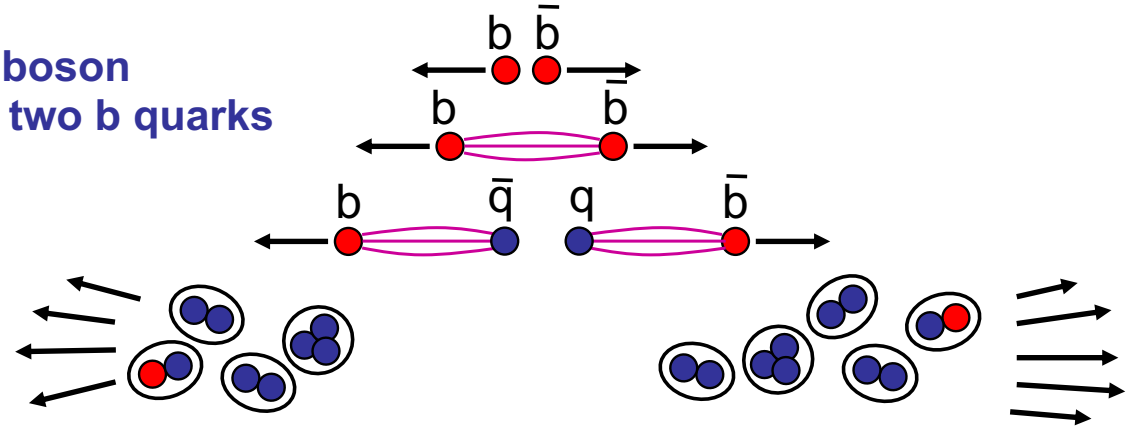
The Higgs predominantly decays to the heaviest particle possible. For Higgs boson masses below 116 GeV the main Higgs decay is therefore to a pair of b-quarks, so LEP Higgs signatures would be:



Tagging b-jets in the Higgs Boson Decays

★ One signature for a Higgs boson decay is the production of two b quarks



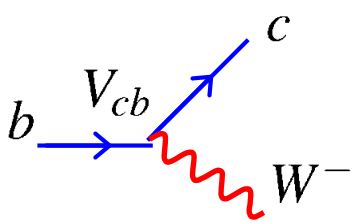


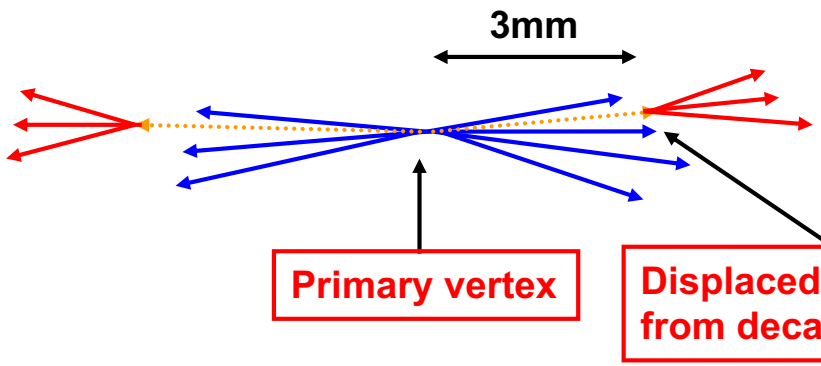
★ Each jet will contain one b-hadron which will decay weakly

★ Because V_{cb} is small $(V_{cb} \approx 0.04)$ hadrons containing b-quarks are relatively long-lived

★ Typical lifetimes of $\tau \sim 1 \times 10^{-12}\,\mathrm{s}$

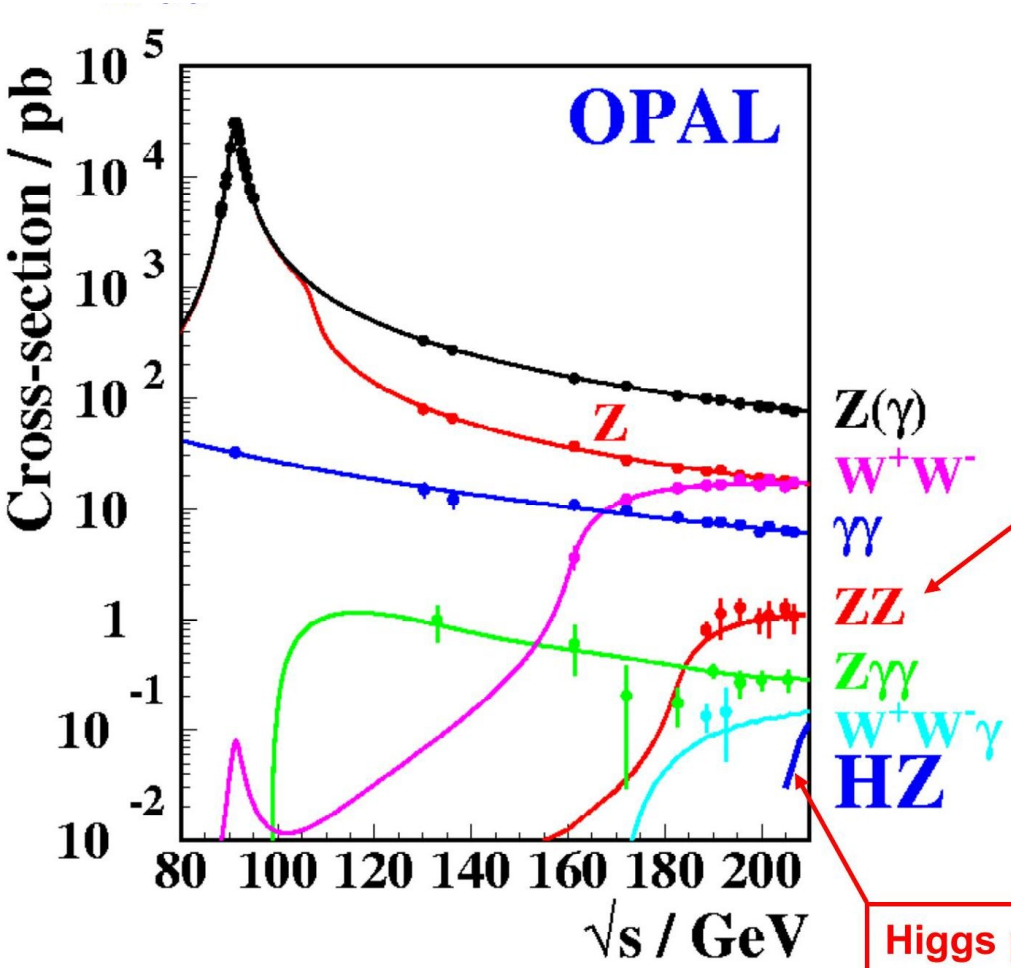
★ At LEP b-hadrons travel approximately **3mm** before decaying



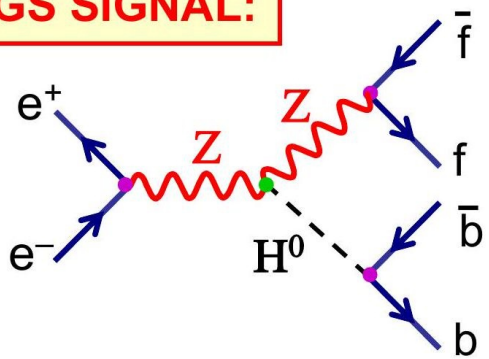


★ Can efficiently identify jets containing b quarks

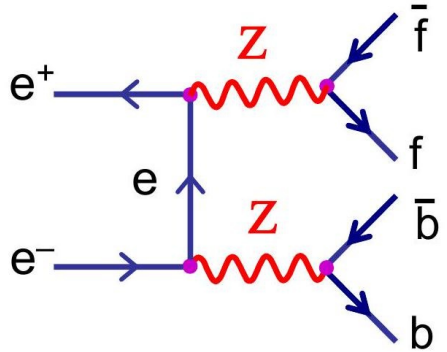
- Clear experimental signature, but small cross section, e.g. for $m_H \approx 115$ GeV would only produce a few tens of $e^+e^- \to H^0$ events at LEP.
- In addition, there are large backgrounds.



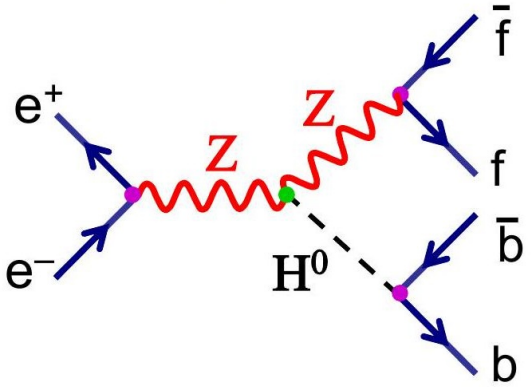
HIGGS SIGNAL:

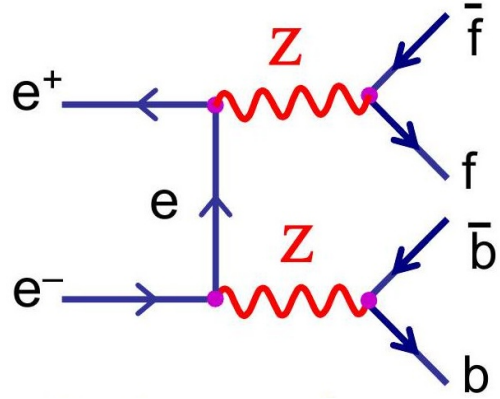


MAIN BACKGROUND:



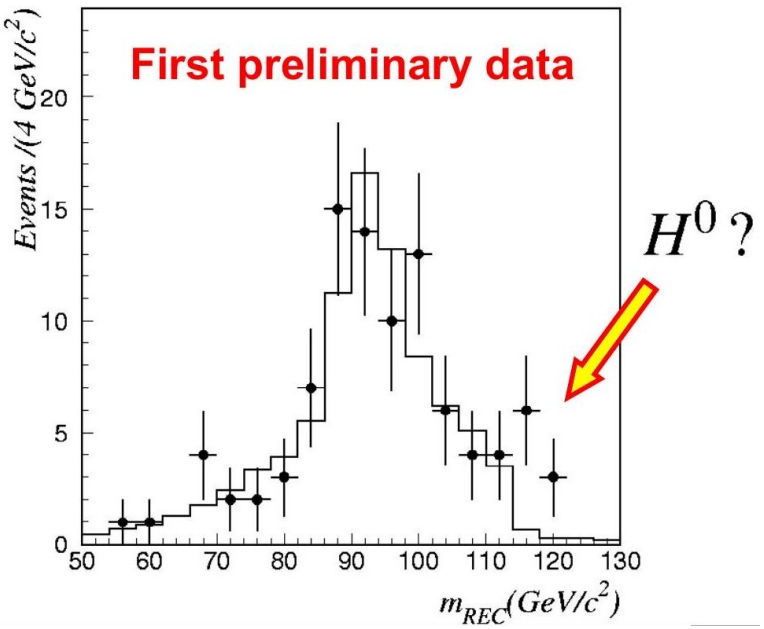
Higgs production cross section (m_H=115 GeV)

- The only way to distinguish 

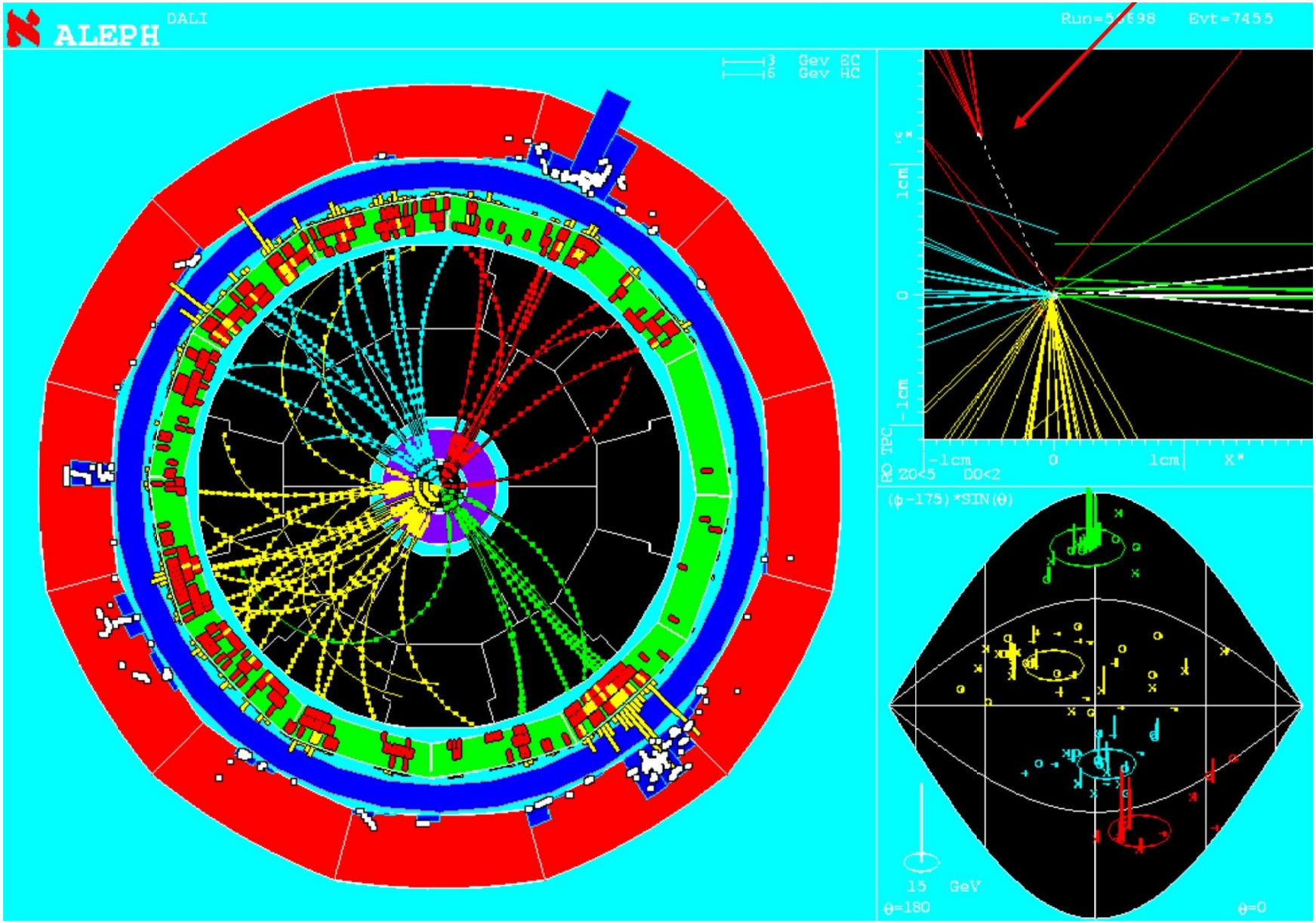
from 

is via the invariant mass of the pair of jets from the boson decays.

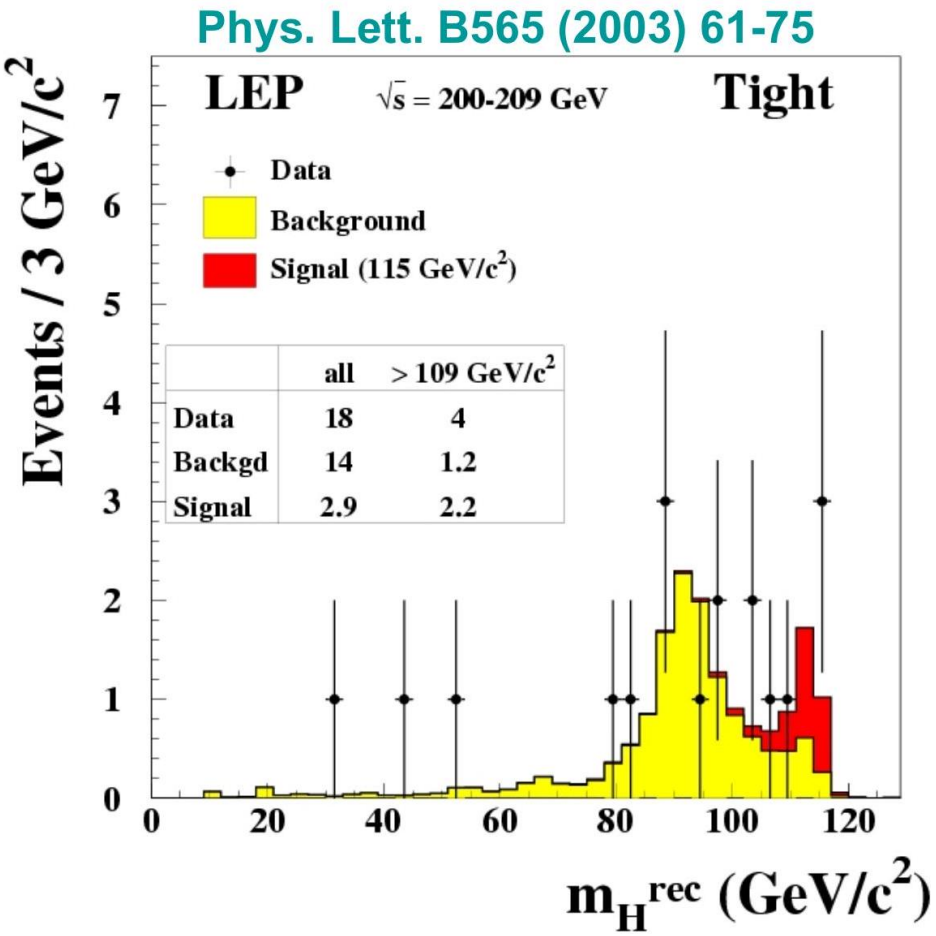
- In 2000 (the last year of LEP running) the ALEPH experiment reported an excess of events consistent with being a Higgs boson with mass 115 GeV.
- ALEPH found 3 events which were high relative probability of being signal.
- L3 found 1 event with high relative probability of being signal.
- OPAL and DELPHI found none.



Example event: Displaced vertex from b-decay



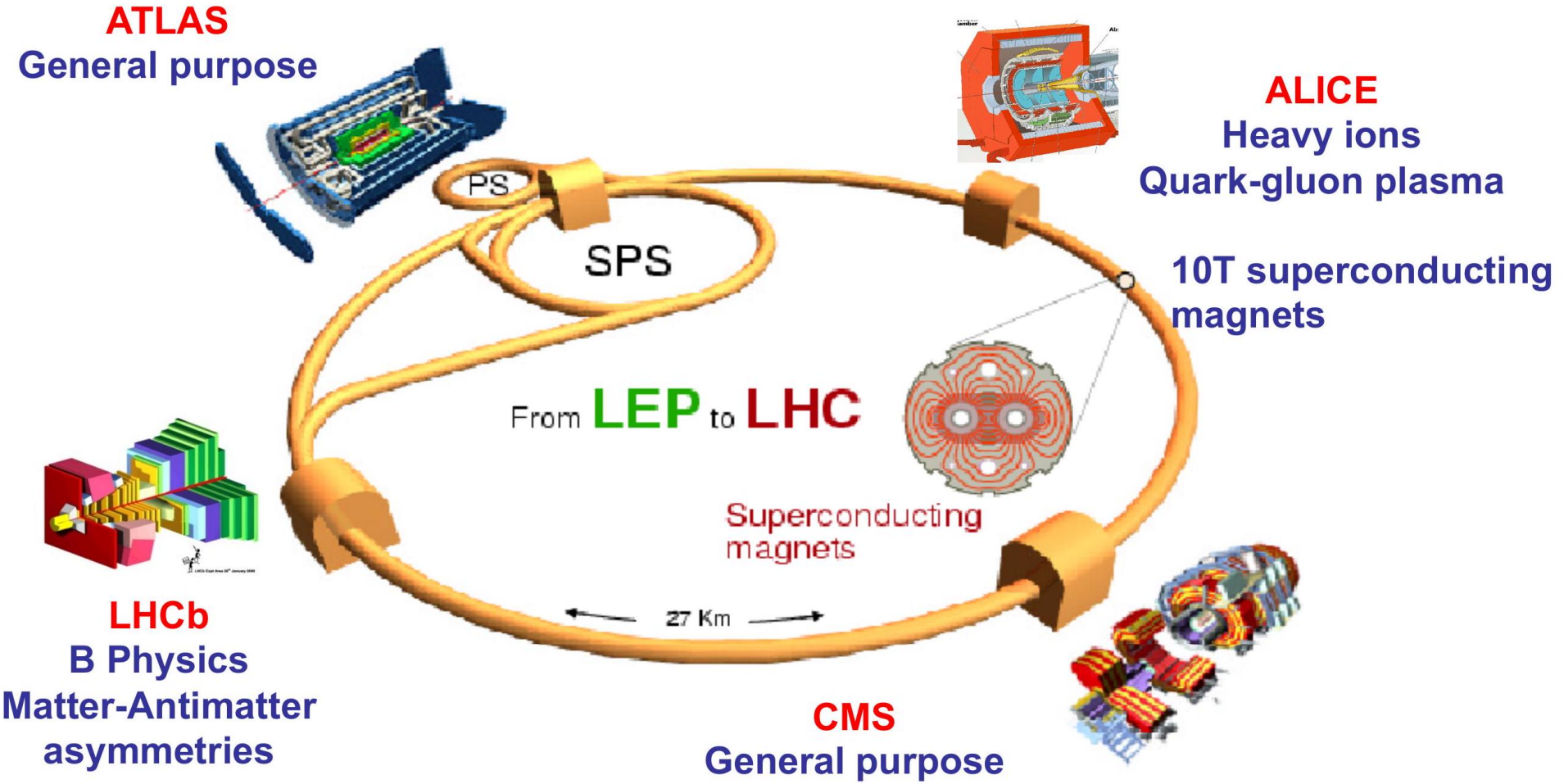
Combined LEP Results



- Final combined LEP results showed $m_H > 114$ GeV.

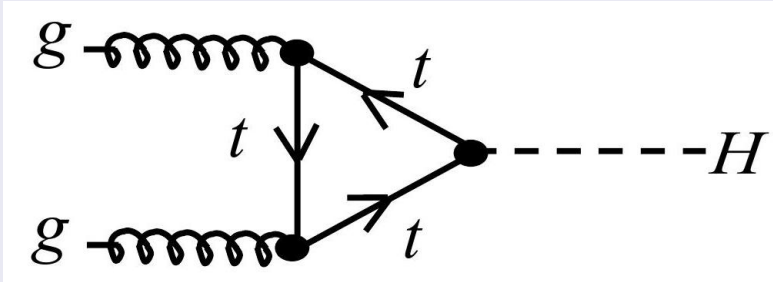
The Large Hadron Collider

The LHC is a proton-proton collider now running in the old LEP tunnel at CERN.

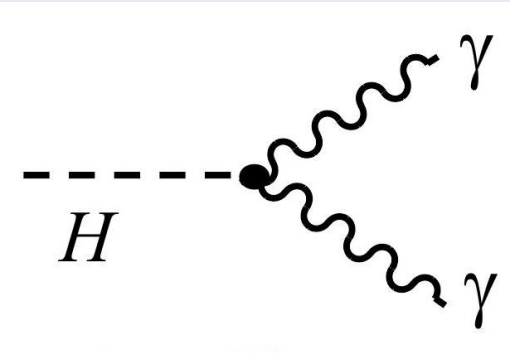


Higgs Production and Decay at the Large Hadron Collider

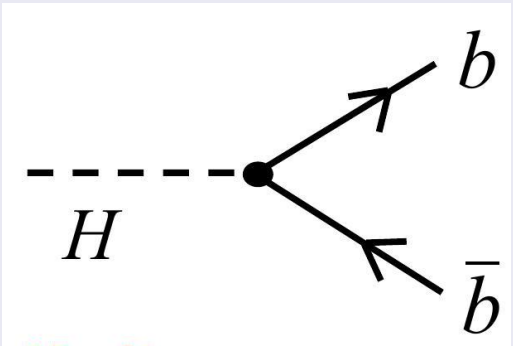
The dominant Higgs production mechanism at the LHC is 'gluon fusion':



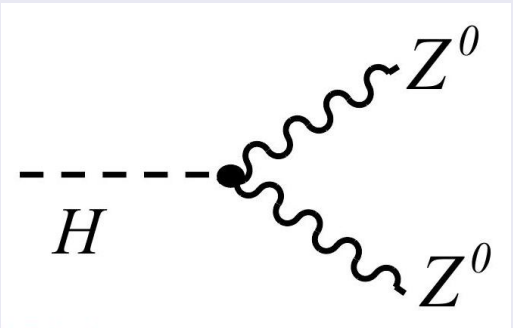
Depending on the mass of the Higgs boson, m_H, it could decay in different ways:



Low Mass



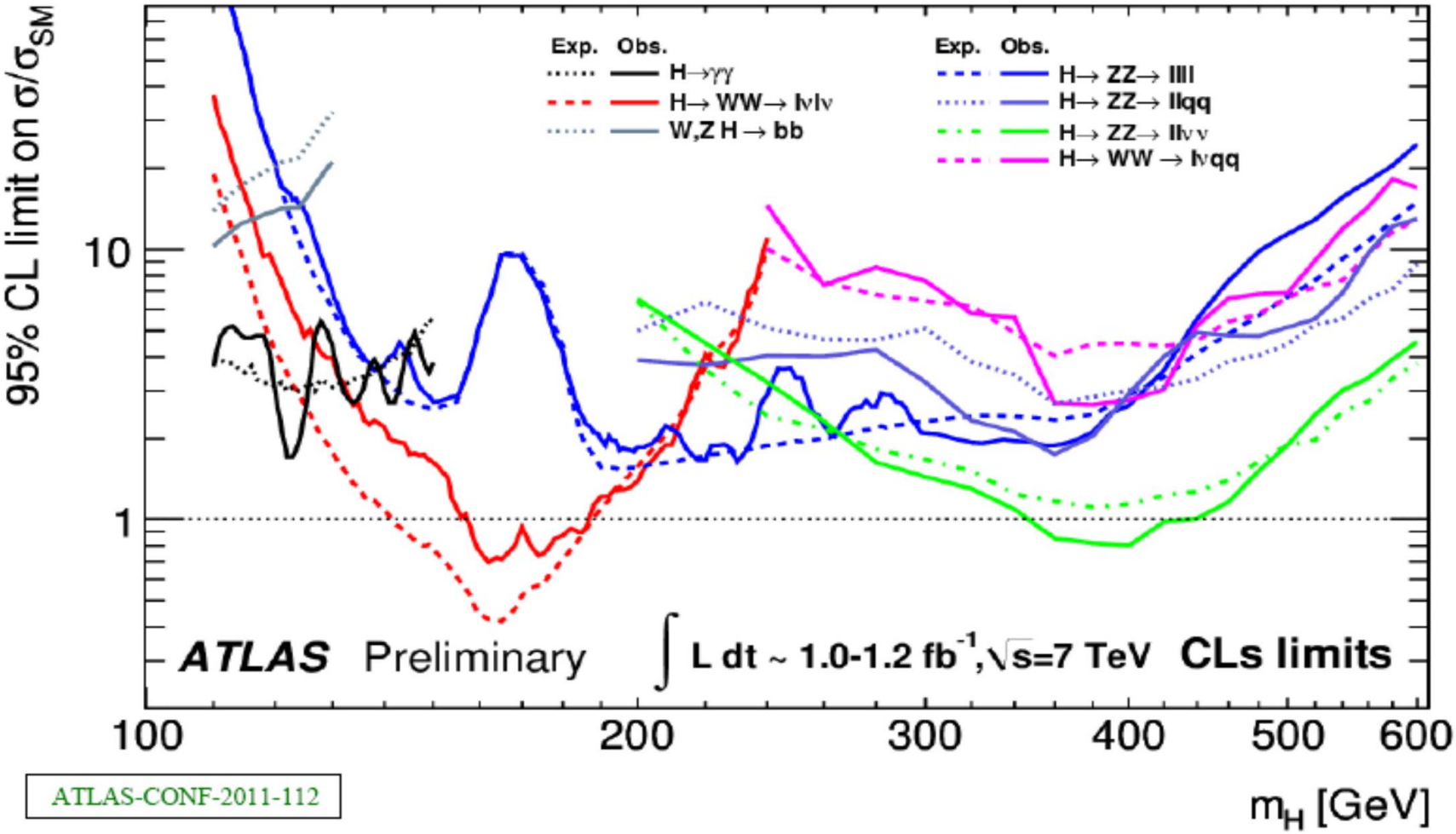
Medium mass



High mass

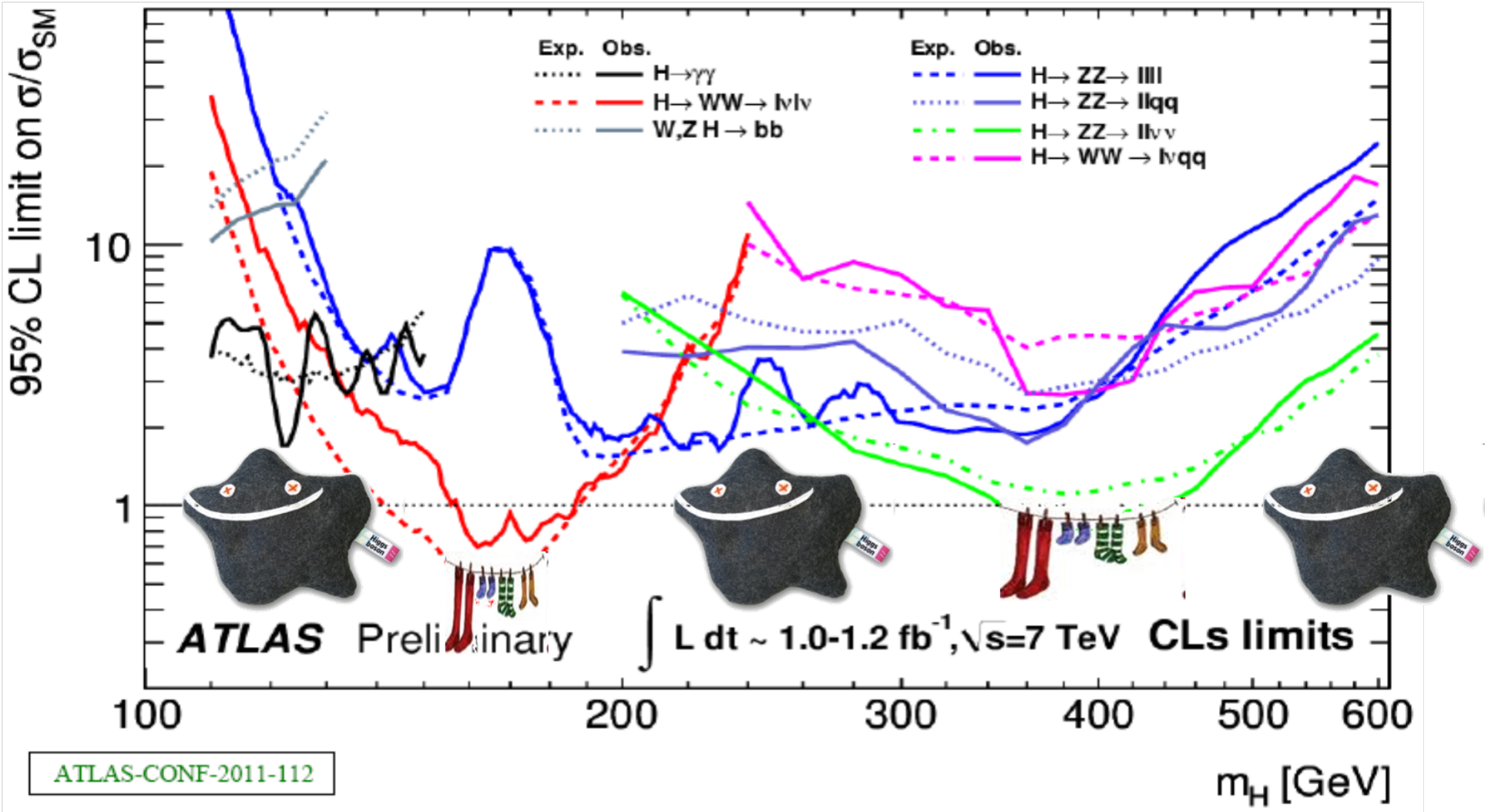
Higgs Boson mass (and number of Higgs Bosons!) was not known before LHC start.

So LHC experiments needed to design **separate search strategies** for each m_H.



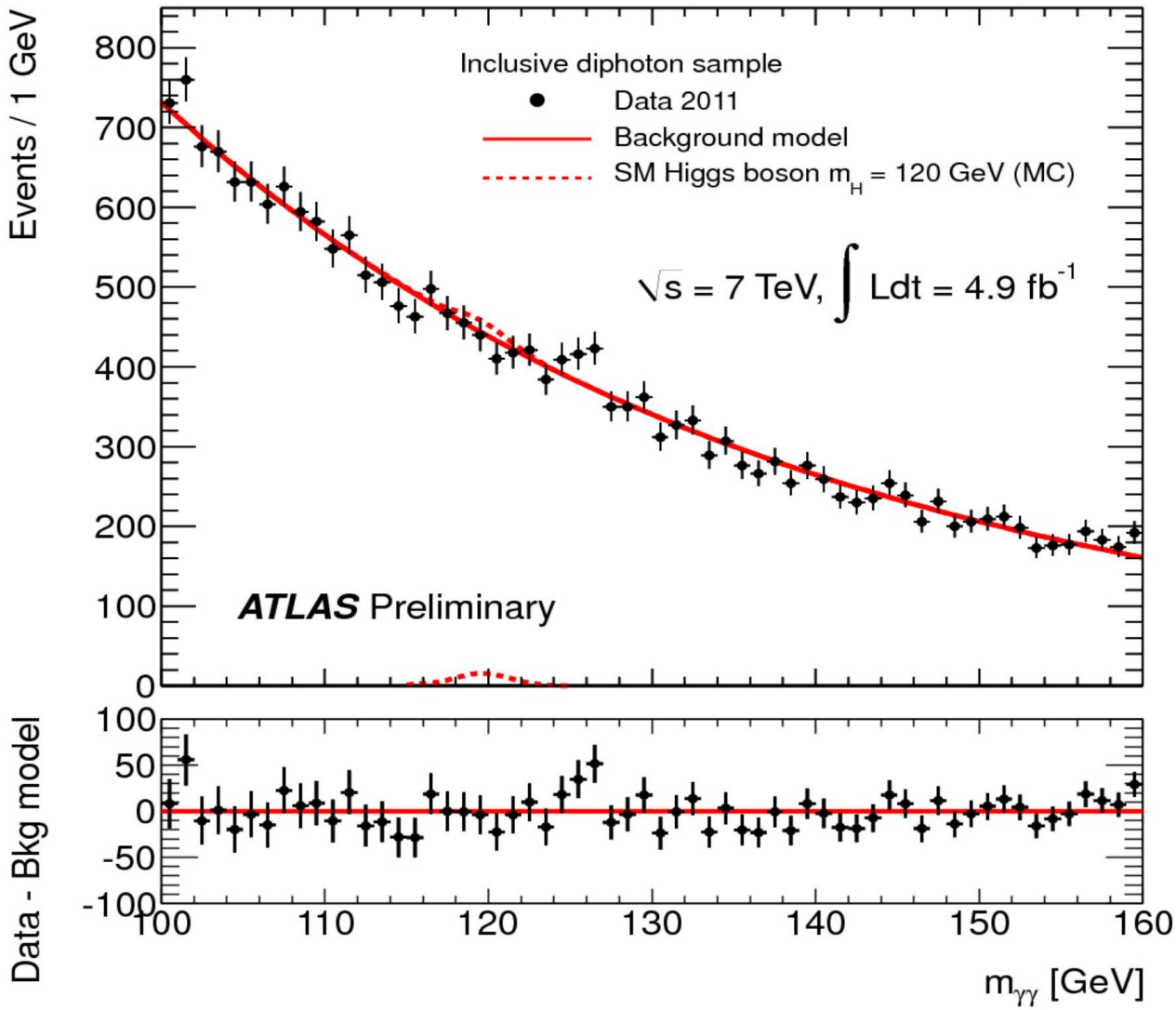
At each possible Higgs mass, m_H, each individual LHC search places a 95% confidence upper bound on the cross section for Higgs-Boson-Like events divided by the expected Higgs cross section at that mass. I.e. a curve passing through '10' on the y-axis at $m_H = 125\ \mathrm{GeV}$ means that a Higgs boson at this mass could have been produced at up to 10 times the rate expected in the Standard Model and yet could still (just) have gone un-noticed, at 95% confidence, by that search.

As data arrives it should lower the curves, unless support from a Higgs boson can prevent curve from passing through dotted line at " 1 "



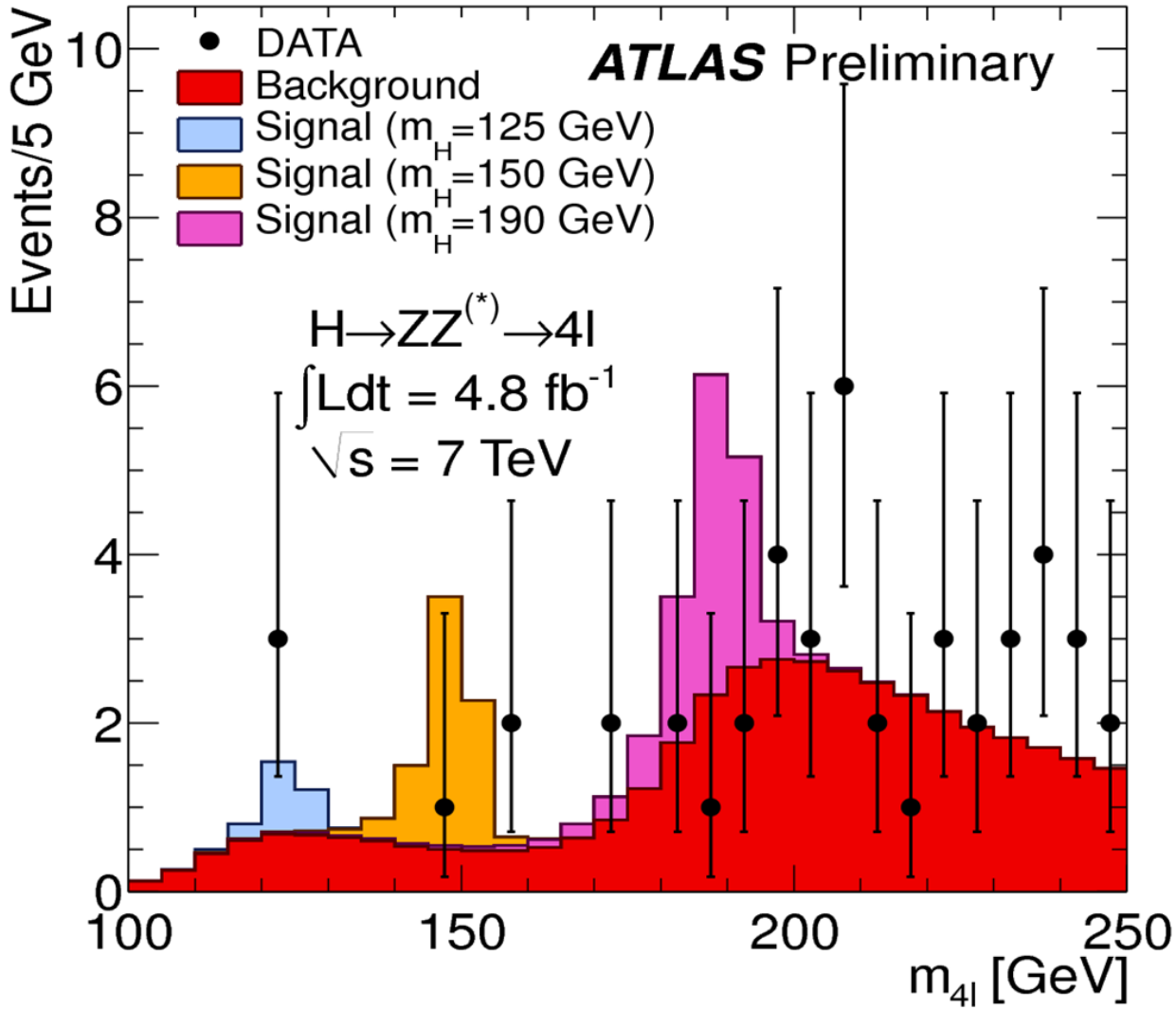
Here is the (unconvincing) data that was made public in February 2012

- The black blobs are data. The smooth curve is the expected background shape.
- The small dotted "bump" indicate how a Higgs signal might change the shape of the distribution if the Higgs boson mass was 120 GeV.
- The variable on the x axis is the invariant mass two photons.

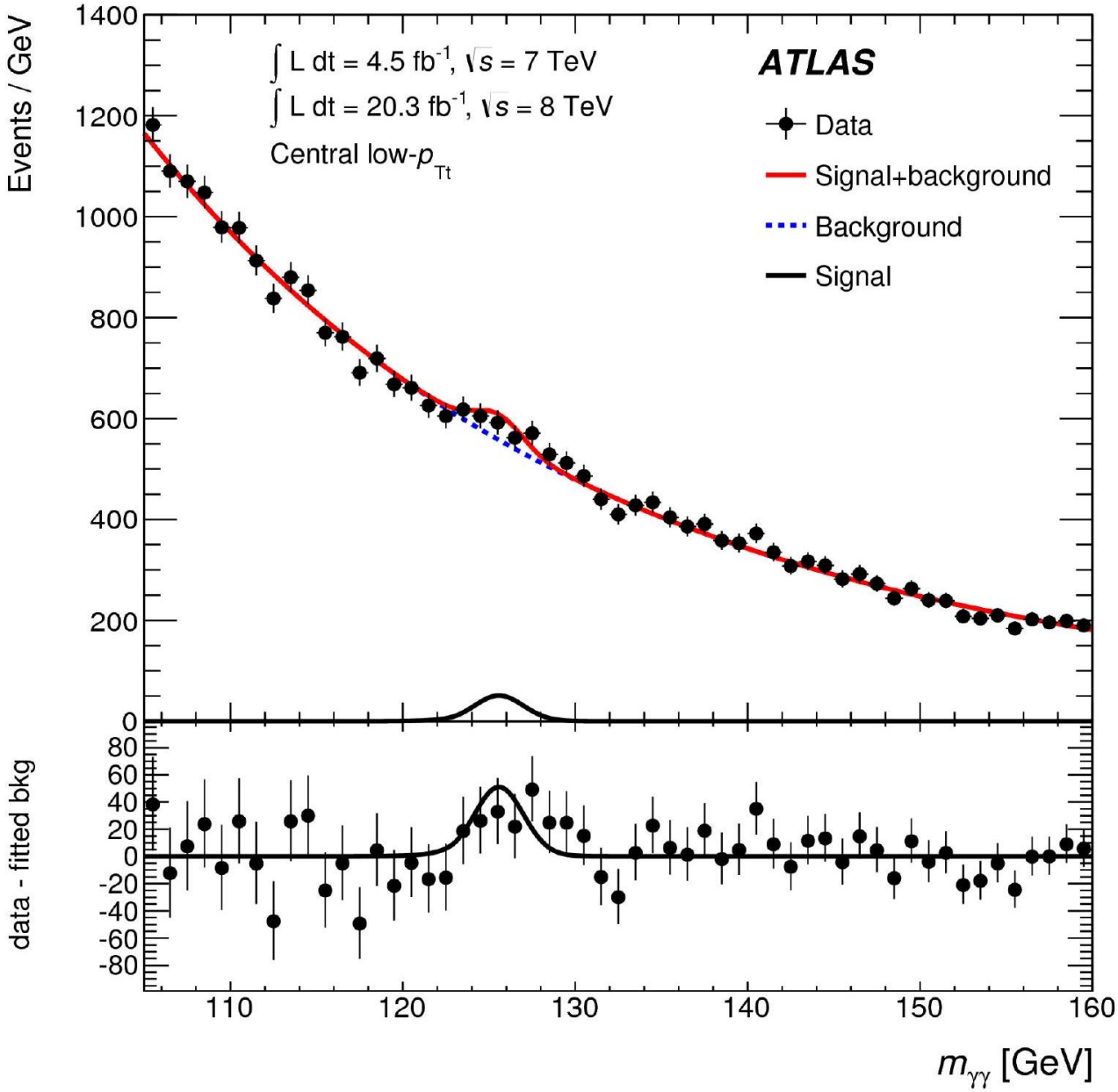


The astonishingly (un?)convincing evidence in the analysis looking for Higgs decays to pairs of Z bosons

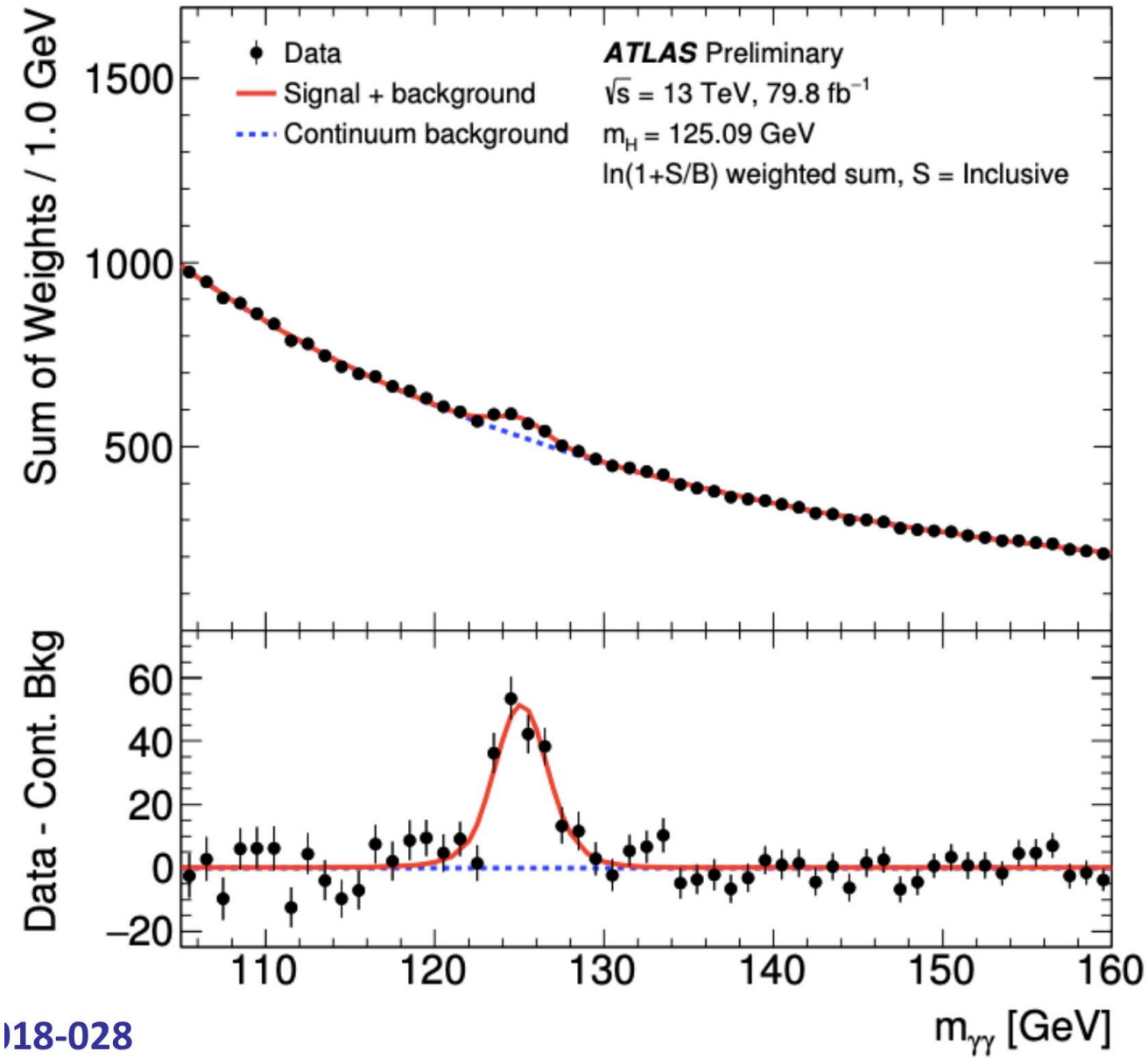
- The black blobs are data.
- The three triangular lumps indicate what a Higgs signal might look at (for three different Higgs boson masses).
- The variable on the x axis is the invariant mass of four leptons which seem to have come from two Z bosons.



The 2015 public ATLAS data for Higgs turning into two photons

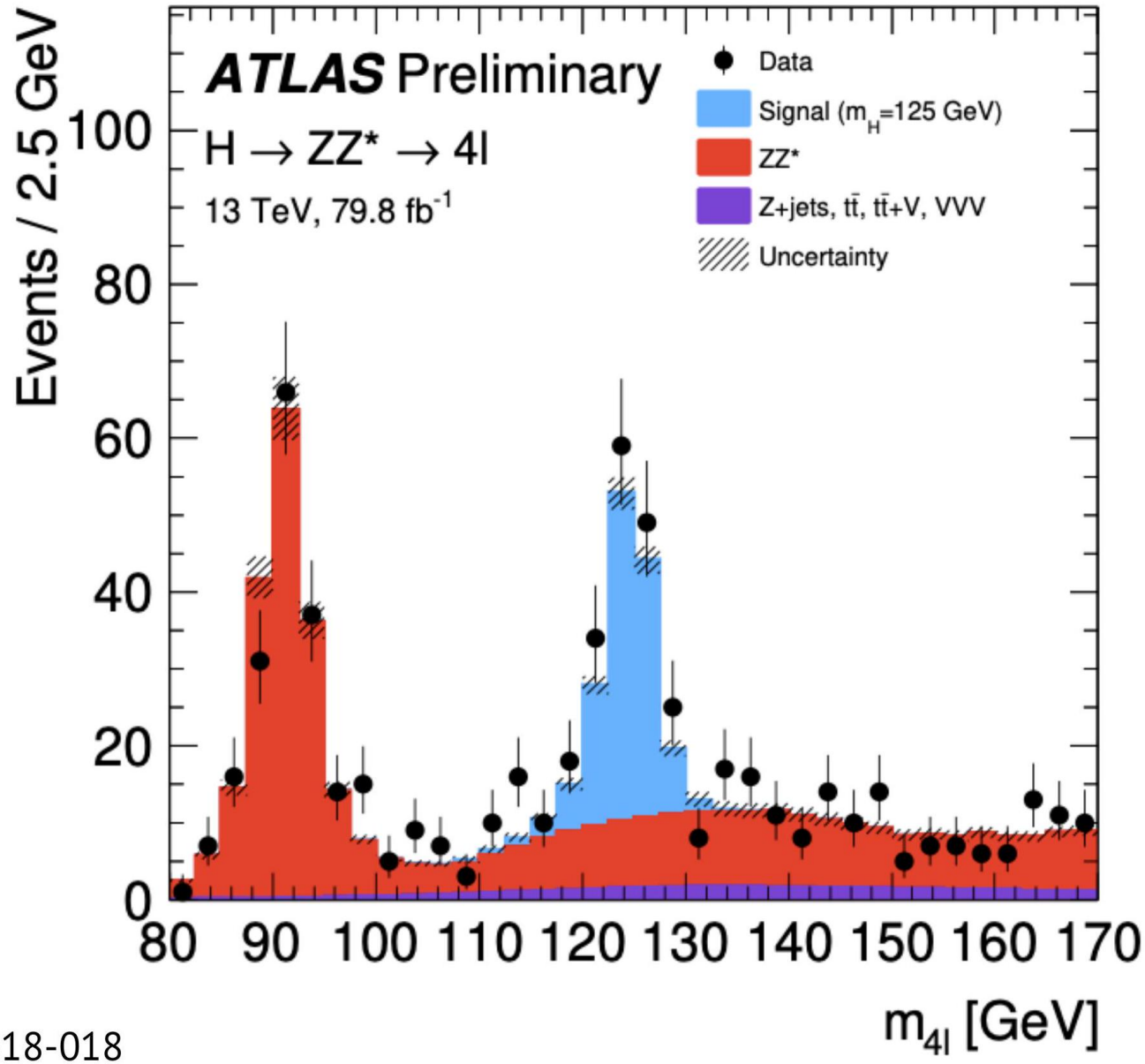


The 2018 public ATLAS data for Higgs turning into two photons



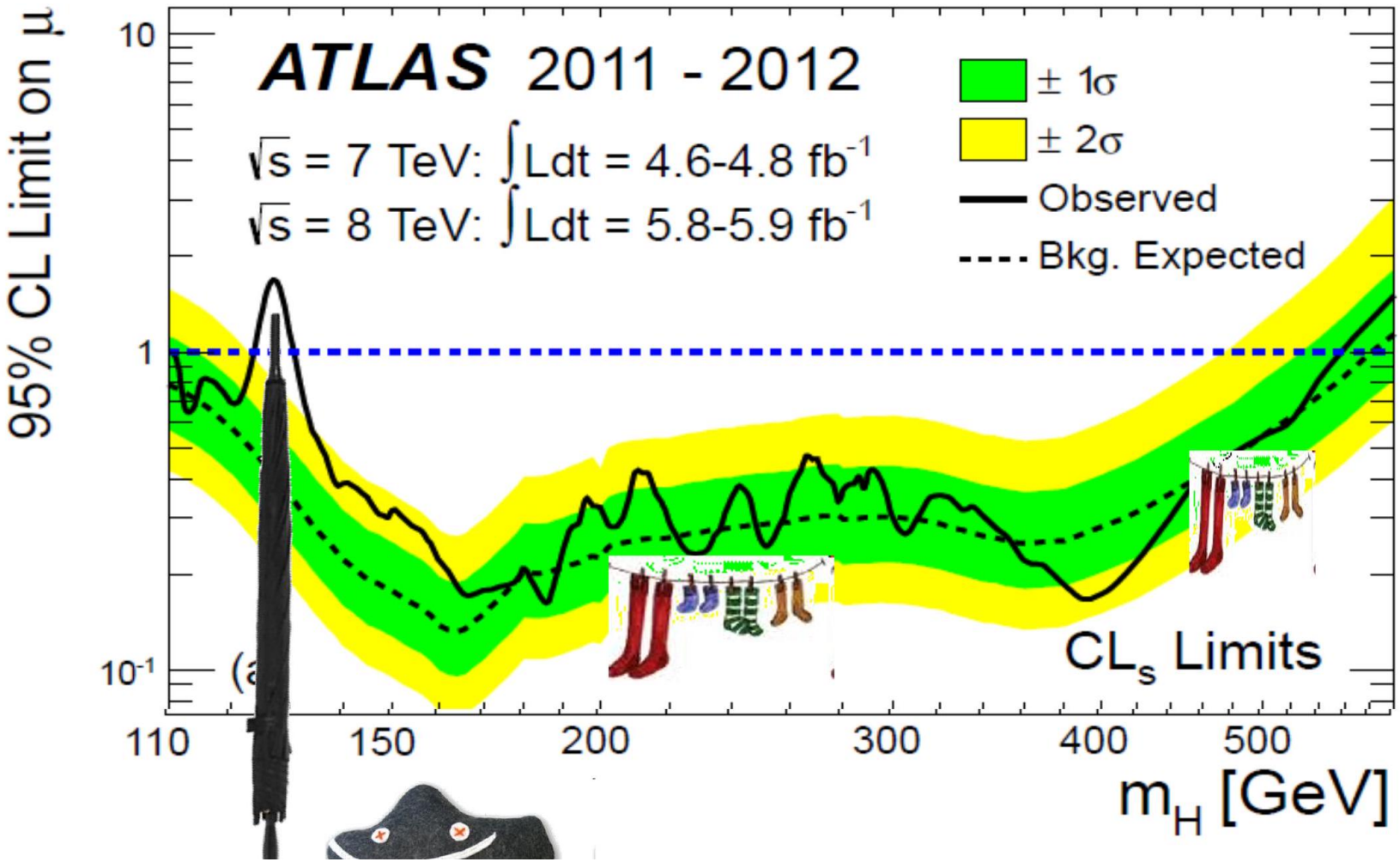
)18-028

... or turning into four leptons



)18-018

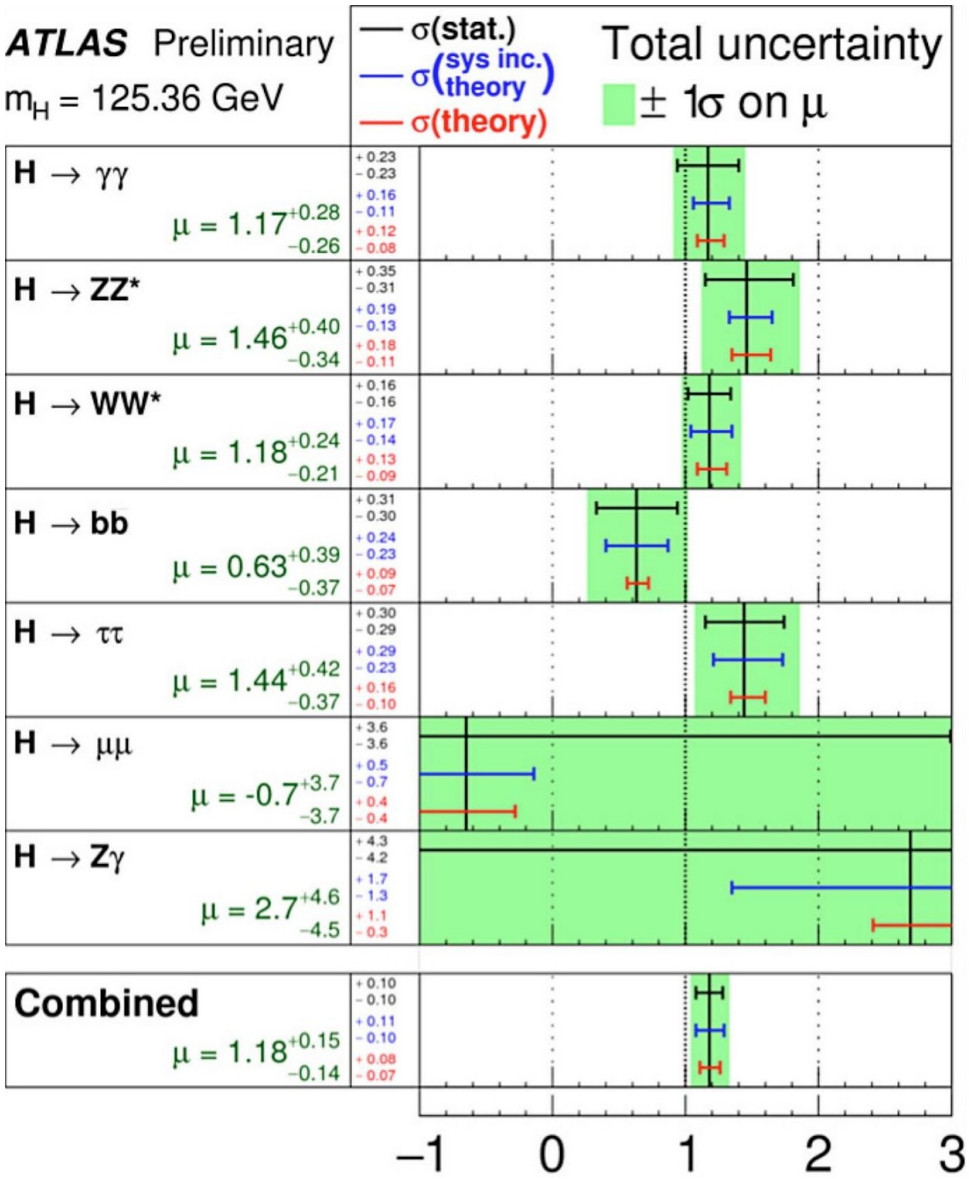
The discovery plot. Supposedly 2×10^{-9} probability of fluctuation!



Spring 2012 data ... this is the data that took us past the 5-sigma 'discovery' threshold.

But is it 'really' the (a?) Higgs boson?

- It is considered to be 'discovered'. Nobel Prize 2013.
- What has been discovered is a bump in the sort of place you'd expect to find a Higgs Boson. In other words, a particle consistent with the Higgs Boson.
- To be really sure its **The** Higgs Boson, we are acquiring more information on its spin and couplings (e.g. data shown to the right). So far everything checks out. The Higgs looks 'as expected'. Nonetheless, other (non-standard) Higgs Bosons could yet be found.



Concluding Remarks (1 of 2) — concerns/questions

The Standard Model with massless neutrinos is controlled by 19 free parameters, e.g.:

- three lepton masses, $(m_e,\ m_\mu,\ m_\tau)$
- six quark masses, $(m_d,\ m_u,\ m_s,\ m_c,\ m_b,\ m_t)$
- four CKM mixing matrix parameters (θ_{12}, θ_{13}, θ_{23} and δ_{CP}),
- three gauge couplings (g', g_w, g_s),
- two boson-sector parameters (e.g. Higgs mass plus one of Higgs vev, m_W or m_Z), and
- the QCD theta parameter (not discussed in this course).

Massive neturinos would add at least another seven (three masses and four mixing angles).

Some open questions:

- Why three generations ?
- Why $SU(3)_c \times SU(2)_L \times U(1)_Y$?
- No quantum gravity?
- Origin of CP violation in early universe?
- What is Dark Matter made of?
- Why are neutrinos so light?

Concluding Remarks (2 of 2) — the future

The main aims of this course have been:

- to show the main experimental methods and results which have lead to **The Standard Model** being considered (at present) to be our best description of fundamental physics,
- to emphasise that **the evidence is sometimes messy or indirect**, and
- to make the point that there is considerable scope for people of the future (you) to make discoveries!

All you have to do is find something better than The Standard Model which is able to retrodict results of all past experiments.

- Be sceptical of The Standard Model, but mindful of the evidence!
- Look for the gaps in that evidence!
- Don't waste your time on untestable meta theories.

The End

The End